Basic Properties of Real Numbers: Symbols and Notations*

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Abstract

This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. The symbols, notations, and properties of numbers that form the basis of algebra, as well as exponents and the rules of exponents, are introduced in this chapter. Each property of real numbers and the rules of exponents are expressed both symbolically and literally. Literal explanations are included because symbolic explanations alone may be difficult for a student to interpret. Topics covered in this module: understand the difference between variables and constants, be familiar with the symbols of operation, equality, and inequality, be familiar with grouping symbols, be able to correctly use the order of operations.

1 Overview

- Variables and Constants
- Symbols of Operation, Equality, and Inequality
- Grouping Symbols
- The Order of Operations

2 Variables and Constants

A basic characteristic of algebra is the use of symbols (usually letters) to represent numbers.
Variable
A letter or symbol that represents any member of a collection of two or more numbers is called a **variable**.
Constant
A letter or symbol that represents a specific number, known or unknown is called a **constant**.

In the following examples, the letter x is a variable since it can be any member of the collection of numbers $\{35, 25, 10\}$. The letter h is a constant since it can assume only the value 5890.

Example 1
Suppose that the streets on your way from home to school have speed limits of 35 mph, 25 mph, and 10 mph. In algebra we can let the letter x represent our speed as we travel from home to

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school. The maximum value of x depends on what section of street we are on. The letter x can assume any one of the **various** values 35,25,10.

Example 2

Suppose that in writing a term paper for a geography class we need to specify the height of Mount Kilimanjaro. If we do not happen to know the height of the mountain, we can represent it (at least temporarily) on our paper with the letter h. Later, we look up the height in a reference book and find it to be 5890 meters. The letter h can assume only the one value, 5890, and no others. The value of h is **constant**.

3 Symbols of Operation, Equality, and Inequality

Binary Operation

A **binary operation** on a collection of numbers is a process that assigns a number to two given numbers in the collection. The binary operations used in algebra are addition, subtraction, multiplication, and division.

Symbols of Operation

If we let x and y each represent a number, we have the following notations:

Addition	$x+y$			
Subtraction	$x-y$			
Multiplication	$x\cdot y$	$(x)(y)$	$x\,(y)$	xy
Division	$\frac{x}{y}$	x/y	$x\div y$	$y\sqrt{x}$

4 Sample Set A

Example 3

$a+b$ represents the **sum** of a and b.

Example 4

$4+y$ represents the **sum** of 4 and y.

Example 5

$8-x$ represents the **difference** of 8 and x.

Example 6

$6x$ represents the **product** of 6 and x.

Example 7

ab represents the **product** of a and b.

Example 8

$h3$ represents the **product** of h and 3.

Example 9

$(14.2)\,a$ represents the **product** of 14.2 and a.

Example 10

$(8)\ (24)$ represents the **product** of 8 and 24.

Example 11

$5\cdot 6\,(b)$ represents the **product** of 5,6, and b.

Example 12

$\frac{6}{x}$ represents the **quotient** of 6 and x.

5 Practice Set A

Exercise 1 *(Solution on p. 10.)*

Represent the product of 29 and x five different ways.

If we let a and b represent two numbers, then a and b are related in exactly one of three ways:

Equality and Inequality Symbols

$a = b$ a and b are equal

$a > b$ a is strictly greater than b

$a < b$ a is strictly less than b

Some variations of these symbols include

$a \neq b$ a is not equal to b

$a \geq b$ a is greater than or equal to b

$a \leq b$ a is less than or equal to b

The last five of the above symbols are inequality symbols. We can **negate** (change to the opposite) any of the above statements by drawing a line through the relation symbol (as in $a \neq b$), as shown below:

a is not greater than b can be expressed as either

$a\overline{) > b}$ or $a \leq$ b.

a is not less b than can be expressed as either

$a\overline{) < b}$ or $a \geq$ b.

$a < b$ and $a\overline{) \geq} b$ both indicate that a is less than b.

6 Grouping Symbols

Grouping symbols are used to indicate that a particular collection of numbers and meaningful operations are to be grouped together and considered as one number. The grouping symbols commonly used in algebra are

Parentheses: ( )

Brackets: [ ]

Braces: { }

Bar:

Ψ

In a computation in which more than one operation is involved, grouping symbols help tell us which operations to perform first. If possible, we perform operations inside grouping symbols first.

7 Sample Set B

Example 13

$(4 + 17) - 6 = 21 - 6 = 15$

Example 14

$8\,(3 + 6) = 8\,(9) = 72$

Example 15

$5\,[8 + (10 - 4)] = 5\,[8 + 6] = 5\,[14] = 70$

Example 16

$2\{3\,[4\,(17 - 11)]\} = 2\{3\,[4\,(6)]\} = 2\{3\,[24]\} = 2\{72\} = 144$

Example 17
$\frac{9(5+1)}{24+3}$.

The fraction bar separates the two groups of numbers $9(5+1)$ and $24+3$. Perform the operations in the numerator and denominator separately.
$\frac{9(5+1)}{24+3} = \frac{9(6)}{24+3} = \frac{54}{24+3} = \frac{54}{27} = 2$

8 Practice Set B

Use the grouping symbols to help perform the following operations.

Exercise 2 *(Solution on p. 10.)*
$3(1+8)$

Exercise 3 *(Solution on p. 10.)*
$4[2(11-5)]$

Exercise 4 *(Solution on p. 10.)*
$6\{2[2(10-9)]\}$

Exercise 5 *(Solution on p. 10.)*
$\frac{1+19}{2+3}$

The following examples show how to use algebraic notation to write each expression.

Example 18
9 minus y becomes $9-y$

Example 19
46 times x becomes 46x

Example 20
7 times $(x+y)$ becomes $7(x+y)$

Example 21
4 divided by 3, times z becomes $\left(\frac{4}{3}\right) z$

Example 22
$(a-b)$ times $(b-a)$ divided by (2 times a) becomes $\frac{(a-b)(b-a)}{2a}$

Example 23
Introduce a variable (**any** letter will do but here we'll let x represent the number) and use appropriate algebraic symbols to write the statement: A number plus 4 is strictly greater than 6. The answer is $x+4>6$.

9 The Order of Operations

Suppose we wish to find the value of $16+4\cdot 9$. We could

1. add 16 and 4, then multiply this sum by 9.
   $16+4\cdot 9 = 20\cdot 9 = 180$
2. multiply 4 and 9, then add 16 to this product.
   $16+4\cdot 9 = 16+36 = 52$

We now have two values for one number. To determine the correct value we must use the standard **order of operations**.

Order of Operations

1. Perform all operations inside grouping symbols, beginning with the innermost set.
2. Perform all multiplications and divisions, as you come to them, moving left-to-right.
3. Perform all additions and subtractions, as you come to them, moving left-to-right.

As we proceed in our study of algebra, we will come upon another operation, exponentiation, that will need to be inserted before multiplication and division. (See Section .)

10 Sample Set C

USe the order of operations to find the value of each number.

Example 24

$$\begin{array}{ll} 16+4\cdot 9 & \text{Multiply first.} \\ =16+36 & \text{Now add.} \\ =52 & \end{array}$$

Example 25

$$\begin{array}{ll} (27-8)+7\,(6+12) & \text{Combine within parentheses.} \\ =19+7\,(18) & \text{Multiply.} \\ =19+126 & \text{Now add.} \\ =145 & \end{array}$$

Example 26

$$\begin{array}{ll} 8+2\,[4+3\,(6-1)] & \text{Begin with the innermost set of grouping symbols, } \left(\quad \right). \\ =8+2\,[4+3\,(5)] & \text{Now work within the next set of grouping symbols, } \left[\quad \right]. \\ =8+2\,[4+15] & \\ =8+2\,[19] & \\ =8+38 & \\ =46 & \end{array}$$

Example 27

$$\begin{array}{rl} \frac{6+4[2+3(19-17)]}{18-2[2(3)+2]} & =\frac{6+4[2+3(2)]}{18-2[6+2]} \\ & =\frac{6+4[2+6]}{18-2[8]} \\ & =\frac{6+4[8]}{18-16} \\ & =\frac{6+32}{2} \\ & =\frac{38}{2} \\ & =19 \end{array}$$

11 Practice Set C

Use the order of operations to find each value.

Exercise 6 *(Solution on p. 10.)*
$25 + 8\,(3)$

Exercise 7 *(Solution on p. 10.)*
$2 + 3\,(18 - 5 \cdot 2)$

Exercise 8 *(Solution on p. 10.)*
$4 + 3\,[2 + 3\,(1 + 8 \div 4)]$

Exercise 9 *(Solution on p. 10.)*
$\frac{19+2\{5+2[18+6(4+1)]\}}{5\cdot 6-3(5)-2}$

12 Exercises

For the following problems, use the order of operations to find each value.

Exercise 10 *(Solution on p. 10.)*
$2 + 3\,(6)$

Exercise 11
$18 - 7\,(8 - 3)$

Exercise 12 *(Solution on p. 10.)*
$8 \cdot 4 \div 16 + 5$

Exercise 13
$(21 + 4) \div 5 \cdot 2$

Exercise 14 *(Solution on p. 10.)*
$3\,(8 + 2) \div 6 + 3$

Exercise 15
$6\,(4 + 1) \div (16 \div 8) - 15$

Exercise 16 *(Solution on p. 10.)*
$6\,(4 - 1) + 8\,(3 + 7) - 20$

Exercise 17
$(8)\ \ (5) + 2\,(14) + (1)\ \ (10)$

Exercise 18 *(Solution on p. 10.)*
$61 - 22 + 4\,[3\,(10) + 11]$

Exercise 19
$\frac{(1+16-3)}{7} + 5\,(12)$

Exercise 20 *(Solution on p. 10.)*
$\frac{8(6+20)}{8} + \frac{3(6+16)}{22}$

Exercise 21
$18 \div 2 + 55$

Exercise 22 *(Solution on p. 10.)*
$21 \div 7 \div 3$

Exercise 23
$85 \div 5 \cdot 5 - 85$

Exercise 24 *(Solution on p. 10.)*
$(300 - 25) \div (6 - 3)$

Exercise 25
$4 \cdot 3 + 8 \cdot 28 - (3 + 17) + 11\,(6)$

Exercise 26 *(Solution on p. 10.)*

$2\{(7+7)+6\left[4\left(8+2\right)\right]\}$

Exercise 27

$0+10\left(0\right)+15\left[4\left(3\right)+1\right]$

Exercise 28 *(Solution on p. 10.)*

$6.1\left(2.2+1.8\right)$

Exercise 29

$\frac{5.9}{2}+0.6$

Exercise 30 *(Solution on p. 10.)*

$\left(4+7\right)\ \left(8-3\right)$

Exercise 31

$\left(10+5\right)\ \left(10+5\right)-4\left(60-4\right)$

Exercise 32 *(Solution on p. 10.)*

$\left(\frac{5}{12}-\frac{1}{4}\right)+\left(\frac{1}{6}+\frac{2}{3}\right)$

Exercise 33

$4\left(\frac{3}{5}-\frac{8}{15}\right)+9\left(\frac{1}{3}+\frac{1}{4}\right)$

Exercise 34 *(Solution on p. 10.)*

$\frac{0}{5}+\frac{0}{1}+0\left[2+4\left(0\right)\right]$

Exercise 35

$0\cdot 9+4\cdot 0\div 7+0\left[2\left(2-2\right)\right]$

For the following problems, state whether the given statements are the same or different.

Exercise 36 *(Solution on p. 10.)*

$x\geq y\quad \text{and}\quad x>y$

Exercise 37

$x<y\quad \text{and}\quad x\ngeq y$

Exercise 38 *(Solution on p. 10.)*

$x=y\quad \text{and}\quad y=x$

Exercise 39

Represent the product of 3 and x five different ways.

Exercise 40 *(Solution on p. 10.)*

Represent the sum of a and b two different ways.

For the following problems, rewrite each phrase using algebraic notation.

Exercise 41

Ten minus three

Exercise 42 *(Solution on p. 11.)*

x plus sixteen

Exercise 43

51 divided by a

Exercise 44 *(Solution on p. 11.)*

81 times x

Exercise 45

3 times $(x+y)$

Exercise 46 *(Solution on p. 11.)*
$(x+b)$ times $(x+7)$

Exercise 47
3 times x times y

Exercise 48 *(Solution on p. 11.)*
x divided by (7 times b)

Exercise 49
$(a+b)$ divided by $(a+4)$

For the following problems, introduce a variable (any letter will do) and use appropriate algebraic symbols to write the given statement.

Exercise 50 *(Solution on p. 11.)*
A number minus eight equals seventeen.

Exercise 51
Five times a number, minus one, equals zero.

Exercise 52 *(Solution on p. 11.)*
A number divided by six is greater than or equal to forty-four.

Exercise 53
Sixteen minus twice a number equals five.

Determine whether the statements for the following problems are true or false.

Exercise 54 *(Solution on p. 11.)*
$6 - 4(4)\ \ (1) \leq 10$

Exercise 55
$5(4 + 2 \cdot 10) \geq 110$

Exercise 56 *(Solution on p. 11.)*
$8 \cdot 6 - 48 \leq 0$

Exercise 57
$\frac{20+4.3}{16} < 5$

Exercise 58 *(Solution on p. 11.)*
$2[6(1+4) - 8] > 3(11+6)$

Exercise 59
$6[4 + 8 + 3(26 - 15)] \neq 3[7(10 - 4)]$

Exercise 60 *(Solution on p. 11.)*
The number of different ways 5 people can be arranged in a row is $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$. How many ways is this?

Exercise 61
A box contains 10 computer chips. Three chips are to be chosen at random. The number of ways this can be done is

$$\frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

How many ways is this?

Exercise 62 *(Solution on p. 11.)*
The probability of obtaining four of a kind in a five-card poker hand is

$$\frac{13 \cdot 48}{(52 \cdot 51 \cdot 50 \cdot 49 \cdot 48) \div (5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)}$$

What is this probability?

Exercise 63

Three people are on an elevator in a five story building. If each person randomly selects a floor on which to get off, the probability that at least two people get off on the same floor is

$$1-\frac{5\cdot 4\cdot 3}{5\cdot 5\cdot 5}$$

What is this probability?

Solutions to Exercises in this Module

Solution to Exercise (p. 3)

$29 \cdot x, \quad 29x, \quad (29)(x), \quad 29(x), \quad (29)x$

Solution to Exercise (p. 4)

27

Solution to Exercise (p. 4)

48

Solution to Exercise (p. 4)

24

Solution to Exercise (p. 4)

4

Solution to Exercise (p. 5)

49

Solution to Exercise (p. 6)

26

Solution to Exercise (p. 6)

37

Solution to Exercise (p. 6)

17

Solution to Exercise (p. 6)

20

Solution to Exercise (p. 6)

7

Solution to Exercise (p. 6)

8

Solution to Exercise (p. 6)

78

Solution to Exercise (p. 6)

203

Solution to Exercise (p. 6)

29

Solution to Exercise (p. 6)

1

Solution to Exercise (p. 6)

$91\frac{2}{3}$

Solution to Exercise (p. 6)

508

Solution to Exercise (p. 7)

24.4

Solution to Exercise (p. 7)

55

Solution to Exercise (p. 7)

1

Solution to Exercise (p. 7)

0

Solution to Exercise (p. 7)

different

Solution to Exercise (p. 7)

same

Solution to Exercise (p. 7)
$a + b$, $b + a$

Solution to Exercise (p. 7)
$x + 16$

Solution to Exercise (p. 7)
$81x$

Solution to Exercise (p. 7)
$(x + b)(x + 7)$

Solution to Exercise (p. 8)
$\frac{x}{7b}$

Solution to Exercise (p. 8)
$x - 8 = 17$

Solution to Exercise (p. 8)
$\frac{x}{6} \geq 44$

Solution to Exercise (p. 8)
true

Solution to Exercise (p. 8)
true

Solution to Exercise (p. 8)
false

Solution to Exercise (p. 8)
120

Solution to Exercise (p. 8)
0.00024, or $\frac{1}{4165}$