

ARITHMETIC REVIEW: FACTORS, PRODUCTS, AND EXPONENTS*

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Abstract

This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. This chapter contains many examples of arithmetic techniques that are used directly or indirectly in algebra. Since the chapter is intended as a review, the problem-solving techniques are presented without being developed. Therefore, no work space is provided, nor does the chapter contain all of the pedagogical features of the text. As a review, this chapter can be assigned at the discretion of the instructor and can also be a valuable reference tool for the student.

1 Overview

- Factors
- Exponential Notation

2 Factors

Let's begin our review of arithmetic by recalling the meaning of multiplication for whole numbers (the counting numbers and zero).

Multiplication

Multiplication is a description of repeated addition.

In the addition

$$7 + 7 + 7 + 7$$

the number 7 is repeated as an **addend*** 4 **times**. Therefore, we say we have **four times seven** and describe it by writing

$$4 \cdot 7$$

The raised dot between the numbers 4 and 7 indicates multiplication. The dot directs us to multiply the

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two numbers that it separates. In algebra, the dot is preferred over the symbol \times to denote multiplication because the letter x is often used to represent a number. Thus,

$$4 \cdot 7 = 7 + 7 + 7 + 7$$

Factors and Products

In a multiplication, the numbers being multiplied are called **factors**. The result of a multiplication is called the **product**. For example, in the multiplication

$$4 \cdot 7 = 28$$

the numbers 4 and 7 are factors, and the number 28 is the product. We say that 4 and 7 are factors of 28. (They are not the only factors of 28. Can you think of others?)

Now we know that

$$(\text{factor}) \cdot (\text{factor}) = \text{product}$$

This indicates that a first number is a factor of a second number if the first number divides into the second number with no remainder. For example, since

$$4 \cdot 7 = 28$$

both 4 and 7 are factors of 28 since both 4 and 7 divide into 28 with no remainder.

3 Exponential Notation

Quite often, a particular number will be repeated as a factor in a multiplication. For example, in the multiplication

$$7 \cdot 7 \cdot 7 \cdot 7$$

the number 7 is repeated as a factor 4 times. We describe this by writing 7^4 . Thus,

$$7 \cdot 7 \cdot 7 \cdot 7 = 7^4$$

The repeated factor is the lower number (the base), and the number recording how many times the factor is repeated is the higher number (the superscript). The superscript number is called an **exponent**.

Exponent

An **exponent** is a number that records how many times the number to which it is attached occurs as a factor in a multiplication.

4 Sample Set A

For Examples 1, 2, and 3, express each product using exponents.

Example 1

$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$. Since 3 occurs as a factor 6 times,

$$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^6$$

Example 2

$8 \cdot 8$. Since 8 occurs as a factor 2 times,

$$8 \cdot 8 = 8^2$$

Example 3

$5 \cdot 5 \cdot 5 \cdot 9 \cdot 9$. Since 5 occurs as a factor 3 times, we have 5^3 . Since 9 occurs as a factor 2 times, we have 9^2 . We should see the following replacements.

$$\underbrace{5 \cdot 5 \cdot 5}_{5^3} \cdot \underbrace{9 \cdot 9}_{9^2}$$

Then we have

$$5 \cdot 5 \cdot 5 \cdot 9 \cdot 9 = 5^3 \cdot 9^2$$

Example 4

Expand 3^5 . The base is 3 so it is the repeated factor. The exponent is 5 and it records the number of times the base 3 is repeated. Thus, 3 is to be repeated as a factor 5 times.

$$3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$$

Example 5

Expand $6^2 \cdot 10^4$. The notation $6^2 \cdot 10^4$ records the following two facts: 6 is to be repeated as a factor 2 times and 10 is to be repeated as a factor 4 times. Thus,

$$6^2 \cdot 10^4 = 6 \cdot 6 \cdot 10 \cdot 10 \cdot 10 \cdot 10$$

5 Exercises

For the following problems, express each product using exponents.

Exercise 1

$$8 \cdot 8 \cdot 8$$

(Solution on p. 5.)

Exercise 2

$$12 \cdot 12 \cdot 12 \cdot 12 \cdot 12$$

Exercise 3

$$5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$$

(Solution on p. 5.)

Exercise 4

$$1 \cdot 1$$

Exercise 5

$$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 4 \cdot 4$$

(Solution on p. 5.)

Exercise 6

$$8 \cdot 8 \cdot 8 \cdot 15 \cdot 15 \cdot 15 \cdot 15$$

Exercise 7

$$2 \cdot 2 \cdot 2 \cdot 9 \cdot 9$$

(Solution on p. 5.)

Exercise 8

$$3 \cdot 3 \cdot 10 \cdot 10 \cdot 10$$

Exercise 9

Suppose that the letters x and y are each used to represent numbers. Use exponents to express the following product.

(Solution on p. 5.)

$$x \cdot x \cdot x \cdot y \cdot y$$

Exercise 10

Suppose that the letters x and y are each used to represent numbers. Use exponents to express the following product.

$$x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y$$

For the following problems, expand each product (do not compute the actual value).

Exercise 11 *(Solution on p. 5.)*
 3^4

Exercise 12
 4^3

Exercise 13 *(Solution on p. 5.)*
 2^5

Exercise 14
 9^6

Exercise 15 *(Solution on p. 5.)*
 $5^3 \cdot 6^2$

Exercise 16
 $2^7 \cdot 3^4$

Exercise 17 *(Solution on p. 5.)*
 $x^4 \cdot y^4$

Exercise 18
 $x^6 \cdot y^2$

For the following problems, specify all the whole number factors of each number. For example, the complete set of whole number factors of 6 is 1, 2, 3, 6.

Exercise 19 *(Solution on p. 5.)*
20

Exercise 20
14

Exercise 21 *(Solution on p. 5.)*
12

Exercise 22
30

Exercise 23 *(Solution on p. 5.)*
21

Exercise 24
45

Exercise 25 *(Solution on p. 5.)*
11

Exercise 26
17

Exercise 27 *(Solution on p. 5.)*
19

Exercise 28
2

Solutions to Exercises in this Module

Solution to Exercise (p. 3)

$$8^3$$

Solution to Exercise (p. 3)

$$5^7$$

Solution to Exercise (p. 3)

$$3^5 \cdot 4^2$$

Solution to Exercise (p. 3)

$$2^3 \cdot 9^8$$

Solution to Exercise (p. 3)

$$x^3 \cdot y^2$$

Solution to Exercise (p. 4)

$$3 \cdot 3 \cdot 3 \cdot 3$$

Solution to Exercise (p. 4)

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

Solution to Exercise (p. 4)

$$5 \cdot 5 \cdot 5 \cdot 6 \cdot 6$$

Solution to Exercise (p. 4)

$$x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$$

Solution to Exercise (p. 4)

$$1, 2, 4, 5, 10, 20$$

Solution to Exercise (p. 4)

$$1, 2, 3, 4, 6, 12$$

Solution to Exercise (p. 4)

$$1, 3, 7, 21$$

Solution to Exercise (p. 4)

$$1, 11$$

Solution to Exercise (p. 4)

$$1, 19$$