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Beti Andonovic

Abstract

solved problems of vector product

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1. λ, $\vec{p}=\lambda\vec{a}-5\vec{b}$ $\vec{q}=3\vec{a}-\vec{b}$, $\vec{a}$ $\vec{b}$.

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$\vec{p}$ $\vec{q}$, $\vec{p}\times\vec{q}=\vec{0}$. ,

$\left(\lambda\vec{a}-5\vec{b}\right)\times\left(3\vec{a}-\vec{b}\right)=\vec{0}$.

$3\lambda\vec{a}\times\vec{a}-\lambda\vec{a}\times\vec{b}-15\vec{b}\times\vec{a}+5\vec{b}\times\vec{b}=\vec{0}$.

´ $\vec{a}\times\vec{a}=\vec{0}$ $\vec{b}\times\vec{b}=\vec{0}$,

$-\lambda\vec{a}\times\vec{b}+15\vec{a}\times\vec{b}=\vec{0}$.

$(15-\lambda)\vec{a}\times\vec{b}=\vec{0}$.

$\vec{a}$ $\vec{b}$, $15-\lambda=0$, .. $\lambda=15$.

2. $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\vec{0}$, $\vec{a}$, $\vec{b}$ $\vec{c}$.

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$$\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}= \tag{1}$$

$$=\vec{a}\times\vec{b}-\vec{c}\times\vec{b}+\vec{c}\times\vec{a}= \tag{2}$$

$$=\left(\vec{a}-\vec{c}\right)\times\vec{b}+\vec{c}\times\vec{a}= \tag{3}$$

$=\left(\vec{a}-\vec{c}\right)\times\vec{b}+\vec{c}\times\vec{a}-\vec{a}\times\vec{a}=$

$$=\left(\vec{a}-\vec{c}\right)\times\vec{b}-\left(\vec{a}-\vec{c}\right)\times\vec{a}= \tag{4}$$

$=\left(\vec{a}-\vec{c}\right)\times\left(\vec{b}-\vec{a}\right)=\vec{0}$.

$\vec{a}-\vec{c}$ $\vec{b}-\vec{a}$, ..

$\vec{a}-\vec{c}=\lambda\left(\vec{b}-\vec{a}\right)$.

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$\vec{a} - \vec{c} = \lambda \vec{b} - \lambda \vec{a}$.

$(1+\lambda)\vec{a} - \lambda \vec{b} - \vec{c} = \vec{0}$, $\vec{a}$, $\vec{b}$ $\vec{c}$.

3. :

$\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$ $\vec{b} = -3\vec{i} - \vec{j} + \vec{k}$.

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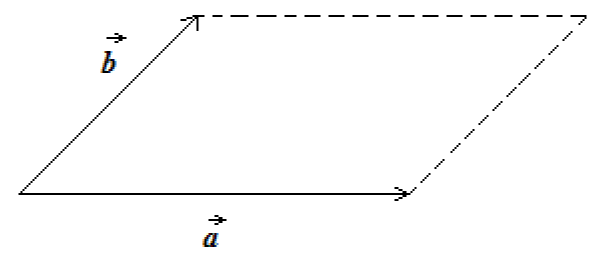
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Table 1

$$\vec{a} = \{2, 3, -1\} \quad (5)$$

$$\vec{b} = \{-3, -1, 1\} \quad (6)$$

$$\vec{a} \times \vec{b} = \left\{\left|\begin{matrix} 3 & -1 \\ -1 & 1 \end{matrix}\right|, \left|\begin{matrix} -1 & 2 \\ 1 & -3 \end{matrix}\right|, \left|\begin{matrix} 2 & 3 \\ -3 & 1 \end{matrix}\right|\right\} = \{2, 1, 7\}.$$

$P = |\vec{a} \times \vec{b}| = \sqrt{2^2 + 1^2 + 7^2} = \sqrt{54} = 3\sqrt{6}$.

4. ABC :

$A(4, -1, 2)$, $B(-8, 0, 4)$ $C(8, 2, 3)$.

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$$P_{\mathrm{ABC}} = \frac{1}{2} P_{\mathrm{ABDC}} = \frac{1}{2} |\vec{\mathrm{AB}} \times \vec{\mathrm{AC}}| \quad (7)$$

$$\vec{\mathrm{AB}} = \{-12, 1, 2\} \quad (8)$$

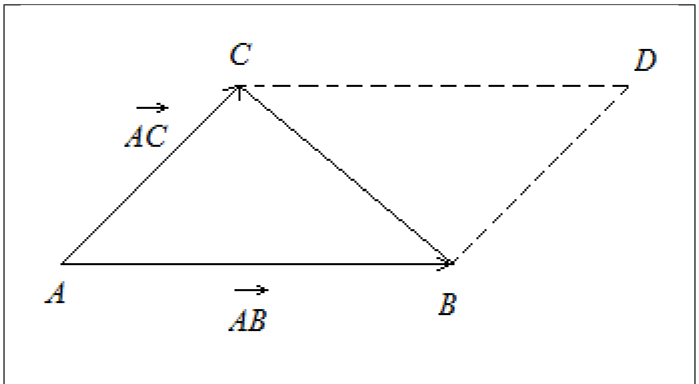

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Table 2

$\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}} = \{\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}, \begin{vmatrix} 2 & -12 \\ 1 & 4 \end{vmatrix}, \begin{vmatrix} -12 & 1 \\ 4 & 3 \end{vmatrix}\} = \{-5, 20, -40\}.$

$P_{\mathrm{ABC}} = \frac{1}{2} |\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}}| = \frac{1}{2}\sqrt{(-5)^2 + 20^2 + (-40)^2} = \frac{\sqrt{25+400+1600}}{2} = \frac{\sqrt{2025}}{2} = \frac{45}{2}.$

5. B AC ABC, :

$A(1,-1,2)$, $B(5,-6,2)$ $C(1,3,-1)$.

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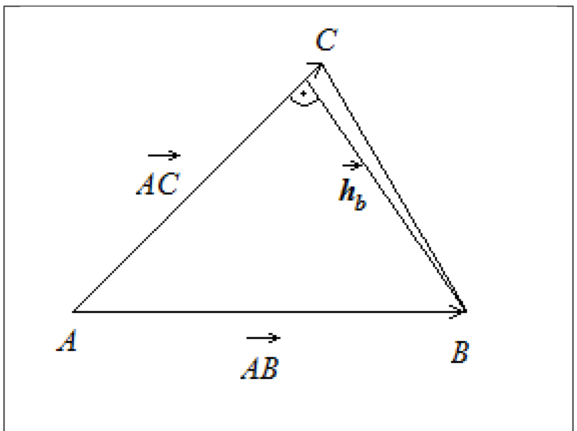

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Table 3

$$P_{\mathrm{ABC}} = \frac{1}{2} |\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}}| \quad (9)$$

$$\overrightarrow{\mathrm{AB}} = \{4, -5, 0\} \quad (10)$$

$$\overrightarrow{\mathrm{AC}} = \{0, 4, -3\} \quad (11)$$

$\vec{\mathrm{AB}} \times \vec{\mathrm{AC}} = \{\begin{vmatrix} -5 & 0 \\ 4 & -3 \end{vmatrix}, \begin{vmatrix} 0 & 4 \\ -3 & 0 \end{vmatrix}, \begin{vmatrix} 4 & -5 \\ 0 & 4 \end{vmatrix}\} = \{15, 12, 16\}.$

$P_{\mathrm{ABC}} = \frac{1}{2} |\vec{\mathrm{AB}} \times \vec{\mathrm{AC}}| = \frac{\sqrt{225+144+256}}{2} = \frac{\sqrt{625}}{2} = \frac{25}{2}.$

, $P_{\mathrm{ABC}} = \frac{|\vec{\mathrm{AC}}| \cdot |\vec{h_b}|}{2}$, $|\vec{h_b}| = \frac{2P_{\mathrm{ABC}}}{|\vec{\mathrm{AC}}|}$.

$|\vec{h_b}| = \frac{2 \cdot \frac{25}{2}}{\sqrt{0+16+9}} = \frac{25}{\sqrt{25}} = \frac{25}{5} = 5.$