Generating Sets and Bases*

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Abstract

This module defines generating sets and bases in linear algebra.

Given $\{x_1, \ldots, x_k\} \in \mathbb{R}$, we can define a linear space (vector space) X as

$$\begin{aligned} X &= \operatorname{span}(\{x_1, \ldots, x_k\}) \\ &= \left\{ \sum_{i=1}^{k} \alpha_i x_i \mid \alpha_i \in \mathbb{R} \right\} \end{aligned} \tag{1}$$

In this case, $\{x_1, \ldots, x_k\}$ form what is known as a **generating set** for the space X. That is to say that any vector in X can be generated by a linear combination of the vectors $\{x_1, \ldots, x_k\}$.

If $\{x_1, \ldots, x_k\}$ happen to be linearly independent, then they also form a **basis** for the space X. When $\{x_1, \ldots, x_k\}$ define a basis for X, k is the dimension of X. A basis is a special subset of a generating set. Every generating set includes a set of basis vectors.

Example 1

The following three vectors form a generating set for the linear space $\mathbb{R}^2$.

$$x_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, x_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, x_3 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

It is obvious that these three vectors can be combined to form any other two dimensional vector; in fact, we don't need this many vectors to completely define the space. As these vectors are not linearly independent, we can eliminate one of them. Seeing that x_3 is equal to $x_1 + x_2$, we can get rid of it and say that our **basis** for $\mathbb{R}^2$ is formed by x_1 and x_2.

*Version 2.9: Nov 18, 2003 8:50 pm -0600
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