Matrices – Solving Linear Equations*

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Abstract

A teacher's guide to solving linear equations.

Just let them start this assignment, it should explain itself.

This is the coolest thing in our whole unit on matrices. It is also the most dangerous.

The cool thing is, you can solve real-world problems very quickly, thanks to matrices—so matrices (and matrix multiplication and the inverse matrix and so on) prove their worth. If you are given:

$2x - 5y + z = 1$
$6x - y + 2 = 4$
$4x - 10y + 2z = 2$

you plug into your calculator $[A] = \begin{bmatrix} 2 & -5 & 1 \\ 6 & -1 & 2 \\ 4 & -10 & 2 \end{bmatrix}$, $[B] = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$, ask for $A^{-1}B$, and the answer pops out! They should be able to do this process quickly and mechanically.

But the quick, mechanical nature of the process is also its great danger. I want them to see the logic of it. I want them to see exactly why the equation $\begin{bmatrix} 2 & -5 & 1 \\ 6 & -1 & 2 \\ 4 & -10 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$ is exactly like those three separate equations up there. I want them to be able to solve like this:

$\mathrm{AX} = B$
$A^{-1}\mathrm{AX} = A^{-1}B$
$\mathrm{IX} = A^{-1}B$
$X = A^{-1}B$

to see why it comes out as it does. If they see that this is all perfectly logical, and they know how to do it, the day is a big success—and in fact, this sort of justifies the whole unit on matrices.

As a final point, mention what happens if the equations were unsolvable: matrix A will have a 0 determinant, and will therefore have no inverse, so the equation won't work. (You get an error on the calculator.)

Homework

"Homework—Solving Linear Equations"

1 Time for Another Test!

And, time to conclude our unit on matrices.

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