Quadratic Homework – The "Generic" Quadratic Equation*

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Abstract

Finding a general quadratic equation.

OK, let's say I wanted to solve a quadratic equation by completing the square. Here are the steps I would take, illustrated on an example problem. (These steps are exactly the same for any problem that you want to solve by completing the square.)

Note that as I go along, I simplify things—for instance, rewriting $3\frac{1}{2}+9$ as $12\frac{1}{2}$, or $\sqrt{12\frac{1}{2}}$ as $\frac{5}{\sqrt{2}}$. It is **always** a good idea to simplify as you go along!

Step	**Example**
The problem itself	$2x^2-3x-7=9x$
Put all the x terms on one side, and the number on the other	$2x^2-12x=7$
Divide both sides by the coefficient of x^2	$x^2-6x=31/2$
Add the same number to both side. What number? **Half the coefficient of x, squared.** (The coefficient of x is -6. Half of that is -3. So we add 9 to both sides.)	$x^2-6x\underline{+9}=3\frac{1}{2}\underline{+9}$
Rewrite the left side as a perfect square	$(x-3)^2=12\frac{1}{2}$
Square root—but with a "plus or minus"! (*Remember, if x^2 is 25, x may be 5 or -5	$x-3=\pm\sqrt{12\frac{1}{2}}=\pm\sqrt{\frac{25}{2}}=\pm\frac{5}{\sqrt{2}}$
	continued on next page

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Finally, add or subtract the number next to the x	$x = 3 \pm \frac{5}{\sqrt{2}}$ $(\approx -.5, 6.5)$

Table 1

Now, you're going to go through that same process, only you're going to start with the "generic" quadratic equation:

$$\mathrm{ax}^2 + \mathrm{bx} + c = 0 \tag{1}$$

As you know, once we solve this equation, we will have a formula that can be used to solve **any** quadratic equation—since every quadratic equation is just a specific case of that one!

Walk through each step. Remember to simplify things as you go along!

Exercise 1
Put all the x terms on one side, and the number on the other.

Exercise 2
Divide both sides by the coefficient of x^2.

Exercise 3
Add the same number to both sides. What number? **Half the coefficient of x, squared**.

- What is the coefficient of x?
- What is $\frac{1}{2}$ of that?
- What is that squared?

OK, now add that to both sides of the equation.

Exercise 4
>This brings us to a "rational expressions moment"—on the right side of the equation you will be adding two fractions. Go ahead and add them!

Exercise 5
Rewrite the left side as a perfect square.

Exercise 6
Square root—but with a "plus or minus"! (*Remember, if $x^2 = 25$, x may be 5 or –5!)

Exercise 7
Finally, add or subtract the number next to the x.

Did you get the good old quadratic formula? If not, go back and see what's wrong. If you did, give it a try on these problems! (Don't solve these by factoring or completing the square, solve them using the quadratic formula that you just derived!)

Exercise 8
$4x^2 + 5x + 1 = 0$

Exercise 9
$9x^2 + 12x + 4 = 0$

Exercise 10
$2x^2 + 2x + 1 = 0$

Exercise 11
In general, a quadratic equation may have **two real roots**, or it may have **one real root**, or it may have **no real roots**. Based on the quadratic formula, and your experience with the previous three problems, how can you look at a quadratic equation $\mathrm{ax}^2 + \mathrm{bx} + c = 0$ and tell what kind of roots it will have?