Sequences and Series Guide – Arithmetic and Geometric Sequences*

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Abstract

A teacher's guide for lecturing on arithmetic and geometric sequences.

The in-class assignment does not need any introduction. Most of them will get the numbers, but they may need help with the last row, with the letters.

After this assignment, however, there is a fair bit of talking to do. They have all the concepts; now we have to dump a lot of words on them.

A "sequence" is a list of numbers. In principal, it could be anything: the phone number 8,6,7,5,3,0,9 is a sequence.

Of course, we will not be focusing on random sequences like that one. Our sequences will usually be expressed by a formula: for instance, "the xxxnth terms of this sequence is given by the formula $100+3(n-1)$" (or $3n+97$ in the case of the first problem on the worksheet. This is a lot like expressing the **function**$y=100+3(x-1)$, but it is not exactly the same. In the function $y=3x+97$, the variable x can be literally any number. But in a **sequence**, xxxn must be a positive integer; you do not have a "minus third term" or a "two-and-a-halfth term."

The first term in the sequence is referred to as t_1 and so on. So in our first example, $t_5=112$.

The number of terms in a sequence, or the particular term you want, is often designated by the letter n.

Our first sequence adds the same amount every time. This is called an **arithmetic sequence**. The amount it goes up by is called the **common difference**d (since it is the difference between any two adjacent terms). Note the relationship to linear functions, and slope.

Exercise 1

If I want to know all about a given arithmetic sequence, what do I need to know? Answer: I need to know t_1 and d.

Exercise 2

OK, so if I **have**t_1 and d for the arithmetic sequence, give me a formula for the n^{th} term in the sequence. (Answer: $t_n=t_1+d(n-1)$. Talk through this carefully before proceeding.)

Time for some more words. A **recursive definition** of a sequence defines each term in terms of the previous. For an arithmetic sequence, the recursive definition is $t_{n+1}=t_n+d$. (For instance, in our

*Version 1.1: Jan 10, 2009 1:38 pm -0600 example, $t_{n+1} = t_n + 3$. An **explicit definition** defines each term as an absolute formula, like the $3n + 97$ or the more general $t_n = t_1 + d\,(n-1)$ we came up with.

Our second sequence multiplies by the same amount every time. This is called a **geometric sequence**. The amount it multiplies by is called the **common ratio** r (since it is the ratio of any two adjacent terms).

Exercise 3

Find the recursive definition of a geometric sequence. (Answer: $t_{n+1} = rt_n$. They will do the explicit definition in the homework.)

Exercise 4

Question: How do you make an arithmetic sequence go **down**? Answer: $d < 0$

Exercise 5

Question: How do you make a geometric series go down? Answer: $0 < r < 1$. (Negative r values get weird and interesting in their own way...why?)

Homework

"Homework: Arithmetic and Geometric Sequences"