Imaginary Concepts – Equality and Inequality in Complex Numbers*

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Abstract

This module introduces the concept of equality and inequality in complex numbers.

What does it mean for two complex numbers to be **equal?** As always, equality asserts that two things are exactly the same. $7 + 3\text{i}$ is **not** equal to 7, or to 3i, or to $7 - 3\text{i}$, or to $3 + 7\text{i}$. It is not equal to anything except $7 + 3\text{i}$.

Definition of Equality

Two complex numbers are **equal** to each other only if **their real parts are equal, and their imaginary parts are equal.**

So if we say that two complex numbers equal each other, we are actually making two separate, independent statements. We can use this, for instance, to solve for two separate variables.

Example: Complex Equality	
If $3\text{x} + 4\text{yi} + 7 = 4\text{x} + 8\text{i}$, what are x and y?	
Normally, it is impossible to solve one equation for two unknowns. But this is really two separate equations!	
Real part on the left = real part on the right:	$3\text{x} + 7 = 4\text{x}$
Imaginary part on the left = imaginary part on the right:	$4\text{y} = 8$
We can now solve both of these equations trivially.	$x = 7$, $y = 2$

Table 1

And what about **inequalities?** The answer may surprise you: **there are no inequalities with complex numbers,** at least not in the form we're seeing.

The real numbers have the property that for any two real numbers a and b, exactly one of the following three statements must be true: $a = b$, $a > b$, or $a < b$. This is one of those properties that seems almost too obvious to bother with. But it becomes more interesting when you realize that the complex numbers do **not** have that property. Consider two simple numbers, i and 1. Which of the following is true?

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- $i = 1$ ☒
- $i > 1$ ☒
- $i < 1$ ☒

None of them is true. It is not generally possible to describe two complex numbers as being "greater than" or "less than" each other.

Visually, this corresponds to the fact that all the real numbers can be laid out on a number line: "greater than" means "to the right of" and so on. The complex numbers cannot be laid out on a number line. They are sometimes pictured on a 2-dimensional graph, where the real part is the x coordinate and the imaginary part is the y coordinate. But one point on a graph is neither greater than, nor less than, another point!