Basic Properties of Real Numbers: Summary of Key Concepts*

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Abstract

This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. The symbols, notations, and properties of numbers that form the basis of algebra, as well as exponents and the rules of exponents, are introduced in this chapter. Each property of real numbers and the rules of exponents are expressed both symbolically and literally. Literal explanations are included because symbolic explanations alone may be difficult for a student to interpret. This module provides a summary of the key concepts of the chapter "Basic Properties of Real Numbers".

1 Summary of Key Concepts

Variables and Constants (here[1])
A **variable** is a letter or symbol that represents any member of a collection of two or more numbers. A **constant** is a letter or symbol that represents a specific number.
Binary Operation (here[2])
A **binary operation** is a process that assigns two numbers to a single number. $+, -, \times, \div$ are binary operations.
Grouping Symbols (here[3])
Grouping symbols are used to indicate that a particular collection of numbers and meaningful operations is to be considered as a single number ($5 \div 0$ is not meaningful). Grouping symbols can also direct us in operations when more than two operations are to be performed. Common algebraic grouping symbols are

Parentheses : $\left(\quad \right)$

Brackets : $\left[\quad \right]$

Braces : $\{ \quad \}$

Bar :

Ψ

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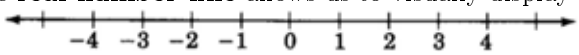
[1]"Basic Properties of Real Numbers: Symbols and Notations" <http://cnx.org/content/m18872/latest/>
[2]"Basic Properties of Real Numbers: Symbols and Notations" <http://cnx.org/content/m18872/latest/>
[3]"Basic Properties of Real Numbers: Symbols and Notations" <http://cnx.org/content/m18872/latest/>

Order of Operations (here[4], here[5])
When two or more operations are to be performed on a collection of numbers, the correct value can be obtained only by using the correct order of operations.
The Real Number Line (here[6])
The **real number line** allows us to visually display some of the numbers in which we are interested.



Coordinate and Graph (here[7])
The number associated with a point on the number line is called the **coordinate** of the point. The point associated with a number is called the **graph** of the number.
Real Number (here[8])
A **real number** is any number that is the coordinate of a point on the real number line.
Types of Real Numbers (here[9])
The collection of real numbers has many subcollections. The ones of most interest to us are

the natural numbers: $\{1, 2, 3, \ldots\}$
the whole numbers: $\{0, 1, 2, 3, \ldots\}$
the integers: $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
the rational numbers: {all numbers that can be expressed as the quotient of two integers}
the irrational numbers: {all numbers that have nonending and nonrepeating decimal representations}

Properties of Real Numbers (here[10])

Closure: If a and b are real numbers, then $a + b$ and $a \cdot b$ are unique real numbers.
Commutative: $a + b = b + a$ and $a \cdot b = b \cdot a$
Associative: $a + (b + c) = (a + b) + c$ and $a \cdot (b \cdot c) = (a \cdot b) \cdot c$
Distributive : $a\,(b + c) = a \cdot b + a \cdot c$
Additive identity: 0 is the additive identity. $a + 0 = a$ and $0 + a = a$.
Multiplicative identity: 1 is the multiplicative identity. $a \cdot 1 = a$ and $1 \cdot a = a$.
Additive inverse: For each real number a there is exactly one number $-$a such that $a + (-\text{a}) = 0$and $(-\text{a}) + a = 0$.
Multiplicative inverse: For each nonzero real number a there is exactly one nonzero real number $\frac{1}{a}$ such that $a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1$.

Exponents (here[11])
Exponents record the number of identical factors that appear in a multiplication.

$$\underbrace{x \cdot x \cdot x \cdot \ \ldots \ \cdot x}_{n \text{ factors of } x} = x^n$$

Rules of Exponents (here[12], here[13])
If x is a real number and n and m are natural numbers, then

[4] "Basic Properties of Real Numbers: Symbols and Notations" <http://cnx.org/content/m18872/latest/>
[5] "Basic Properties of Real Numbers: Exponents" <http://cnx.org/content/m21883/latest/>
[6] "Basic Properties of Real Numbers: The Real Number Line and the Real Numbers" <http://cnx.org/content/m21895/latest/>
[7] "Basic Properties of Real Numbers: The Real Number Line and the Real Numbers" <http://cnx.org/content/m21895/latest/>
[8] "Basic Properties of Real Numbers: The Real Number Line and the Real Numbers" <http://cnx.org/content/m21895/latest/>
[9] "Basic Properties of Real Numbers: The Real Number Line and the Real Numbers" <http://cnx.org/content/m21895/latest/>
[10] "Basic Properties of Real Numbers: Properties of the Real Numbers" <http://cnx.org/content/m21894/latest/>
[11] "Basic Properties of Real Numbers: Exponents" <http://cnx.org/content/m21883/latest/>
[12] "Basic Properties of Real Numbers: Rules of Exponents" <http://cnx.org/content/m21900/latest/>
[13] "Basic Properties of Real Numbers: The Power Rules for Exponents" <http://cnx.org/content/m21897/latest/>

: $x^n \cdot x^m = x^{n+m}$
: $\frac{x^n}{x^m} = x^{n-m}$, $x \neq 0$
: $x^0 = 1$, $x \neq 0$
: $(x^n)^m = x^{n \cdot m}$
: $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$, $y \neq 0$