Constructing the Operator (Design)*

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Solving the third problem posed in the introduction to these notes is rather different from the other two. Here we want to find an operator or matrix that when multiplied by $\mathbf{x}$ gives $\mathbf{b}$. Clearly a solution to this problem would not be unique as stated. In order to pose a better defined problem, we generally give a set or family of inputs $\mathbf{x}$ and the corresponding outputs $\mathbf{b}$. If these families are independent, and if the number of them is the same as the size of the matrix, a unique matrix is defined and can be found by solving simultaneous equations. If a smaller number is given, the remaining degrees of freedom can be used to satisfy some other criterion. If a larger number is given, there is probably no exact solution and some approximation will be necessary.

Alternatively, the matrix may be constrained by structure to have less than N^2 degrees of freedom. It may be a cyclic convolution, a non cyclic convolution, a Toeplitz, a Hankel, or a Toeplitz plus Hankel matrix.

This problem came up in research on designing efficient prime length fast Fourier transform (FFT) algorithms where $\mathbf{x}$ is the data and $\mathbf{b}$ is the FFT of $\mathbf{x}$. The problem was to derive an operator that would make this calculation using the least amount of arithmetic. We solved it using a special formulation [1] and Matlab.

References

[1] Howard W. Johnson and C. S. Burrus. On the structure of efficient dft algorithms. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, 33(1):248–254, February 1985.

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