Rational Expression Concepts – Adding and Subtracting Rational Expressions*

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Abstract

This module covers the addition and subtraction of rational expressions.

Adding and subtracting fractions is harder—but once again, it is a familiar process.

$$\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6} \tag{1}$$

The key is finding the **least common denominator**: the smallest multiple of both denominators. Then you rewrite the two fractions with this denominator. Finally, you add the fractions by **adding the numerators and leaving the denominator alone**.

But how do you find the least common denominator? Consider this problem:

$$\frac{5}{12}+\frac{7}{30} \tag{2}$$

You could probably find the least common denominator if you played around with the numbers long enough. But what I want to show you is a **systematic method** for finding least common denominators—a method that works with rational expressions just as well as it does with numbers. We start, as usual, by factoring. For each of the denominators, we find all the **prime factors**, the prime numbers that multiply to give that number.

$$\frac{5}{2\cdot 2\cdot 3}+\frac{7}{2\cdot 3\cdot 5} \tag{3}$$

If you are not familiar with the concept of prime factors, it may take a few minutes to get used to. $2\times 2\times 3$ is 12, broken into its **prime factors**: that is, it is the list of prime numbers that multiply to give 12. Similarly, the prime factors of 30 are $2\times 3\times 5$.

Why does that help? Because $12=2\times 2\times 3$, any number whose prime factors include two 2s and one 3 will be a multiple of 12. Similarly, any number whose prime factors include a 2, a 3, and a 5 will be a multiple of 30.

*Version 1.2: Apr 5, 2010 5:11 pm -0500

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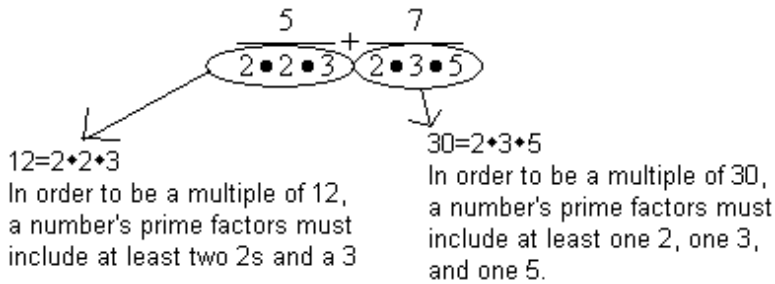

Figure 1

The least common denominator is the smallest number that meets both these criteria: it must have two 2s, one 3, and one 5. Hence, the least common denominator must be $2 \times 2 \times 3 \times 5$, and we can finish the problem like this.

$$\frac{5}{2 \cdot 2 \cdot 3} + \frac{7}{2 \cdot 3 \cdot 5} = \frac{5\underline{5}}{(2 \cdot 2 \cdot 3)\,\underline{5}} + \frac{7\underline{2}}{(2 \cdot 3 \cdot 5)\,\underline{2}} = \frac{25}{60} + \frac{14}{60} = \frac{39}{60} \tag{4}$$

This may look like a very strange way of solving problems that you've known how to solve since the third grade. However, I would urge you to spend a few minutes carefully following that solution, focusing on the question: **why is $2 \times 2 \times 3 \times 5$ guaranteed to be the least common denominator**? Because once you understand that, you have the key concept required to add and subtract rational expressions.

Example 1: Subtracting Rational Expressions

$\frac{3}{x^2+12x+36} - \frac{4x}{x^3+4x^2-12x}$	**The problem**
$= \frac{3}{(x+6)^2} - \frac{4x}{x(x+6)(x-2)}$	**Always begin rational expression problems by factoring! The least common denominator must have two $(x+6)$ s, one x , and one $(x-2)$.**
$= \frac{3(x)(x-2)}{(x+6)^2}(x)(x-2) - \frac{4x(x+6)}{x(x+6)^2}(x-2)$	**Rewrite both fractions with the common denominator.**
$= \frac{3(x)(x-2)-4x(x+6)}{x(x-2)(x+6)^2}$	**Subtracting fractions is easy when you have a common denominator! It's best to leave the bottom alone, since it is factored. The top, however, consists of two separate factored pieces, and will be simpler if we multiply them out so we can combine them.**
	continued on next page

$= \frac{3x^2-6x-\left(4x^2+24x\right)}{x(x-2)(x+6)}$	**A common student mistake here is forgetting the parentheses. The entire second term is subtracted; without the parentheses, the $24x$ ends up being added.**
$= \frac{-x^2-30x}{x(x-2)(x+6)}$	**Almost done! But finally, we note that we can factor the top again. If we factor out an x it will cancel with the x in the denominator.**
$= \frac{-x-30}{(x-2)(x+6)}$	**A lot simpler than where we started, isn't it?**

Table 1

The problem is long, and the math is complicated. So after following all the steps, it's worth stepping back to realize that even this problem results simply from the two rules we started with.

First, always factor rational expressions before doing anything else.

Second, follow the regular processes for fractions: in this case, the procedure for subtracting fractions, which involves finding a common denominator. After that, you subtract the numerators while leaving the denominator alone, and then simplify.