

EXPONENTS HOMEWORK – HOMEWORK: “REAL LIFE” EXPONENTIAL CURVES*

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Abstract

This module provides practice problems designed to explore realistic applications of exponents.

Radioactive substances decay according to a “half-life.” The half-life is the period of time that it takes for half the substance to decay. For instance, if the half-life is 20 minutes, then every 20 minutes, half the remaining substance decays.

As you can see, this is the sort of exponential curve that goes down instead of up: at each step (or half-life) the total amount **divides by 2**; or, to put it another way, **multiplies by $\frac{1}{2}$** .

Exercise 1

First “Radioactive Decay” Case

You have 1 gram of a substance with a half-life of 1 minute. Fill in the following table.

Time	Substance remaining
0	1 gram
1 minute	$\frac{1}{2}$ gram
2 minutes	
3 minutes	
4 minutes	
5 minutes	

Table 1

- After n minutes, how many grams are there? Give me an equation.
- Use **that equation** to answer the question: after 5 minutes, how many grams of substance are there? Does your answer agree with what you put under “5 minutes” above? (If not, something’s wrong somewhere—find it and fix it!)
- How much substance will be left after $4\frac{1}{2}$ minutes?
- How much substance will be left after half an hour?

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- e. How long will it be before only one one-millionth of a gram remains?
- f. Finally, on the attached graph paper, do a graph of this function, where the “minute” is on the x-axis and the “amount of stuff left” is on the y-axis (so you are graphing grams as a function of minutes). Obviously, your graph won’t get past the fifth or sixth minute or so, but try to get an idea for what the shape looks like.

Exercise 2

Second “Radioactive Decay” Case

Now, we’re going to do a more complicated example. Let’s say you start with 1000 grams of a substance, and its half-life is 20 minutes; that is, every 20 minutes, half the substance disappears. Fill in the following chart.

Time	Half-Lives	Substance remaining
0	0	1000 grams
20 minutes	1	500 grams
40 minutes		
60 minutes		
80 minutes		
100 minutes		

Table 2

- a. After n half-lives, how many grams are there? Give me an equation.
- b. After n half-lives, how many **minutes** have gone by? Give me an equation.
- c. Now, let’s look at that equation the other way. After t minutes (for instance, after 60 minutes, or 80 minutes, **etc**), how many half-lives have gone by? Give me an equation.
- d. Now we need to put it all together. After t minutes, how many grams are there? This equation should take you directly from the first column to the third: for instance, it should turn 0 into 1000, and 20 into 500. (*Note: you can build this as a **composite function**, starting from two of your previous answers!)
- e. Test that equation to see if it gives you the same result you gave above after 100 minutes.
- f. Once again, graph that do a graph on the graph paper. The x-axis should be minutes. The y-axis should be the total amount of substance. In the space below, answer the question: how is it like, and how is it unlike, the previous graph?
- g. How much substance will be left after 70 minutes?
- h. How much substance will be left after two hours? (*Not two minutes, two hours!)
- i. How long will it be before only one gram of the original substance remains?

Exercise 3

Compound Interest

Finally, a bit more about compound interest

If you invest \$ A into a bank with $i\%$ interest compounded n times per year, after t years your bank account is worth an amount M given by:

$$M = A\left(1 + \frac{i}{n}\right)^{nt}$$

For instance, suppose you invest \$1,000 in a bank that gives 10% interest, compounded “semi-annually” (twice a year). So A , your initial investment, is \$1,000. i , the interest rate, is 10%, or 0.10. n , the number of times compounded per year, is 2. So after 30 years, you would have:

$\$1,000 \left(1 + \frac{0.10}{2}\right)^{2 \times 30} = \$18,679$. (Not bad for a \$1,000 investment!)

Now, suppose you invest \$1.00 in a bank that gives 100% interest (nice bank!). How much do you have after one year if the interest is...

- a. Compounded annually (once per year)?
- b. Compounded quarterly (four times per year)?
- c. Compounded daily?
- d. Compounded every second?