

RATIONAL EXPRESSION CONCEPTS – RATIONAL EQUATIONS*

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Abstract

This module introduces rational expressions in equations.

1 Rational Equations

A **rational equation** means that you are setting two rational **expressions** equal to each other. The goal is to solve for x ; that is, find the x value(s) that make the equation true.

Suppose I told you that:

$$\frac{x}{8} = \frac{3}{8} \quad (1)$$

If you think about it, the x in this equation has to be a 3. That is to say, if $x=3$ then this equation is true; for any other x value, this equation is false.

This leads us to a very general rule.

A very general rule about rational equations

If you have a rational equation where the **denominators** are the same, then the **numerators** must be the same.

This in turn suggests a strategy: find a common denominator, and then set the numerators equal.

Example: Rational Equation	
$\frac{3}{x^2+12x+36} = \frac{4x}{x^3+4x^2-12x}$	Same problem we worked before, but now we are equating these two fractions, instead of subtracting them.
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$\frac{3(x)(x-2)^2}{(x+6)} (x)(x-2) = \frac{4x(x+6)^2}{x(x+6)} (x-2)$	Rewrite both fractions with the common denominator.
$3x(x-2) = 4x(x+6)$	Based on the rule above—since the denominators are equal, we can now assume the numerators are equal.
$3x^2 - 6x = 4x^2 + 24x$	Multiply it out
$x^2 + 30x = 0$	What we're dealing with, in this case, is a quadratic equation. As always, move everything to one side...
$x(x+30) = 0$...and then factor. A common mistake in this kind of problem is to divide both sides by x; this loses one of the two solutions.
$x=0$ or $x = -30$	Two solutions to the quadratic equation. However, in this case, $x = 0$ is not valid, since it was not in the domain of the original right-hand fraction. (Why?) So this problem actually has only one solution, $x = -30$.

Table 1

As always, it is vital to remember what we have found here. We started with the equation $\frac{3}{x^2+12x+36} = \frac{4x}{x^3+4x^2-12x}$. We have concluded now that if you plug $x = -30$ into that equation, you will get a true equation (you can verify this on your calculator). For any other value, this equation will evaluate false.

To put it another way: if you graphed the functions $\frac{3}{x^2+12x+36}$ and $\frac{4x}{x^3+4x^2-12x}$, the two graphs would intersect at one point only: the point when $x = -30$.