Imaginary Concepts – Complex Numbers*

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Abstract

This module introduces the concept of complex numbers in Algebra.

A "complex number" is the sum of two parts: a real number by itself, and a real number multiplied by i. It can therefore be written as $a + \mathrm{bi}$, where a and b are real numbers.

The first part, a, is referred to as the **real part**. The second part, bi, is referred to as the **imaginary part**.

Examples of complex numbers $a + \mathrm{bi}$ (a is the "real part"; bi is the "imaginary part")	
$3 + 2\mathrm{i}$	$a = 3$, $b = 2$
π	$a = \pi$, $b = 0$(no imaginary part: a "pure real number")
$-i$	$a = 0$, $b = -1$ (no real part: a "pure imaginary number")

Table 1

Some numbers are not obviously in the form $a + \mathrm{bi}$. However, any number can be **put** in this form.

Example 1: Putting a fraction into $a + \mathrm{bi}$ form (i in the numerator)
$\frac{3-4i}{5}$ is a valid complex number. But it is not in the form $a + \mathrm{bi}$, and we cannot immediately see what the real and imaginary parts are.
To see the parts, we rewrite it like this:
$\frac{3-4i}{5} = \frac{3}{5} - \frac{4}{5}i$
Why does that work? It's just the ordinary rules of fractions, applied backward. (Try multiplying and then subtracting on the right to confirm this.) But now we have a form we can use:
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$\frac{3-4i}{5} a = \frac{3}{5}$, $b = - - \frac{4}{5}$
So we see that fractions are very easy to break up, if the i is in the numerator. An i in the denominator is a bit trickier to deal with.

Table 2

Example 2: Putting a fraction into $a + \mathrm{bi}$ form (i in the denominator)	
$\frac{1}{i} = \frac{1 \cdot i}{i \cdot i}$	**Multiplying the top and bottom of a fraction by the same number never changes the value of the fraction: it just rewrites it in a different form.**
$= \frac{i}{-1}$	**Because $i \bullet i$ is i^2 , or –1.**
$= -\mathrm{i}$	**This is not a property of i, but of –1. Similarly,** $\frac{5}{-1} = - - 5$.
$\frac{1}{i}$: $a = 0$, $b = -1$	**since we rewrote it as**$-\mathrm{i}$, **or**$0 - 1\mathrm{i}$

Table 3

Finally, what if the denominator is a more complicated complex number? The trick in this case is similar to the trick we used for rationalizing the denominator: we multiply by a quantity known as the **complex conjugate of the denominator**.

Definition of Complex Conjugate

The complex conjugate of the number $a + \mathrm{bi}$ is $a - \mathrm{bi}$. In words, you leave the real part alone, and change the sign of the imaginary part.

Here is how we can use the "complex conjugate" to simplify a fraction.

Example: Using the Complex Conjugate to put a fraction into $a + \mathrm{bi}$ form	
$\frac{5}{3-4i}$	**The fraction: a complex number not currently in the form $a + bi$**
$= \frac{5(3+4i)}{(3-4i)(3+4i)}$	**Multiply the top and bottom by the complex conjugate of the denominator**
$= \frac{15+20i}{3^2-(4i)^2}$	**Remember,** $(x + a)(x - -a) = x^2 - - \mathrm{a}^2$
$= \frac{15+20i}{9+16}$	$(4\mathrm{i})^2 = 4^2 i^2 = 16(- - 1) = - - 16$, **which we are subtracting from 9**
$= \frac{15+20i}{25}$	**Success! The top has**i, **but the bottom doesn't. This is easy to deal with.**
$= \frac{15}{25} + \frac{20i}{25}$	**Break the fraction up, just as we did in a previous example.**
$= \frac{3}{5} + \frac{4}{5} i$	**So we're there!**$a = \frac{3}{5}$ and $b = \frac{4}{5}$

Table 4

Any number of any kind can be written as $a + \mathrm{bi}$. The above examples show how to rewrite fractions in this form. In the text, you go through a worksheet designed to rewrite $\sqrt[3]{-1}$ as three different complex numbers. Once you understand this exercise, you can rewrite other radicals, such as $\sqrt{i}$, in $a + \mathrm{bi}$ form.