Working with Numbers and Letters in Algebra*

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Abstract

This module explains the procedures for working with numbers and letters in algebra.

This module explains the procedures for working with numbers and letters in algebra. Some algebra terms are explained.

Math in elementary and middle school is about specific cases of numbers doing specific things. In algebra, we begin to look at mathematics in a general sense, generalizing the behavior of numbers for all numbers, not just secific cases. $2 + 3 = 5$ is a specific case of something mathematical happening. $x + y = 5$ is 5 expressed in general terms, where x and y could be any two real numbers. In this general case, many pairs of real numbers could be adding to 5. In algebra, letters stand in place for specific numbers. If you want to look at a specific case, numbers can be assigned to the letters. Once a number is assigned to a letter, that number is used everywhere that letter appears in an expression. A different letter would indicate the presence of a different number.

Numbers and letters go together to make up a term. Numbers always come first followed by the letters in alphabetical order.

An expression with a single term is called a monomial. An expression with two terms is called a binomial. An expression with three terms is called a trinomial. All other expressions are simply called polynomials.

The number in a term is called the coefficient. If no number precedes the letters the coefficient is understood to be 1. All terms have at least the number 1 as a coefficient.

The letters are called variables because they can take on various values depending on the situation being modeled. It is customary to arrange the letters so that an expression involving numbers, letters and operations is on one side of an equal sign making up an equation and a single letter with a coefficent of 1 is on the other side of an equal sign. The letters in the equation are called independent variables because they can take on values independently. The single variable by itself is called a dependent variable because its value depends on the values of the other variables.

Because letters are standing in for real numbers, they can be treated as real numbers. We can add them, subtract them, multiply them, divide them, raise them to powers, or find roots of them. They obey all the properties of real numbers. They are commutative, associative, distributive, and they obey the laws of exponents and roots.

In a term, the operation between numbers and variables and variables and variables is understood to be multiplication. Multiplication symbols are not used between the coefficient and the various variables in the term. Exponents are understood to apply only to the coefficient or variable they are to the right of. For example, the term $3xy^2$ could be written out as $3 \times x \times y^2$ and only the y variable is squared.

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When simplifying algebraic terms, coefficients can be multiplied or divided by other coefficients or numbers, and variables of one letter can be multiplied or divided by variables of the same letter. You can only add or subtract letters to each other if they are the same letter and have the same exponent. You can't add x^2 and x^3. Think of x^2 as being a two dimensional square and x^3 being a three dimensional cube. They are similar but not the same. You can't add squares and cubes; terms have to have the same dimension. If multiple letters are involved in a term, you can only add or subtract other terms that have the same letters with the same exponents. When you add or subtract like terms, you add or subtract the coefficients and don't change the variables in any way. Terms don't have to be alike to multiply or divide. You simply follow the laws of exponents for each variable in the terms. If radicals are involved, it is easiest to convert the radicals into rational exponents and deal only with the laws of exponents when multiplying or dividing. Convert any remaining rational exponents back into radicals when you are done.

Let's get the equation $x^2+2y^3=5$ into an expression with independent and dependent variables. Keep in mind that what ever you do to one side of an equation, you have to do to the other side of the equation. This means we are adding only additive inverses and multiplying only by the multiplicative inverses. Remember that if we add zero to somehting we don't change it. Likewise, multiplying something by 1 doesn't change it. Whatever we do to an equation, we must be careful not to change it in any way.

Let's start by adding the additive inverse of x^2 to both sides of the equation.

$x^2 - x^2 + 2y^3 = 5 - x^2$

This eliminates the x^2 term from the left side.

$2y^3 = 5 - x^2$

Now let's get the coefficient of the y term to be 1 by dividing both sides by 2.

$\frac{2y^3}{2} = \frac{5-x^2}{2}$

$y^3 = \frac{5-x^2}{2}$

Now we have to get rid of the cube on the left side. To do that, let's take the cube root of both sides.

$\sqrt[3]{y^3} = \sqrt[3]{\frac{5-x^2}{2}}$

$y = \sqrt[3]{\frac{5-x^2}{2}}$

The way we have written this equation, the value of y depends on the value of x, so x is the independent variable and y is the dependent variable. We can pick specific values for x to see what y becomes in specific situations.

Notice how we went backwards through the order of operations to isolate the y term. It's easiest to do addition and subtraction first, then multiplication and division, then exponents and radicals last. If the y term had been inside parenthesis, we would have worked our way from the outside in instead of inside out.