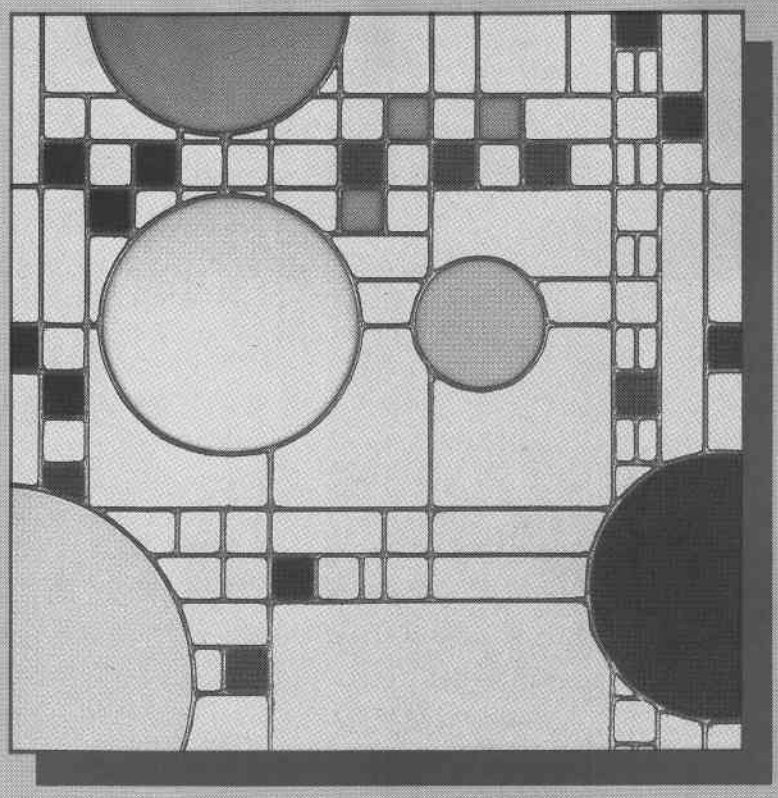
7 Ratios and Rates



After completing this chapter, you should

Section 7.1 Ratios and Rates
- be able to distinguish between denominate and pure numbers and between ratios and rates

Section 7.2 Proportions
- be able to describe proportions and find the missing factor in a proportion
- be able to work with proportions involving rates

Section 7.3 Applications of Proportions
- solve proportion problems using the five-step method

Section 7.4 Percent
- understand the relationship between ratios and percents
- be able to make conversions between fractions, decimals, and percents

Section 7.5 Fractions of One Percent
- understand the meaning of a fraction of one percent
- be able to make conversions involving fractions of one percent

Section 7.6 Applications of Percents
- be able to distinguish between base, percent, and percentage
- be able to find the percentage, the percent, and the base

7.1 Ratios and Rates

Section Overview

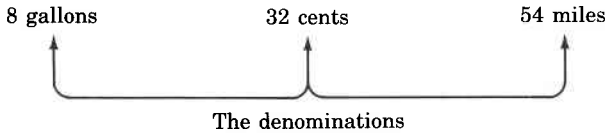
- ❑ DENOMINATE NUMBERS AND PURE NUMBERS
- ❑ RATIOS AND RATES

❑ DENOMINATE NUMBERS AND PURE NUMBERS

Denominate Numbers

Like and Unlike Denominate Numbers

It is often necessary or convenient to compare two quantities. **Denominate numbers** are numbers together with some specified unit. If the units being compared are alike, the denominate numbers are called **like denominate numbers.** If units are not alike, the numbers are called **unlike denominate numbers.** Examples of denominate numbers are shown in the diagram:



Pure Numbers

Numbers that exist purely as numbers and do *not* represent amounts of quantities are called **pure numbers.** Examples of pure numbers are 8, 254, 0, $21\frac{5}{8}$, $\frac{2}{5}$, and 0.07.

Numbers can be *compared* in two ways: subtraction and division.

Comparing Numbers by Subtraction and Division

Comparison of two numbers by subtraction indicates how *much more* one number is than another.

Comparison by division indicates how *many times* larger or smaller one number is than another.

Comparing Pure or Like Denominate Numbers by Subtraction

Numbers can be compared by subtraction if and only if they both are like denominate numbers or both pure numbers.

☆ SAMPLE SET A

1. Compare 8 miles and 3 miles by subtraction.

8 miles − 3 miles = 5 miles

This means that 8 miles is 5 miles more than 3 miles.

Examples of use: I can now jog 8 miles whereas I used to jog only 3 miles. So, I can now jog 5 miles more than I used to.

2. Compare 12 and 5 by subtraction.

12 − 5 = 7

This means that 12 is 7 more than 5.

3. Comparing 8 miles and 5 gallons by subtraction makes no sense.

8 miles − 5 gallons = ?

4. Compare 36 and 4 by division.

36 ÷ 4 = 9

This means that 36 is 9 times as large as 4. Recall that 36 ÷ 4 = 9 can be expressed as $\frac{36}{4} = 9$.

5. Compare 8 miles and 2 miles by division.

$$\frac{8 \text{ miles}}{2 \text{ miles}} = 4$$

This means that 8 miles is 4 times as large as 2 miles.

Example of use: I can jog 8 miles to your 2 miles. Or, for every 2 miles that you jog, I jog 8. So, I jog 4 times as many miles as you jog.

Notice that when like quantities are being compared by division, we drop the units. Another way of looking at this is that the units divide out (cancel).

6. Compare 30 miles and 2 gallons by division.

$$\frac{30 \text{ miles}}{2 \text{ gallons}} = \frac{15 \text{ miles}}{1 \text{ gallon}}$$

Example of use: A particular car goes 30 miles on 2 gallons of gasoline. This is the same as getting 15 miles to 1 gallon of gasoline.

Notice that when the quantities being compared by division are unlike quantities, we do not drop the units.

★ PRACTICE SET A

Make the following comparisons and interpret each one.

1. Compare 10 diskettes to 2 diskettes by
   (a) subtraction:

   (b) division:

2. Compare, if possible, 16 bananas and 2 bags by
   (a) subtraction:

   (b) division.

❑ RATIOS AND RATES

Ratio

A comparison, by division, of two pure numbers or two like denominate numbers is a **ratio.**

The comparison by division of the pure numbers $\frac{36}{4}$ and the like denominate numbers $\frac{8 \text{ miles}}{2 \text{ miles}}$ are examples of ratios.

Rate

A comparison, by division, of two unlike denominate numbers is a **rate.**

The comparison by division of two unlike denominate numbers, such as

$$\frac{55 \text{ miles}}{1 \text{ gallon}} \quad \text{and} \quad \frac{40 \text{ dollars}}{5 \text{ tickets}}$$

are examples of rates.

Let's agree to represent two numbers (pure or denominate) with the letters a and b. This means that we're letting a represent some number and b represent some, perhaps different, number. With this agreement, we can write the ratio of the two numbers a and b as

$$\frac{a}{b} \quad \text{or} \quad \frac{b}{a}$$

The ratio $\frac{a}{b}$ is read as "a to b."

The ratio $\frac{b}{a}$ is read as "b to a."

Since a ratio or a rate can be expressed as a fraction, it may be reducible.

☆ SAMPLE SET B

1. The ratio 30 to 2 can be expressed as $\frac{30}{2}$. Reducing, we get $\frac{15}{1}$.

The ratio 30 to 2 is *equivalent* to the ratio 15 to 1.

2. The rate "4 televisions to 12 people" can be expressed as $\frac{4 \text{ televisions}}{12 \text{ people}}$. The meaning of this rate is that "for every 4 televisions, there are 12 people."

Reducing, we get $\frac{1 \text{ television}}{3 \text{ people}}$. The meaning of this rate is that "for every 1 television, there are 3 people."

Thus, the rate of "4 televisions to 12 people" is the *same* as the rate of "1 television to 3 people."

★ PRACTICE SET B

Write the following ratios and rates as fractions.

1. 3 to 2

2. 1 to 9

3. 5 books to 4 people

4. 120 miles to 2 hours

5. 8 liters to 3 liters