

# BASIC OPERATIONS WITH REAL NUMBERS: ABSOLUTE VALUE\*

Wade Ellis  
Denny Burzynski

This work is produced by The Connexions Project and licensed under the Creative Commons Attribution License †

## Abstract

This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. The basic operations with real numbers are presented in this chapter. The concept of absolute value is discussed both geometrically and symbolically. The geometric presentation offers a visual understanding of the meaning of  $|x|$ . The symbolic presentation includes a literal explanation of how to use the definition. Negative exponents are developed, using reciprocals and the rules of exponents the student has already learned. Scientific notation is also included, using unique and real-life examples. Objectives of this module: understand the geometric and algebraic definitions of absolute value.

## 1 Overview

- Geometric Definition of Absolute Value
- Algebraic Definition of Absolute Value

## 2 Geometric Definition of Absolute Value

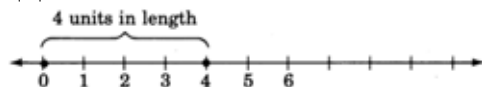
### Absolute Value—Geometric Approach

The **absolute value** of a number  $a$ , denoted  $|a|$ , is the distance from  $a$  to 0 on the number line.

Absolute value speaks to the question of "how far," and not "which way." The phrase how far implies length, and length is always a nonnegative (zero or positive) quantity. Thus, the absolute value of a number is a nonnegative number. This is shown in the following examples:

#### Example 1

$$|4| = 4$$

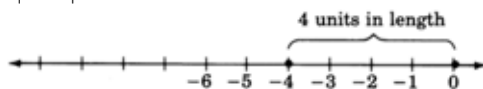


\*Version 1.4: May 28, 2009 4:42 pm GMT-5

†<http://creativecommons.org/licenses/by/3.0/>

**Example 2**

$$|-4| = 4$$

**Example 3**

$$|0| = 0$$

**Example 4**

$$-|5| = -5.$$

The quantity on the left side of the equal sign is read as "negative the absolute value of 5." The absolute value of 5 is 5. Hence, negative the absolute value of 5 is  $-5$ .

**Example 5**

$$-|-3| = -3.$$

The quantity on the left side of the equal sign is read as "negative the absolute value of  $-3$ ." The absolute value of  $-3$  is 3. Hence, negative the absolute value of  $-3$  is  $-(3) = -3$ .

### 3 Algebraic Definition of Absolute Value

The problems in the first example may help to suggest the following algebraic definition of absolute value. The definition is interpreted below. Examples follow the interpretation.

#### Absolute Value—Algebraic Approach

The **absolute value** of a number  $a$  is

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

The algebraic definition takes into account the fact that the number  $a$  could be either positive or zero ( $\geq 0$ ) or negative ( $< 0$ ).

1. If the number  $a$  is positive or zero ( $\geq 0$ ), the first part of the definition applies. The first part of the definition tells us that if the number enclosed in the absolute bars is a nonnegative number, the absolute value of the number is the number itself.
2. If the number  $a$  is negative ( $< 0$ ), the second part of the definition applies. The second part of the definition tells us that if the number enclosed within the absolute value bars is a negative number, the absolute value of the number is the opposite of the number. The opposite of a negative number is a positive number.

### 4 Sample Set A

Use the algebraic definition of absolute value to find the following values.

**Example 6**

$$|8|.$$

The number enclosed within the absolute value bars is a nonnegative number so the first part of the definition applies. This part says that the absolute value of 8 is 8 itself.

$$|8| = 8$$

**Example 7**

$$|-3|.$$

The number enclosed within absolute value bars is a negative number so the second part of the

definition applies. This part says that the absolute value of  $-3$  is the opposite of  $-3$ , which is  $-(-3)$ . By the double-negative property,  $-(-3) = 3$ .

$$|-3| = 3$$

## 5 Practice Set A

Use the algebraic definition of absolute value to find the following values.

**Exercise 1** *(Solution on p. 7.)*

$$|7|$$

**Exercise 2** *(Solution on p. 7.)*

$$|9|$$

**Exercise 3** *(Solution on p. 7.)*

$$|-12|$$

**Exercise 4** *(Solution on p. 7.)*

$$|-5|$$

**Exercise 5** *(Solution on p. 7.)*

$$-|8|$$

**Exercise 6** *(Solution on p. 7.)*

$$-|1|$$

**Exercise 7** *(Solution on p. 7.)*

$$-|-52|$$

**Exercise 8** *(Solution on p. 7.)*

$$-|-31|$$

## 6 Exercises

For the following problems, determine each of the values.

**Exercise 9** *(Solution on p. 7.)*

$$|5|$$

**Exercise 10**

$$|3|$$

**Exercise 11** *(Solution on p. 7.)*

$$|6|$$

**Exercise 12**

$$|14|$$

**Exercise 13** *(Solution on p. 7.)*

$$|-8|$$

**Exercise 14**

$$|-10|$$

**Exercise 15** *(Solution on p. 7.)*

$$|-16|$$

**Exercise 16**

$$-|8|$$

- Exercise 17** *(Solution on p. 7.)*  
-|12|
- Exercise 18**  
-|47|
- Exercise 19** *(Solution on p. 7.)*  
-|9|
- Exercise 20**  
|-9|
- Exercise 21** *(Solution on p. 7.)*  
|-1|
- Exercise 22**  
|-4|
- Exercise 23** *(Solution on p. 7.)*  
-|3|
- Exercise 24**  
-|7|
- Exercise 25** *(Solution on p. 7.)*  
-|-14|
- Exercise 26**  
-|-19|
- Exercise 27** *(Solution on p. 7.)*  
-|-28|
- Exercise 28**  
-|-31|
- Exercise 29** *(Solution on p. 7.)*  
-|-68|
- Exercise 30**  
|0|
- Exercise 31** *(Solution on p. 7.)*  
|-26|
- Exercise 32**  
-|-26|
- Exercise 33** *(Solution on p. 7.)*  
-|-(-8)|
- Exercise 34**  
-|-(-4)|
- Exercise 35** *(Solution on p. 7.)*  
-|-(-1)|
- Exercise 36**  
-|-(-7)|
- Exercise 37** *(Solution on p. 7.)*  
-(-|4|)
- Exercise 38**  
-(-|2|)

**Exercise 39** *(Solution on p. 7.)*  
 $-(-|-6|)$

**Exercise 40**  
 $-(-|-42|)$

**Exercise 41** *(Solution on p. 7.)*  
 $|-|-3||$

**Exercise 42**  
 $|-|-15||$

**Exercise 43** *(Solution on p. 8.)*  
 $|-|-12||$

**Exercise 44**  
 $|-|-29||$

**Exercise 45** *(Solution on p. 8.)*  
 $|6-|-2||$

**Exercise 46**  
 $|18-|-11||$

**Exercise 47** *(Solution on p. 8.)*  
 $|5-|-1||$

**Exercise 48**  
 $|10-|-3||$

**Exercise 49** *(Solution on p. 8.)*  
 $|- (17-|-12|) |$

**Exercise 50**  
 $|- (46-|-24|) |$

**Exercise 51** *(Solution on p. 8.)*  
 $|5|-|-2|$

**Exercise 52**  
 $|-2|^3$

**Exercise 53** *(Solution on p. 8.)*  
 $|- (2 \cdot 3) |$

**Exercise 54**  
 $|-2|+|-9|$

**Exercise 55** *(Solution on p. 8.)*  
 $(|-6|+|4|)^2$

**Exercise 56**  
 $(|-1|-|1|)^3$

**Exercise 57** *(Solution on p. 8.)*  
 $(|4|+|-6|)^2 - (|-2|)^3$

**Exercise 58**  
 $-[|-10|-6]^2$

**Exercise 59** *(Solution on p. 8.)*  
 $-[-(-|-4|+|-3|)^3]^2$

**Exercise 60**

A Mission Control Officer at Cape Canaveral makes the statement "lift-off,  $T$  minus 50 seconds."  
How long before lift-off?

**Exercise 61** *(Solution on p. 8.)*

Due to a slowdown in the industry, a Silicon Valley computer company finds itself in debt \$2,400,000. Use absolute value notation to describe this company's debt.

**Exercise 62**

A particular machine is set correctly if upon action its meter reads 0 units. One particular machine has a meter reading of  $-1.6$  upon action. How far is this machine off its correct setting?

## 7 Exercises for Review

**Exercise 63** *(Solution on p. 8.)*

( [here](#)<sup>1</sup>) Write the following phrase using algebraic notation: "four times  $(a + b)$ ."

**Exercise 64**

( [here](#)<sup>2</sup>) Is there a smallest natural number? If so, what is it?

**Exercise 65** *(Solution on p. 8.)*

( [here](#)<sup>3</sup>) Name the property of real numbers that makes  $5 + a = a + 5$  a true statement.

**Exercise 66**

( [here](#)<sup>4</sup>) Find the quotient of  $\frac{x^6 y^8}{x^4 y^3}$ .

**Exercise 67**

( [here](#)<sup>5</sup>) Simplify  $-(-4)$ . *(Solution on p. 8.)*

---

<sup>1</sup>"Basic Properties of Real Numbers: Symbols and Notations" <<http://cnx.org/content/m18872/latest/>>

<sup>2</sup>"Basic Properties of Real Numbers: The Real Number Line and the Real Numbers"

<<http://cnx.org/content/m21895/latest/>>

<sup>3</sup>"Basic Properties of Real Numbers: Properties of the Real Numbers" <<http://cnx.org/content/m21894/latest/>>

<sup>4</sup>"Basic Properties of Real Numbers: Rules of Exponents" <<http://cnx.org/content/m21900/latest/>>

<sup>5</sup>"Basic Operations with Real Numbers: Signed Numbers" <<http://cnx.org/content/m18874/latest/>>

## Solutions to Exercises in this Module

**Solution to Exercise 1 (p. 3)**

7

**Solution to Exercise 2 (p. 3)**

9

**Solution to Exercise 3 (p. 3)**

12

**Solution to Exercise 4 (p. 3)**

5

**Solution to Exercise 5 (p. 3)**

-8

**Solution to Exercise 6 (p. 3)**

-1

**Solution to Exercise 7 (p. 3)**

-52

**Solution to Exercise 8 (p. 3)**

-31

**Solution to Exercise 9 (p. 3)**

5

**Solution to Exercise 11 (p. 3)**

6

**Solution to Exercise 13 (p. 3)**

8

**Solution to Exercise 15 (p. 3)**

16

**Solution to Exercise 17 (p. 3)**

-12

**Solution to Exercise 19 (p. 4)**

-9

**Solution to Exercise 21 (p. 4)**

1

**Solution to Exercise 23 (p. 4)**

-3

**Solution to Exercise 25 (p. 4)**

-14

**Solution to Exercise 27 (p. 4)**

-28

**Solution to Exercise 29 (p. 4)**

-68

**Solution to Exercise 31 (p. 4)**

26

**Solution to Exercise 33 (p. 4)**

-8

**Solution to Exercise 35 (p. 4)**

-1

**Solution to Exercise 37 (p. 4)**

4

**Solution to Exercise 39 (p. 4)**

6

**Solution to Exercise 41 (p. 5)**

3

**Solution to Exercise 43 (p. 5)**

12

**Solution to Exercise 45 (p. 5)**

4

**Solution to Exercise 47 (p. 5)**

4

**Solution to Exercise 49 (p. 5)**

5

**Solution to Exercise 51 (p. 5)**

3

**Solution to Exercise 53 (p. 5)**

6

**Solution to Exercise 55 (p. 5)**

100

**Solution to Exercise 57 (p. 5)**

92

**Solution to Exercise 59 (p. 5)**

-1

**Solution to Exercise 61 (p. 6)**

$|\$ - 2,400,000|$

**Solution to Exercise 63 (p. 6)**

$4(a + b)$

**Solution to Exercise 65 (p. 6)**

commutative property of addition

**Solution to Exercise 67 (p. 6)**

4