The Concept of State*

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Abstract

Memory and state in systems.

In order to characterize the memory of a dynamical system, we use a concept known as **state**.

NOTE: A system's state is defined as the minimal set of variables evaluated at $t = t_0$ needed to determine the future evolution of the system for $t > t_0$, given the excitation $u(t)$ for $t > t_0$

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Example 1

We are given the following differential equation describing a system. Note that $u(t) = 0$.

$$\frac{d^1 y(t)}{dt^1} + y(t) = 0 \tag{1}$$

Using the Laplace transform techniques described in the module on Linear Systems with Constant Coefficients, we can find a solution for $y(t)$:

$$y(t) = y(t_0) e^{t_0 - t} \tag{2}$$

As we need the information contained in $y(t_0)$ for this solution, $y(t)$ defines the state.

Example 2

The differential equation describing an unforced system is:

$$\frac{d^2 y(t)}{dt^2} + 3\frac{d^1 y(t)}{dt^1} + 2y(t) = 0 \tag{3}$$

Finding the $q(s)$ function, we have

$$q(s) = s^2 + 3s + 2 \tag{4}$$

The roots of this function are $\lambda_1 = -1$ and $\lambda_2 = -2$. These values are used in the solution to the differential equation as the exponents of the exponential functions:

$$y(t) = c_1 e^{-t} + c_2 e^{-2t} \tag{5}$$

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where c_1 and c_2 are constants. To determine the values of these constants we would need two equations (with two equations and two unknowns, we can find the unknowns). If we knew $y(0)$ and $\frac{d}{dt}y(0)$ we could find two equations, and we could then solve for $y(t)$. Therefore the system's state, $x(t)$, is

$$x(t) = \begin{pmatrix} y(t) \\ \frac{d^1 y(t)}{dt^1} \end{pmatrix} \tag{6}$$

In fact, the state can also be defined as any two non-trivial (i.e. independent) linear combinations of $y(t)$ and $\frac{d}{dt}y(t)$.

NOTE: **Basically, a system's state summarizes its entire past. It describes the memory-side of dynamical systems.**