Basic Operations with Real Numbers: Negative Exponents*

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Abstract

This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. The basic operations with real numbers are presented in this chapter. The concept of absolute value is discussed both geometrically and symbolically. The geometric presentation offers a visual understanding of the meaning of |x|. The symbolic presentation includes a literal explanation of how to use the definition. Negative exponents are developed, using reciprocals and the rules of exponents the student has already learned. Scientific notation is also included, using unique and real-life examples. Objectives of this module: understand the concepts of reciprocals and negative exponents, be able to work with negative exponents.

1 Overview

- Reciprocals
- Negative Exponents
- Working with Negative Exponents

2 Reciprocals

Reciprocals

Two real numbers are said to be **reciprocals** of each other if their product is 1. Every nonzero real number has exactly one reciprocal, as shown in the examples below. Zero has no reciprocal.

Example 1

$4 \cdot \frac{1}{4} = 1.$ This means that 4 and $\frac{1}{4}$ are reciprocals.

Example 2

$6 \cdot \frac{1}{6} = 1.$ Hence, 6 and $\frac{1}{6}$ are reciprocals.

Example 3

$-2 \cdot \frac{-1}{2} = 1.$ Hence, -2 and $-\frac{1}{2}$ are reciprocals.

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Example 4

$a \cdot \frac{1}{a} = 1$. Hence, a and $\frac{1}{a}$ are reciprocals if $a \neq 0$.

Example 5

$x \cdot \frac{1}{x} = 1$. Hence, x and $\frac{1}{x}$ are reciprocals if $x \neq 0$.

Example 6

$x^3 \cdot \frac{1}{x^3} = 1$. Hence, x^3 and $\frac{1}{x^3}$ are reciprocals if $x \neq 0$.

3 Negative Exponents

We can use the idea of reciprocals to find a meaning for negative exponents.

Consider the product of x^3 and x^{-3}. Assume $x \neq 0$.

$x^3 \cdot x^{-3} = x^{3+(-3)} = x^0 = 1$

Thus, since the product of x^3 and x^{-3} is 1, x^3 and x^{-3} must be reciprocals.

We also know that $x^3 \cdot \frac{1}{x^3} = 1$. (See problem 6 above.) Thus, x^3 and $\frac{1}{x^3}$ are also reciprocals.

Then, since x^{-3} and $\frac{1}{x^3}$ are both reciprocals of x^3 and a real number can have only one reciprocal, it must be that $x^{-3} = \frac{1}{x^3}$.

We have used -3 as the exponent, but the process works as well for all other negative integers. We make the following definition.

If n is any natural number and x is any nonzero real number, then

$x^{-n} = \frac{1}{x^n}$

4 Sample Set A

Write each of the following so that only positive exponents appear.

Example 7

$x^{-6} = \frac{1}{x^6}$

Example 8

$a^{-1} = \frac{1}{a^1} = \frac{1}{a}$

Example 9

$7^{-2} = \frac{1}{7^2} = \frac{1}{49}$

Example 10

$(3a)^{-6} = \frac{1}{(3a)^6}$

Example 11

$(5x-1)^{-24} = \frac{1}{(5x-1)^{24}}$

Example 12

$(k+2z)^{-(-8)} = (k+2z)^8$

5 Practice Set A

Write each of the following using only positive exponents.

Exercise 1 *(Solution on p. 12.)*

y^{-5}

Exercise 2 *(Solution on p. 12.)*

m^{-2}

Exercise 3 *(Solution on p. 12.)*

3^{-2}

Exercise 4 *(Solution on p. 12.)*

5^{-1}

Exercise 5 *(Solution on p. 12.)*

2^{-4}

Exercise 6 *(Solution on p. 12.)*

$(xy)^{-4}$

Exercise 7 *(Solution on p. 12.)*

$(a+2b)^{-12}$

Exercise 8 *(Solution on p. 12.)*

$(m-n)^{-(-4)}$

CAUTION

It is important to note that a^{-n} is not necessarily a negative number. For example,

$3^{-2} = \frac{1}{3^2} = \frac{1}{9} \quad 3^{-2} \neq -9$

6 Working with Negative Exponents

The problems of Sample Set A suggest the following rule for working with exponents:

Moving Factors Up and Down

In a fraction, a **factor** can be moved from the numerator to the denominator or from the denominator to the numerator by changing the sign of the exponent.

7 Sample Set B

Write each of the following so that only positive exponents appear.

Example 13

$x^{-2}y^5$. The *factor* x^{-2} can be moved from the numerator to the denominator by changing the exponent -2 to $+2$.

$x^{-2}y^5 = \frac{y^5}{x^2}$

Example 14

a^9b^{-3}. The *factor* b^{-3} can be moved from the numerator to the denominator by changing the exponent -3 to $+3$.

$a^9b^{-3} = \frac{a^9}{b^3}$

Example 15

$\frac{a^4b^2}{c^{-6}}$. This fraction can be written without any negative exponents by moving the *factor* c^{-6} into the numerator. We must change the -6 to $+6$ to make the move legitimate.

$\frac{a^4b^2}{c^{-6}} = a^4b^2c^6$

Example 16

$\frac{1}{x^{-3}y^{-2}z^{-1}}$. This fraction can be written without negative exponents by moving all the *factors* from the denominator to the numerator. Change the sign of each exponent: -3 to $+3$, -2 to $+2$, -1 to $+1$.

$\frac{1}{x^{-3}y^{-2}z^{-1}} = x^3y^2z^1 = x^3y^2z$

8 Practice Set B

Write each of the following so that only positive exponents appear.

Exercise 9 ***(Solution on p. 12.)***

$x^{-4}y^7$

Exercise 10 ***(Solution on p. 12.)***

$\frac{a^2}{b^{-4}}$

Exercise 11 ***(Solution on p. 12.)***

$\frac{x^3y^4}{z^{-8}}$

Exercise 12 ***(Solution on p. 12.)***

$\frac{6m^{-3}n^{-2}}{7k^{-1}}$

Exercise 13 ***(Solution on p. 12.)***

$\frac{1}{a^{-2}b^{-6}c^{-8}}$

Exercise 14 ***(Solution on p. 12.)***

$\frac{3a(a-5b)^{-2}}{5b(a-4b)^5}$

9 Sample Set C

Example 17

Rewrite $\frac{24a^7b^9}{2^3a^4b^{-6}}$ in a simpler form.

Notice that we are dividing powers with the same base. We'll proceed by using the rules of exponents.

$$\begin{aligned}\frac{24a^7b^9}{2^3a^4b^{-6}} &= \frac{24a^7b^9}{8a^4b^{-6}} &= 3a^{7-4}b^{9-(-6)}\\ &&= 3a^3b^{9+6}\\ &&= 3a^3b^{15}\end{aligned}$$

Example 18

Write $\frac{9a^5b^3}{5x^3y^2}$ so that no denominator appears.

We can eliminate the denominator by moving all factors that make up the denominator to the numerator.

$9a^5b^35^{-1}x^{-3}y^{-2}$

Example 19
Find the value of $\frac{1}{10^{-2}} + \frac{3}{4^{-3}}$.
We can evaluate this expression by eliminating the negative exponents.

$$\begin{aligned}\frac{1}{10^{-2}} + \frac{3}{4^{-3}} &= 1 \cdot 10^2 + 3 \cdot 4^3 \\ &= 1 \cdot 100 + 3 \cdot 64 \\ &= 100 + 192 \\ &= 292\end{aligned}$$

10 Practice Set C

Exercise 15 *(Solution on p. 12.)*
Rewrite $\frac{36x^8b^3}{3^2x^{-2}b^{-5}}$ in a simpler form.

Exercise 16 *(Solution on p. 12.)*
Write $\frac{2^4m^{-3}n^7}{4^{-1}x^5}$ in a simpler form and one in which no denominator appears.

Exercise 17 *(Solution on p. 12.)*
Find the value of $\frac{2}{5^{-2}} + 6^{-2} \cdot 2^3 \cdot 3^2$.

11 Exercises

Write the following expressions using only positive exponents. Assume all variables are nonzero.

Exercise 18 *(Solution on p. 12.)*
x^{-2}

Exercise 19
x^{-4}

Exercise 20 *(Solution on p. 12.)*
x^{-7}

Exercise 21
a^{-8}

Exercise 22 *(Solution on p. 12.)*
a^{-10}

Exercise 23
b^{-12}

Exercise 24 *(Solution on p. 12.)*
b^{-14}

Exercise 25
y^{-1}

Exercise 26 *(Solution on p. 12.)*
y^{-5}

Exercise 27
$(x+1)^{-2}$

Exercise 28 *(Solution on p. 12.)*
$(x-5)^{-3}$

Exercise 29
$(y-4)^{-6}$

Exercise 30 *(Solution on p. 12.)*
$(a+9)^{-10}$

Exercise 31
$(r+3)^{-8}$

Exercise 32 *(Solution on p. 12.)*
$(a-1)^{-12}$

Exercise 33
x^3y^{-2}

Exercise 34 *(Solution on p. 13.)*
x^7y^{-5}

Exercise 35
a^4b^{-1}

Exercise 36 *(Solution on p. 13.)*
a^7b^{-8}

Exercise 37
$a^2b^3c^{-2}$

Exercise 38 *(Solution on p. 13.)*
$x^3y^2z^{-6}$

Exercise 39
$x^3y^{-4}z^2w$

Exercise 40 *(Solution on p. 13.)*
$a^7b^{-9}zw^3$

Exercise 41
$a^3b^{-1}zw^2$

Exercise 42 *(Solution on p. 13.)*
$x^5y^{-5}z^{-2}$

Exercise 43
$x^4y^{-8}z^{-3}w^{-4}$

Exercise 44 *(Solution on p. 13.)*
$a^{-4}b^{-6}c^{-1}d^4$

Exercise 45
$x^9y^{-6}z^{-1}w^{-5}r^{-2}$

Exercise 46 *(Solution on p. 13.)*
$4x^{-6}y^2$

Exercise 47
$5x^2y^2z^{-5}$

Exercise 48 *(Solution on p. 13.)*
$7a^{-2}b^2c^2$

Exercise 49
$4x^3(x+1)^2y^{-4}z^{-1}$

Exercise 50 *(Solution on p. 13.)*
$7a^2(a-4)^3b^{-6}c^{-7}$

Exercise 51
$18b^{-6}\left(b^2-3\right)^{-5}c^{-4}d^5e^{-1}$

Exercise 52 *(Solution on p. 13.)*
$7(w+2)^{-2}(w+1)^3$

Exercise 53
$2(a-8)^{-3}(a-2)^5$

Exercise 54 *(Solution on p. 13.)*
$\left(x^2+3\right)^3\left(x^2-1\right)^{-4}$

Exercise 55
$\left(x^4+2x-1\right)^{-6}(x+5)^4$

Exercise 56 *(Solution on p. 13.)*
$\left(3x^2-4x-8\right)^{-9}(2x+11)^{-3}$

Exercise 57
$\left(5y^2+8y-6\right)^{-2}(6y-1)^{-7}$

Exercise 58 *(Solution on p. 13.)*
$7a\left(a^2-4\right)^{-2}\left(b^2-1\right)^{-2}$

Exercise 59
$(x-5)^{-4}3b^2c^4(x+6)^8$

Exercise 60 *(Solution on p. 13.)*
$\left(y^3+1\right)^{-1}5y^3z^{-4}w^{-2}\left(y^3-1\right)^{-2}$

Exercise 61
$5x^3\left(2x^{-7}\right)$

Exercise 62 *(Solution on p. 13.)*
$3y^{-3}\left(9x\right)$

Exercise 63
$6a^{-4}\left(2a^{-6}\right)$

Exercise 64 *(Solution on p. 13.)*
$4a^2b^2a^{-5}b^{-2}$

Exercise 65
$5^{-1}a^{-2}b^{-6}b^{-11}c^{-3}c^9$

Exercise 66 *(Solution on p. 13.)*
$2^3x^22^{-3}x^{-2}$

Exercise 67
$7a^{-3}b^{-9}\cdot 5a^6bc^{-2}c^4$

Exercise 68 *(Solution on p. 13.)*
$(x+5)^2(x+5)^{-6}$

Exercise 69
$(a-4)^3(a-4)^{-10}$

Exercise 70 *(Solution on p. 13.)*
$8(b+2)^{-8}(b+2)^{-4}(b+2)^3$

Exercise 71
$3a^5b^{-7}\left(a^2+4\right)^{-3}6a^{-4}b\left(a^2+4\right)^{-1}\left(a^2+4\right)$

Exercise 72 *(Solution on p. 13.)*
$-4a^3b^{-5}\left(2a^2b^7c^{-2}\right)$

Exercise 73
$-2x^{-2}y^{-4}z^{4}\left(-6x^{3}y^{-3}z\right)$

Exercise 74 *(Solution on p. 13.)*
$(-5)^{2}(-5)^{-1}$

Exercise 75
$(-9)^{-3}(9)^{3}$

Exercise 76 *(Solution on p. 13.)*
$(-1)^{-1}(-1)^{-1}$

Exercise 77
$(4)^{2}(2)^{-4}$

Exercise 78 *(Solution on p. 13.)*
$\frac{1}{a^{-4}}$

Exercise 79
$\frac{1}{a^{-1}}$

Exercise 80 *(Solution on p. 14.)*
$\frac{4}{x^{-6}}$

Exercise 81
$\frac{7}{x^{-8}}$

Exercise 82 *(Solution on p. 14.)*
$\frac{23}{y^{-1}}$

Exercise 83
$\frac{6}{a^{2}b^{-4}}$

Exercise 84 *(Solution on p. 14.)*
$\frac{3c^{5}}{a^{3}b^{-3}}$

Exercise 85
$\frac{16a^{-2}b^{-6}c}{2yz^{-5}w^{-4}}$

Exercise 86 *(Solution on p. 14.)*
$\frac{24y^{2}z^{-8}}{6a^{2}b^{-1}c^{-9}d^{3}}$

Exercise 87
$\frac{3^{-1}b^{5}(b+7)^{-4}}{9^{-1}a^{-4}(a+7)^{2}}$

Exercise 88 *(Solution on p. 14.)*
$\frac{36a^{6}b^{5}c^{8}}{3^{2}a^{3}b^{7}c^{9}}$

Exercise 89
$\frac{45a^{4}b^{2}c^{6}}{15a^{2}b^{7}c^{8}}$

Exercise 90 *(Solution on p. 14.)*
$\frac{3^{3}x^{4}y^{3}z}{3^{2}xy^{5}z^{5}}$

Exercise 91
$\frac{21x^{2}y^{2}z^{5}w^{4}}{7xyz^{12}w^{14}}$

Exercise 92 *(Solution on p. 14.)*
$\frac{33a^{-4}b^{-7}}{11a^{3}b^{-2}}$

Exercise 93
$\frac{51x^{-5}y^{-3}}{3xy}$

Exercise 94 *(Solution on p. 14.)*
$\frac{2^{6}x^{-5}y^{-2}a^{-7}b^{5}}{2^{-1}x^{-4}y^{-2}b^{6}}$

Exercise 95
$\frac{(x+3)^3(y-6)^4}{(x+3)^5(y-6)^{-8}}$

Exercise 96 *(Solution on p. 14.)*
$\frac{4x^3}{y^7}$

Exercise 97
$\frac{5x^4y^3}{a^3}$

Exercise 98 *(Solution on p. 14.)*
$\frac{23a^4b^5c^{-2}}{x^{-6}y^5}$

Exercise 99
$\frac{2^3b^5c^2d^{-9}}{4b^4cx}$

Exercise 100 *(Solution on p. 14.)*
$\frac{10x^3y^{-7}}{3x^5z^2}$

Exercise 101
$\frac{3x^2y^{-2}(x-5)}{9^{-1}(x+5)^3}$

Exercise 102 *(Solution on p. 14.)*
$\frac{14a^2b^2c^{-12}\left(a^2+21\right)^{-4}}{4^{-2}a^2b^{-1}(a+6)^3}$

For the following problems, evaluate each numerical expression.

Exercise 103
4^{-1}

Exercise 104 *(Solution on p. 14.)*
7^{-1}

Exercise 105
6^{-2}

Exercise 106 *(Solution on p. 14.)*
2^{-5}

Exercise 107
3^{-4}

Exercise 108 *(Solution on p. 14.)*
$6 \cdot 3^{-3}$

Exercise 109
$4 \cdot 9^{-2}$

Exercise 110 *(Solution on p. 14.)*
$28 \cdot 14^{-1}$

Exercise 111
$2^{-3}\left(3^{-2}\right)$

Exercise 112 *(Solution on p. 14.)*
$2^{-1} \cdot 3^{-1} \cdot 4^{-1}$

Exercise 113
$10^{-2} + 3\left(10^{-2}\right)$

Exercise 114 *(Solution on p. 14.)*
$(-3)^{-2}$

Exercise 115
$(-10)^{-1}$

Exercise 116 *(Solution on p. 14.)*

$\frac{3}{2^{-3}}$

Exercise 117

$\frac{4^{-1}}{5^{-2}}$

Exercise 118 *(Solution on p. 14.)*

$\frac{2^4-7}{4^{-1}}$

Exercise 119

$\frac{2^{-1}+4^{-1}}{2^{-2}+4^{-2}}$

Exercise 120 *(Solution on p. 14.)*

$\frac{21^0-2^6}{2\cdot 6-13}$

For the following problems, write each expression so that only positive exponents appear.

Exercise 121

$\left(a^6\right)^{-2}$

Exercise 122 *(Solution on p. 14.)*

$\left(a^5\right)^{-3}$

Exercise 123

$\left(x^7\right)^{-4}$

Exercise 124 *(Solution on p. 14.)*

$\left(x^4\right)^{-8}$

Exercise 125

$\left(b^{-2}\right)^{7}$

Exercise 126 *(Solution on p. 14.)*

$\left(b^{-4}\right)^{-1}$

Exercise 127

$\left(y^{-3}\right)^{-4}$

Exercise 128 *(Solution on p. 15.)*

$\left(y^{-9}\right)^{-3}$

Exercise 129

$\left(a^{-1}\right)^{-1}$

Exercise 130 *(Solution on p. 15.)*

$\left(b^{-1}\right)^{-1}$

Exercise 131

$\left(a^0\right)^{-1}, \quad a \neq 0$

Exercise 132 *(Solution on p. 15.)*

$\left(m^0\right)^{-1}, \quad m \neq 0$

Exercise 133

$\left(x^{-3}y^7\right)^{-4}$

Exercise 134 *(Solution on p. 15.)*

$\left(x^6y^6z^{-1}\right)^{2}$

Exercise 135

$\left(a^{-5}b^{-1}c^0\right)^{6}$

Exercise 136 *(Solution on p. 15.)*

$\left(\frac{y^3}{x^{-4}}\right)^{5}$

Exercise 137
$\left(\frac{a^{-8}}{b^{-6}}\right)^3$

Exercise 138 *(Solution on p. 15.)*
$\left(\frac{2a}{b^3}\right)^4$

Exercise 139
$\left(\frac{3b}{a^2}\right)^{-5}$

Exercise 140 *(Solution on p. 15.)*
$\left(\frac{5^{-1}a^3b^{-6}}{x^{-2}y^9}\right)^2$

Exercise 141
$\left(\frac{4m^{-3}n^6}{2m^{-5}n}\right)^3$

Exercise 142 *(Solution on p. 15.)*
$\left(\frac{r^5s^{-4}}{m^{-8}n^7}\right)^{-4}$

Exercise 143
$\left(\frac{h^{-2}j^{-6}}{k^{-4}p}\right)^{-5}$

12 Exercises for Review

Exercise 144 *(Solution on p. 15.)*
(**here**[1]) Simplify $\left(4x^5y^3z^0\right)^3$

Exercise 145
(**here**[2]) Find the sum. $-15+3$.

Exercise 146 *(Solution on p. 15.)*
(**here**[3]) Find the difference. $8-(-12)$.

Exercise 147
(**here**[4]) Simplify $(-3)(-8)+4(-5)$.

Exercise 148 *(Solution on p. 15.)*
(**here**[5]) Find the value of m if $m=\frac{-3k-5t}{kt+6}$ when $k=4$ and $t=-2$.

[1] "Basic Properties of Real Numbers: The Power Rules for Exponents" <http://cnx.org/content/m21897/latest/>
[2] "Basic Operations with Real Numbers: AddItion of Signed Numbers" <http://cnx.org/content/m21991/latest/>
[3] "Basic Operations with Real Numbers: Subtraction of Signed Numbers" <http://cnx.org/content/m21877/latest/>
[4] "Basic Operations with Real Numbers: Multiplication and Division of Signed Numbers" <http://cnx.org/content/m21872/latest/>
[5] "Basic Operations with Real Numbers: Multiplication and Division of Signed Numbers" <http://cnx.org/content/m21872/latest/>

Solutions to Exercises in this Module

Solution to Exercise (p. 3)
$\frac{1}{y^5}$
Solution to Exercise (p. 3)
$\frac{1}{m^2}$
Solution to Exercise (p. 3)
$\frac{1}{9}$
Solution to Exercise (p. 3)
$\frac{1}{5}$
Solution to Exercise (p. 3)
$\frac{1}{16}$
Solution to Exercise (p. 3)
$\frac{1}{(xy)^4}$
Solution to Exercise (p. 3)
$\frac{1}{(a+2b)^{12}}$
Solution to Exercise (p. 3)
$(m-n)^4$
Solution to Exercise (p. 4)
$\frac{y^7}{x^4}$
Solution to Exercise (p. 4)
a^2b^4
Solution to Exercise (p. 4)
$x^3y^4z^8$
Solution to Exercise (p. 4)
$\frac{6k}{7m^3n^2}$
Solution to Exercise (p. 4)
$a^2b^6c^8$
Solution to Exercise (p. 4)
$\frac{3a}{5b(a-5b)^2(a-4b)^5}$
Solution to Exercise (p. 5)
$4x^{10}b^8$
Solution to Exercise (p. 5)
$64m^{-3}n^7x^{-5}$
Solution to Exercise (p. 5)
52
Solution to Exercise (p. 5)
$\frac{1}{x^2}$
Solution to Exercise (p. 5)
$\frac{1}{x^7}$
Solution to Exercise (p. 5)
$\frac{1}{a^{10}}$
Solution to Exercise (p. 5)
$\frac{1}{b^{14}}$
Solution to Exercise (p. 5)
$\frac{1}{y^5}$
Solution to Exercise (p. 5)
$\frac{1}{(x-5)^3}$
Solution to Exercise (p. 6)
$\frac{1}{(a+9)^{10}}$

Solution to Exercise (p. 6)

$\frac{1}{(a-1)^{12}}$

Solution to Exercise (p. 6)

$\frac{x^7}{y^5}$

Solution to Exercise (p. 6)

$\frac{a^7}{b^8}$

Solution to Exercise (p. 6)

$\frac{x^3y^2}{z^6}$

Solution to Exercise (p. 6)

$\frac{a^7zw^3}{b^9}$

Solution to Exercise (p. 6)

$\frac{x^5}{y^5z^2}$

Solution to Exercise (p. 6)

$\frac{d^4}{a^4b^6c}$

Solution to Exercise (p. 6)

$\frac{4y^2}{x^6}$

Solution to Exercise (p. 6)

$\frac{7b^2c^2}{a^2}$

Solution to Exercise (p. 6)

$\frac{7a^2(a-4)^3}{b^6c^7}$

Solution to Exercise (p. 7)

$\frac{7(w+1)^3}{(w+2)^2}$

Solution to Exercise (p. 7)

$\frac{\left(x^2+3\right)^3}{(x^2-1)^4}$

Solution to Exercise (p. 7)

$\frac{1}{(3x^2-4x-8)^9(2x+11)^3}$

Solution to Exercise (p. 7)

$\frac{7a}{(a^2-4)^2(b^2-1)^2}$

Solution to Exercise (p. 7)

$\frac{5y^3}{(y^3+1)z^4w^2(y^3-1)^2}$

Solution to Exercise (p. 7)

$\frac{27x}{y^3}$

Solution to Exercise (p. 7)

$\frac{4}{a^3}$

Solution to Exercise (p. 7)

1

Solution to Exercise (p. 7)

$\frac{1}{(x+5)^4}$

Solution to Exercise (p. 7)

$\frac{8}{(b+2)^9}$

Solution to Exercise (p. 7)

$\frac{-8a^5b^2}{c^2}$

Solution to Exercise (p. 8)

-5

Solution to Exercise (p. 8)

1

Solution to Exercise (p. 8)
a^4
Solution to Exercise (p. 8)
$4x^6$
Solution to Exercise (p. 8)
$23y$
Solution to Exercise (p. 8)
$\frac{3b^3c^5}{a^3}$
Solution to Exercise (p. 8)
$\frac{4bc^9y^2}{a^2d^3z^8}$
Solution to Exercise (p. 8)
$\frac{4a^3}{b^2c}$
Solution to Exercise (p. 8)
$\frac{3x^3}{y^2z^4}$
Solution to Exercise (p. 8)
$\frac{3}{a^7b^5}$
Solution to Exercise (p. 8)
$\frac{128}{a^7bx}$
Solution to Exercise (p. 9)
$\frac{4x^3}{y^7}$
Solution to Exercise (p. 9)
$\frac{23a^4b^5x^6}{c^2y^5}$
Solution to Exercise (p. 9)
$\frac{10}{3x^2y^7z^2}$
Solution to Exercise (p. 9)
$\frac{224b^3}{c^{12}(a^2+21)^4(a+6)^3}$
Solution to Exercise (p. 9)
$\frac{1}{7}$
Solution to Exercise (p. 9)
$\frac{1}{32}$
Solution to Exercise (p. 9)
$\frac{2}{9}$
Solution to Exercise (p. 9)
2
Solution to Exercise (p. 9)
$\frac{1}{24}$
Solution to Exercise (p. 9)
$\frac{1}{9}$
Solution to Exercise (p. 9)
24
Solution to Exercise (p. 10)
36
Solution to Exercise (p. 10)
63
Solution to Exercise (p. 10)
$\frac{1}{a^{15}}$
Solution to Exercise (p. 10)
$\frac{1}{x^{32}}$

Solution to Exercise (p. 10)
b^4
Solution to Exercise (p. 10)
y^{27}
Solution to Exercise (p. 10)
b
Solution to Exercise (p. 10)
1
Solution to Exercise (p. 10)
$\frac{x^{12}y^{12}}{z^2}$
Solution to Exercise (p. 10)
$x^{20}y^{15}$
Solution to Exercise (p. 11)
$\frac{16a^4}{b^{12}}$
Solution to Exercise (p. 11)
$\frac{a^6x^4}{25b^{12}y^{18}}$
Solution to Exercise (p. 11)
$\frac{n^{28}s^{16}}{m^{32}r^{20}}$
Solution to Exercise (p. 11)
$64x^{15}y^9$
Solution to Exercise (p. 11)
20
Solution to Exercise (p. 11)
1