Basic Operations with Real Numbers: Scientific Notation*

Wade Ellis
Denny Burzynski

Abstract

This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. The basic operations with real numbers are presented in this chapter. The concept of absolute value is discussed both geometrically and symbolically. The geometric presentation offers a visual understanding of the meaning of |x|. The symbolic presentation includes a literal explanation of how to use the definition. Negative exponents are developed, using reciprocals and the rules of exponents the student has already learned. Scientific notation is also included, using unique and real-life examples. Objectives of this module: be able to convert a number from standard form to scientific form and from scientific form to standard form, be able to work with numbers in scientific notation.

1 Overview

- Standard Form to Scientific Form
- Scientific Form to Standard Form
- Working with Numbers in Scientific Notation

2 Standard Form to Scientific Form

Very large numbers such as 43,000,000,000,000,000,000 (the number of different possible configurations of Rubik's cube) and very small numbers such as 0.000000000000000000000340 (the mass of the amino acid tryptophan) are extremely inconvenient to write and read. Such numbers can be expressed more conveniently by writing them as part of a power of 10.

To see how this is done, let us start with a somewhat smaller number such as 2480. Notice that

$$\begin{aligned}\underbrace{2480}_{\text{Standard form}} &= 248.0 \times 10^1\\ &= 24.80 \times 10^2\\ &= \underbrace{2.480 \times 10^3}_{\text{Scientific form}}\end{aligned}$$

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Scientific Form
The last form is called the **scientific form** of the number. There is **one** nonzero digit to the left of the decimal point and the absolute value of the exponent on 10 records the number of places the original decimal point was moved to the **left**.

$$\begin{aligned} 0.00059 &= \frac{0.0059}{10} &= \frac{0.0059}{10^1} &= 0.0059 \times 10^{-1} \\ &= \frac{0.059}{100} &= \frac{0.059}{10^2} &= 0.059 \times 10^{-2} \\ &= \frac{0.59}{1000} &= \frac{0.59}{10^3} &= 0.59 \times 10^{-3} \\ &= \frac{5.9}{10{,}000} &= \frac{5.9}{10^4} &= 5.9 \times 10^{-4} \end{aligned}$$

There is **one** nonzero digit to the left of the decimal point and the absolute value of the exponent of 10 records the number of places the original decimal point was moved to the **right**.

Scientific Notation
Numbers written in scientific form are also said to be written using scientific notation. In **scientific notation**, a number is written as the product of a number between and including 1 and 10 (1 is included, 10 is not) and some power of 10.

Writing a Number in Scientific Notation
To write a number in scientific notation:

1. Move the decimal point so that there is one nonzero digit to its left.
2. Multiply the result by a power of 10 using an exponent whose absolute value is the number of places the decimal point was moved. Make the exponent positive if the decimal point was moved to the left and negative if the decimal point was moved to the right.

3 Sample Set A

Write the numbers in scientific notation.

Example 1
981

The number 981 is actually 981., and it is followed by a decimal point. In integers, the decimal point at the end is usually omitted.

$981 = 981. = 9.81 \times 10^2$

The decimal point is now two places to the left of its original position, and the power of 10 is 2.

Example 2
$54.066 = 5.4066 \times 10^1 = 5.4066 \times 10$

The decimal point is one place to the left of its original position, and the power of 10 is 1.

Example 3
$0.000000000004632 = 4.632 \times 10^{-12}$

The decimal point is twelve places to the right of its original position, and the power of 10 is -12.

Example 4
$0.027 = 2.7 \times 10^{-2}$

The decimal point is two places to the right of its original position, and the power of 10 is -2.

4 Practice Set A

Write the following numbers in scientific notation.

Exercise 1 *(Solution on p. 10.)*

346

Exercise 2 *(Solution on p. 10.)*

72.33

Exercise 3 *(Solution on p. 10.)*

5387.7965

Exercise 4 *(Solution on p. 10.)*

87,000,000

Exercise 5 *(Solution on p. 10.)*

179,000,000,000,000,000,000

Exercise 6 *(Solution on p. 10.)*

100,000

Exercise 7 *(Solution on p. 10.)*

1,000,000

Exercise 8 *(Solution on p. 10.)*

0.0086

Exercise 9 *(Solution on p. 10.)*

0.000098001

Exercise 10 *(Solution on p. 10.)*

0.000000000000000054

Exercise 11 *(Solution on p. 10.)*

0.0000001

Exercise 12 *(Solution on p. 10.)*

0.00000001

5 Scientific Form to Standard Form

A number written in scientific notation can be converted to standard form by reversing the process shown in Sample Set A.

Converting from Scientific Notation

To convert a number written in scientific notation to a number in standard form, move the decimal point the number of places prescribed by the exponent on the 10.

Positive Exponent Negative Exponent

Move the decimal point to the right when you have a positive exponent, and move the decimal point to the left when you have a negative exponent.

6 Sample Set B

Example 5
4.673×10^4.

The exponent of 10 is 4 so we must move the decimal point to the right 4 places (adding 0's if necessary).

$$4.6730 \times 10^4 = 46730$$

Example 6
2.9×10^7.

The exponent of 10 is 7 so we must move the decimal point to the right 7 places (adding 0's if necessary).

$2.9 \times 10^7 = 29000000$

Example 7
1×10^{27}.

The exponent of 10 is 27 so we must move the decimal point to the right 27 places (adding 0's without a doubt).

$1 \times 10^{27} = 1{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000$

Example 8
4.21×10^{-5}.

The exponent of 10 is -5 so we must move the decimal point to the left 5 places (adding 0's if necessary).

$4.21 \times 10^{-5} = 0.0000421$

Example 9
1.006×10^{-18}.

The exponent of 10 is -18 so we must move the decimal point to the left 18 places (adding 0's if necessary).

$1.006 \times 10^{-18} = 0.000000000000000001006$

7 Practice Set B

Convert the following numbers to standard form.

Exercise 13 *(Solution on p. 10.)*
9.25×10^2

Exercise 14 *(Solution on p. 10.)*
4.01×10^5

Exercise 15 *(Solution on p. 10.)*
1.2×10^{-1}

Exercise 16 *(Solution on p. 10.)*
8.88×10^{-5}

8 Working with Numbers in Scientific Notation

Multiplying Numbers Using Scientific Notation

There are many occasions (particularly in the sciences) when it is necessary to find the product of two numbers written in scientific notation. This is accomplished by using two of the basic rules of algebra.

Suppose we wish to find $(a \times 10^n)(b \times 10^m)$. Since the only operation is multiplication, we can use the commutative property of multiplication to rearrange the numbers.

$(a \times 10^n)(b \times 10^m) = (a \times b)(10^n \times 10^m)$

Then, by the rules of exponents, $10^n \times 10^m = 10^{n+m}$. Thus,

$(a \times 10^n)(b \times 10^m) = (a \times b) \times 10^{n+m}$

The product of $(a \times b)$ may not be between 1 and 10, so $(a \times b) \times 10^{n+m}$ may not be in scientific form. The decimal point in $(a \times b)$ may have to be moved. An example of this situation is in Sample Set C, problem 2.

9 Sample Set C

Example 10

$$\begin{aligned}\left(2 \times 10^3\right)\left(4 \times 10^8\right) &= (2 \times 4)\left(10^3 \times 10^8\right) \\ &= 8 \times 10^{3+8} \\ &= 8 \times 10^{11}\end{aligned}$$

Example 11

$$\begin{aligned}\left(5 \times 10^{17}\right)\left(8.1 \times 10^{-22}\right) &= (5 \times 8.1)\left(10^{17} \times 10^{-22}\right) \\ &= 40.5 \times 10^{17-22} \\ &= 40.5 \times 10^{-5}\end{aligned}$$

We need to move the decimal point one place to the **left** to put this number in scientific notation. Thus, we must also change the exponent of 10.

40.5×10^{-5}

$4.05 \times 10^1 \times 10^{-5}$

$4.05 \times \left(10^1 \times 10^{-5}\right)$

$4.05 \times \left(10^{1-5}\right)$

4.05×10^{-4}

Thus,

$\left(5 \times 10^{17}\right)\left(8.1 \times 10^{-22}\right) = 4.05 \times 10^{-4}$

10 Practice Set C

Perform each multiplication.

Exercise 17 *(Solution on p. 10.)*

$\left(3 \times 10^5\right)\left(2 \times 10^{12}\right)$

Exercise 18 *(Solution on p. 10.)*

$\left(1 \times 10^{-4}\right)\left(6 \times 10^{24}\right)$

Exercise 19 *(Solution on p. 10.)*

$\left(5 \times 10^{18}\right)\left(3 \times 10^6\right)$

Exercise 20 *(Solution on p. 10.)*

$\left(2.1 \times 10^{-9}\right)\left(3 \times 10^{-11}\right)$

11 Exercises

Convert the numbers used in the following problems to scientific notation.

Exercise 21 *(Solution on p. 10.)*

Mount Kilimanjaro is the highest mountain in Africa. It is 5890 meters high.

Exercise 22

The planet Mars is about 222,900,000,000 meters from the sun.

Exercise 23 *(Solution on p. 10.)*

There is an irregularly shaped galaxy, named NGC 4449, that is about 250,000,000,000,000,000,000,000 meters from earth.

Exercise 24

The farthest object astronomers have been able to see (as of 1981) is a quasar named 3C427. There seems to be a haze beyond this quasar that appears to mark the visual boundary of the universe. Quasar 3C427 is at a distance of 110,000,000,000,000,000,000,000,000 meters from the earth.

Exercise 25 *(Solution on p. 10.)*

The smallest known insects are about the size of a typical grain of sand. They are about 0.0002 meters in length (2 ten-thousandths of a meter).

Exercise 26

Atoms such as hydrogen, carbon, nitrogen, and oxygen are about 0.0000000001 meter across.

Exercise 27 *(Solution on p. 10.)*

The island of Manhattan, in New York, is about 57,000 square meters in area.

Exercise 28

The second largest moon of Saturn is Rhea. Rhea has a surface area of about 735,000 square meters, roughly the same surface area as Australia.

Exercise 29 *(Solution on p. 10.)*

A star, named Epsilon Aurigae B, has a diameter (distance across) of 2,800,000,000,000 meters. This diameter produces a surface area of about 24,630,000,000,000,000,000,000,000 square meters. This star is what astronomers call a red giant and it is the largest red giant known. If Epsilon Aurigae were placed at the sun's position, its surface would extend out to the planet Uranus.

Exercise 30

The volume of the planet Venus is 927,590,000,000,000,000,000 cubic meters.

Exercise 31 *(Solution on p. 11.)*

The average mass of a newborn American female is about 3360 grams.

Exercise 32

The largest brain ever measured was that of a sperm whale. It had a mass of 9200 grams.

Exercise 33 *(Solution on p. 11.)*

The mass of the Eiffel tower in Paris, France, is 8,000,000 grams.

Exercise 34

In 1981, a Japanese company built the largest oil tanker to date. The ship has a mass of about 510,000,000,000 grams. This oil tanker is more than 6 times as massive as the U.S. aircraft carrier, U.S.S. **Nimitz**.

Exercise 35 *(Solution on p. 11.)*

In the constellation of Virgo, there is a cluster of about 2500 galaxies. The combined mass of these galaxies is 150,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000 grams.

Exercise 36

The mass of an amoeba is about 0.000004 gram.

Exercise 37 *(Solution on p. 11.)*

Cells in the human liver have masses of about 0.000000008 gram.

Exercise 38

The human sperm cell has a mass of about 0.000000000017 gram.

Exercise 39 *(Solution on p. 11.)*

The principal protein of muscle is myosin. Myosin has a mass of 0.00000000000000000103 gram.

Exercise 40

Amino acids are molecules that combine to make up protein molecules. The amino acid tryptophan has a mass of 0.0000000000000000000000340 gram.

Exercise 41 *(Solution on p. 11.)*

An atom of the chemical element bromine has 35 electrons. The mass of a bromine atom is 0.0000000000000000000000000031 gram.

Exercise 42

Physicists are performing experiments that they hope will determine the mass of a small particle called a neutrino. It is suspected that neutrinos have masses of about 0.000000000000000000000000000000001 gram.

Exercise 43 *(Solution on p. 11.)*

The approximate time it takes for a human being to die of asphyxiation is 316 seconds.

Exercise 44

On the average, the male housefly lives 1,468,800 seconds (17 days).

Exercise 45 *(Solution on p. 11.)*

Aluminum-26 has a half-life of 740,000 years.

Exercise 46

Manganese-53 has a half-life of 59,918,000,000,000 seconds (1,900,000 years).

Exercise 47 *(Solution on p. 11.)*

In its orbit around the sun, the earth moves a distance one and one half feet in about 0.0000316 second.

Exercise 48

A pi-meson is a subatomic particle that has a half-life of about 0.0000000261 second.

Exercise 49 *(Solution on p. 11.)*

A subatomic particle called a neutral pion has a half-life of about 0.0000000000000001 second.

Exercise 50

Near the surface of the earth, the speed of sound is 1195 feet per second.

For the following problems, convert the numbers from scientific notation to standard decimal form.

Exercise 51 *(Solution on p. 11.)*

The sun is about 1×10^{8} meters from earth.

Exercise 52

The mass of the earth is about 5.98×10^{27} grams.

Exercise 53 *(Solution on p. 11.)*

Light travels about 5.866×10^{12} miles in one year.

Exercise 54

One year is about 3×10^{7} seconds.

Exercise 55 *(Solution on p. 11.)*

Rubik's cube has about 4.3×10^{19} different configurations.

Exercise 56

A photon is a particle of light. A 100-watt light bulb emits 1×10^{20} photons every second.

Exercise 57 *(Solution on p. 11.)*

There are about 6×10^{7} cells in the retina of the human eye.

Exercise 58

A car traveling at an average speed will travel a distance about equal to the length of the smallest fingernail in 3.16×10^{-4} seconds.

Exercise 59 *(Solution on p. 11.)*

A ribosome of **E. coli** has a mass of about 4.7×10^{-19} grams.

Exercise 60

A mitochondrion is the energy-producing element of a cell. A mitochondrion is about 1.5×10^{-6} meters in diameter.

Exercise 61 *(Solution on p. 11.)*

There is a species of frogs in Cuba that attain a length of at most 1.25×10^{-2} meters.

Perform the following operations.

Exercise 62

$\left(2 \times 10^{4}\right)\left(3 \times 10^{5}\right)$

Exercise 63 *(Solution on p. 11.)*

$\left(4 \times 10^{2}\right)\left(8 \times 10^{6}\right)$

Exercise 64

$\left(6 \times 10^{14}\right)\left(6 \times 10^{-10}\right)$

Exercise 65 *(Solution on p. 11.)*

$\left(3 \times 10^{-5}\right)\left(8 \times 10^{7}\right)$

Exercise 66

$\left(2 \times 10^{-1}\right)\left(3 \times 10^{-5}\right)$

Exercise 67 *(Solution on p. 11.)*

$\left(9 \times 10^{-5}\right)\left(1 \times 10^{-11}\right)$

Exercise 68

$\left(3.1 \times 10^{4}\right)\left(3.1 \times 10^{-6}\right)$

Exercise 69 *(Solution on p. 11.)*

$\left(4.2 \times 10^{-12}\right)\left(3.6 \times 10^{-20}\right)$

Exercise 70

$\left(1.1 \times 10^{6}\right)^{2}$

Exercise 71 *(Solution on p. 11.)*

$\left(5.9 \times 10^{14}\right)^{2}$

Exercise 72

$\left(1.02 \times 10^{-17}\right)^{2}$

Exercise 73 *(Solution on p. 11.)*

$\left(8.8 \times 10^{-50}\right)^{2}$

Exercise 74

If Mount Kilimanjaro was 1,000,000 times as high as it really is, how high would it be? (See problem 1.)

Exercise 75 *(Solution on p. 11.)*

If the planet Mars was 300,000 times as far from the sun as it really is, how far from the sun would it be? (See problem 2.)

Exercise 76
If 800,000,000 of the smallest insects known were lined up head to tail, how far would they stretch? (See problem 5.)

Exercise 77 *(Solution on p. 11.)*
If Rhea, the moon of Saturn, had a surface area 0.00000000002 of its real surface area, what would that surface area be? (See problem 8.)

Exercise 78
If the star Epsilon Aurigae B had a surface area 0.005 of its real surface area, what would that surface area be? (See problem 9.)

Exercise 79 *(Solution on p. 11.)*
If the mass of all the galaxies in the constellation Virgo was only 0.000000000000000000000003 of its real mass, what would that mass be? (See problem 15.)

Exercise 80
What is the mass of 15,000,000,000,000 bromine atoms? (See problem 21.)

12 Exercises for Review

Exercise 81 *(Solution on p. 12.)*
(here[1]) What integers can replace x so that the statement $-6 < x < -2$ is true?

Exercise 82
(here[2]) Simplify $\left(5x^2y^4\right)\left(2xy^5\right)$

Exercise 83 *(Solution on p. 12.)*
(here[3]) Determine the value of $-[-(-|-5|)]$.

Exercise 84
(here[4]) Write $\frac{x^3y^{-5}}{z^{-4}}$ so that only positive exponents appear.

Exercise 85 *(Solution on p. 12.)*
(here[5]) Write $(2z+1)^3(2z+1)^{-5}$ so that only positive exponents appear.

[1]"Basic Properties of Real Numbers: The Real Number Line and the Real Numbers" <http://cnx.org/content/m21895/latest/>
[2]"Basic Properties of Real Numbers: Rules of Exponents" <http://cnx.org/content/m21900/latest/>
[3]"Basic Operations with Real Numbers: Absolute Value" <http://cnx.org/content/m21876/latest/>
[4]"Basic Operations with Real Numbers: Negative Exponents" <http://cnx.org/content/m21882/latest/>
[5]"Basic Operations with Real Numbers: Negative Exponents" <http://cnx.org/content/m21882/latest/>

Solutions to Exercises in this Module

Solution to Exercise (p. 3)
3.46×10^2
Solution to Exercise (p. 3)
7.233×10
Solution to Exercise (p. 3)
5.3877965×10^3
Solution to Exercise (p. 3)
8.7×10^7
Solution to Exercise (p. 3)
1.79×10^{20}
Solution to Exercise (p. 3)
1.0×10^5
Solution to Exercise (p. 3)
1.0×10^6
Solution to Exercise (p. 3)
8.6×10^{-3}
Solution to Exercise (p. 3)
9.8001×10^{-5}
Solution to Exercise (p. 3)
5.4×10^{-17}
Solution to Exercise (p. 3)
1.0×10^{-7}
Solution to Exercise (p. 3)
1.0×10^{-8}
Solution to Exercise (p. 4)
925
Solution to Exercise (p. 4)
401000
Solution to Exercise (p. 4)
0.12
Solution to Exercise (p. 4)
0.0000888
Solution to Exercise (p. 5)
6×10^{17}
Solution to Exercise (p. 5)
6×10^{20}
Solution to Exercise (p. 5)
1.5×10^{25}
Solution to Exercise (p. 5)
6.3×10^{-20}
Solution to Exercise (p. 6)
5.89×10^3
Solution to Exercise (p. 6)
2.5×10^{23}
Solution to Exercise (p. 6)
2×10^{-4}
Solution to Exercise (p. 6)
5.7×10^4

Solution to Exercise (p. 6)
2.8×10^{12}, 2.463×10^{25}
Solution to Exercise (p. 6)
3.36×10^{3}
Solution to Exercise (p. 6)
8×10^{6}
Solution to Exercise (p. 6)
1.5×10^{62}
Solution to Exercise (p. 7)
8×10^{-9}
Solution to Exercise (p. 7)
1.03×10^{-18}
Solution to Exercise (p. 7)
3.1×10^{-26}
Solution to Exercise (p. 7)
3.16×10^{2}
Solution to Exercise (p. 7)
7.4×10^{5}
Solution to Exercise (p. 7)
3.16×10^{-5}
Solution to Exercise (p. 7)
1×10^{-16}
Solution to Exercise (p. 7)
$100,000,000$
Solution to Exercise (p. 7)
$5,866,000,000,000$
Solution to Exercise (p. 7)
$43,000,000,000,000,000,000$
Solution to Exercise (p. 8)
$60,000,000$
Solution to Exercise (p. 8)
0.0000000000000000047
Solution to Exercise (p. 8)
0.0125
Solution to Exercise (p. 8)
3.2×10^{9}
Solution to Exercise (p. 8)
2.4×10^{3}
Solution to Exercise (p. 8)
9×10^{-16}
Solution to Exercise (p. 8)
1.512×10^{-31}
Solution to Exercise (p. 8)
3.481×10^{29}
Solution to Exercise (p. 8)
7.744×10^{-99}
Solution to Exercise (p. 8)
6.687×10^{16}
Solution to Exercise (p. 9)
1.47×10^{-5}

Solution to Exercise (p. 9)
4.5×10^{37}

Solution to Exercise (p. 9)
$-5, \ -4, \ -3$

Solution to Exercise (p. 9)
-5

Solution to Exercise (p. 9)
$\frac{1}{(2z+1)^2}$