Functions Guide – Permutations*

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Abstract

A teacher's guide to permutations and functions.

The end of the "Graphing" homework sets this topic up.

Have one person in the class "be" x^2. He is allowed to use a calculator; so you can, for instance, hand him the number 1.7 and he will square it, producing the point (1.7,2.89).

Another person is $x^2 + 1$. He is *not* allowed to use a calculator, but he *is* allowed to talk to the first person, who is. So if you hand him 1.7, he asks the first person, who says 2.9, and then he comes back with a 3.9. Make sure everyone understands what we have just learned: the graph of $x^2 + 1$ contains the point (1.7,3.9). Do a few points this way.

Another person is $(x + 1)^2$ with the same rules. So if you give him a 1.7 he hands a 2.7 to the calculator person. Make sure everyone understands how this process gives us the point (1.7,7.3).

Talk about the fact that the first graph is a *vertical* permutation: it messed with the y-values that came out of the function. It's easy to understand what it did. It added 1 to every y-value, so the function went up 1.

The second graph is a *horizontal* permutation: it messed with the x-values that went into the function. It's harder to see what that did: why did $(x + 1)^2$ move to the left? Ask them to explain that.

Now hand them the worksheet "Horizontal and Vertical Permutations I." Hand one to each person—they will start in class, but probably finish in the homework. It's on the long side.

The next day, talk it all through very carefully. Key points to bring out:

1. What does $f(x) + 2$ mean? It means first plug a number into $f(x)$, and then add 2.
2. And what does that do to the graph? It means every y-value is two higher than it used to be, so the graph moves up by 2.
3. What does $f(x + 2)$ mean? It means first add 2, then plug a number into $f(x)$.
4. And what does that do to the graph? It means that when $x = 3$ you have the same y-value that the old graph had when $x = 5$. So your new graph is to the *left* of the old one.
5. What does all that have to do with our rock? This should be a long-ish conversation by itself. The vertical and horizontal permutations represent very different types of changes in the life of our rock. Suggest a different scenario, such as our old standard, the number of candy bars in the room as a function of the number of students, $c(s)$. What scenario would $c(s) + 3$ represent? How about $c(s + 3)$?

If you haven't already done so, introduce graphing on the calculator, including how to properly set the window. It only takes 5-10 minutes, but is necessary for the homework.

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Now, put the graph of $y = x^2$ on the board. We saw what $(x+1)^2$ and x^2+1 looked like yesterday. What do you think $-x^2$ would look like? How about $(x+2)^2 - 3$?

At some point, during the first or second day, you can come back to the idea of domain. What is the domain of $y = \sqrt{x+3}$? See if they can see the answer both numerically (you can plug in $x = -3$ but not $x = -4$ and graphically (the graph of $y = \sqrt{x}$ moved three spaces to the left, and its domain moved too).

Homework:

"Horizontal and Vertical Permutations II"