Factoring Polynomials: Factoring by Grouping*

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Abstract

This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. Factoring is an essential skill for success in algebra and higher level mathematics courses. Therefore, we have taken great care in developing the student's understanding of the factorization process. The technique is consistently illustrated by displaying an empty set of parentheses and describing the thought process used to discover the terms that are to be placed inside the parentheses. The factoring scheme for special products is presented with both verbal and symbolic descriptions, since not all students can interpret symbolic descriptions alone. Two techniques, the standard "trial and error" method, and the "collect and discard" method (a method similar to the "ac" method), are presented for factoring trinomials with leading coefficients different from 1. Objectives of this module: know how to factor a polynomial using the grouping method and when to try the grouping method.

1 Overview

- Using Grouping to Factor a Polynomial
- Knowing when to Try the Grouping Method

2 Using Grouping to Factor a Polynomial

Sometimes a polynomial will not have a particular factor common to every term. However, we may still be able to produce a factored form for the polynomial.

The polynomial $x^3+3x^2-6x-18$has no single factor that is common to every term. However, we notice that if we **group** together the first two terms and the second two terms, we see that each resulting binomial has a particular factor common to both terms.

$$\underbrace{x^3+3x^2}_{x^2 \text{ is common}} \quad - \quad \underbrace{6x-18}_{-6 \text{ is common}}$$

Factor x^2 out of the first two terms, and factor -6 out of the second two terms.
$x^2(x+3) \ -6(x+3)$

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Now look closely at this binomial. Each of the two terms contains the factor $(x+3)$.
Factor out $(x+3)$.
$(x+3)\ \left(x^2-6\right)$ is the final factorization.
$x^3+3x^2-6x-18=\ (x+3)\ \left(x^2-6\right)$

3 Knowing when to Try the Grouping Method

We are alerted to the idea of grouping when the polynomial we are considering has **either** of these qualities:

1. no factor common to **all** terms
2. an **even** number of terms

When factoring by grouping, the sign $(+\text{or}-)$of the factor we are taking out will **usually** (but not always) be the same as the sign of the first term in that group.

4 Sample Set A

Example 1
Factor $8a^2b^4-4b^4+14a^2-7$.

1. We notice there is no factor common to all terms.
2. We see there are four terms, an even number.
3. We see that terms 1 and 2 have $+4b^4$in common (since the 1st term in the group is $+8a^2b^4$.
4. We notice that the 3rd and 4th terms have $+7$ in common (since the 1st term in the group is $+14a^2$).

$$8a^2b^4-4b^4+14a^2-7=4b^4(2a^2-1)+7(2a^2-1)$$

$(2a^2-1)$ is common

$8a^2b^4-4b^4+14a^2-7=\quad (2a^2\text{-}1)(4b^4+7)$

5 Practice Set A

Use the grouping method to factor the following polynomials.

Exercise 1 *(Solution on p. 5.)*
$ax+ay+bx+by$

Exercise 2 *(Solution on p. 5.)*
$2am+8m+5an+20n$

Exercise 3 *(Solution on p. 5.)*
$a^2x^3+4a^2y^3+3bx^3+12by^3$

Exercise 4 *(Solution on p. 5.)*
$15mx+10nx-6my-4ny$

Exercise 5 *(Solution on p. 5.)*
$40abx-24abxy-35c^2x+21c^2xy$

Exercise 6 *(Solution on p. 5.)*
When factoring the polynomial $8a^2b^4-4b^4+14a^2-7$ in Sample Set A, we grouped together terms1 and 2 and 3 and 4. Could we have grouped together terms1 and 3 and 2 and 4? Try this.
$8a^2b^4-4b^4+14a^2-7=$

Do we get the same result? If the results do not look precisely the same, recall the commutative property of multiplication.

6 Exercises

For the following problems, use the grouping method to factor the polynomials. Some polynomials may not be factorable using the grouping method.

Exercise 7 *(Solution on p. 5.)*
$2ab + 3a + 18b + 27$

Exercise 8
$xy - 7x + 4y - 28$

Exercise 9 *(Solution on p. 5.)*
$xy + x + 3y + 3$

Exercise 10
$mp + 3mq + np + 3nq$

Exercise 11 *(Solution on p. 5.)*
$ar + 4as + 5br + 20bs$

Exercise 12
$14ax - 6bx + 21ay - 9by$

Exercise 13 *(Solution on p. 5.)*
$12mx - 6bx + 21ay - 9by$

Exercise 14
$36ak - 8ah - 27bk + 6bh$

Exercise 15 *(Solution on p. 5.)*
$a^2b^2 + 2a^2 + 3b^2 + 6$

Exercise 16
$3n^2 + 6n + 9m^3 + 12m$

Exercise 17 *(Solution on p. 5.)*
$8y^4 - 5y^3 + 12z^2 - 10z$

Exercise 18
$x^2 + 4x - 3y^2 + y$

Exercise 19 *(Solution on p. 5.)*
$x^2 - 3x + xy - 3y$

Exercise 20
$2n^2 + 12n - 5mn - 30m$

Exercise 21 *(Solution on p. 5.)*
$4pq - 7p + 3q^2 - 21$

Exercise 22
$8x^2 + 16xy - 5x - 10y$

Exercise 23 *(Solution on p. 5.)*
$12s^2 - 27s - 8st + 18t$

Exercise 24
$15x^2 - 12x - 10xy + 8y$

Exercise 25 *(Solution on p. 5.)*
$a^4b^4 + 3a^5b^5 + 2a^2b^2 + 6a^3b^3$

Exercise 26
$4a^3bc - 14a^2bc^3 + 10abc^2 - 35bc^4$

Exercise 27 *(Solution on p. 5.)*
$5x^2y^3z + 3x^3yw - 10y^3z^2 - 6wxyz$

Exercise 28
$a^3b^2cd + abc^2dx - a^2bxy - cx^2y$

Exercise 29 *(Solution on p. 5.)*
$5m^{10}n^{17}p^3 - m^6n^7p^4 - 40m^4n^{10}qt^2 + 8pqt^2$

7 Exercises for Review

Exercise 30
(**here**[1]) Simplify $\left(x^5y^3\right)\ \left(x^2y\right)$.

Exercise 31 *(Solution on p. 5.)*
(**here**[2]) Use scientific notation to find the product of $\left(3 \times 10^{-5}\right)\left(2 \times 10^2\right)$.

Exercise 32
(**here**[3]) Find the domain of the equation $y = \frac{6}{x+5}$.

Exercise 33 *(Solution on p. 5.)*
(**here**[4]) Construct the graph of the inequality $y \geq -2$.

Exercise 34
(**here**[5]) Factor $8a^4b^4 + 12a^3b^5 - 8a^2b^3$.

[1] "Basic Properties of Real Numbers: Rules of Exponents" <http://cnx.org/content/m21900/latest/>
[2] "Basic Operations with Real Numbers: Scientific Notation" <http://cnx.org/content/m21879/latest/>
[3] "Algebraic Expressions and Equations: Terminology Associated with Equations" <http://cnx.org/content/m21849/latest/>
[4] "Solving Linear Equations and Inequalities: Linear Equations in Two Variables" <http://cnx.org/content/m21982/latest/>
[5] "Factoring Polynomials: The Greatest Common Factor" <http://cnx.org/content/m21913/latest/>

Solutions to Exercises in this Module

Solution to Exercise (p. 2)
$(a+b)\ (x+y)$
Solution to Exercise (p. 2)
$(2m+5n)\ (a+4)$
Solution to Exercise (p. 2)
$\left(a^2+3b\right)\ \left(x^3+4y^3\right)$
Solution to Exercise (p. 2)
$(5x-2y)\ (3m+2n)$
Solution to Exercise (p. 2)
$x\left(8ab-7c^2\right)\ (5-3y)$
Solution to Exercise (p. 2)
yes
Solution to Exercise (p. 3)
$(2b+3)(a+9)$
Solution to Exercise (p. 3)
$(y+1)(x+3)$
Solution to Exercise (p. 3)
$(a+5b)(r+4s)$
Solution to Exercise (p. 3)
$3(4mx-2bx+7ay-3by)$ Not factorable by grouping
Solution to Exercise (p. 3)
$\left(a^2+3\right)\left(b^2+2\right)$
Solution to Exercise (p. 3)
Not factorable by grouping
Solution to Exercise (p. 3)
$(x+y)(x-3)$
Solution to Exercise (p. 3)
Not factorable by grouping
Solution to Exercise (p. 3)
$(4s-9)(3s-2t)$
Solution to Exercise (p. 3)
$a^2b^2\left(a^2b^2+2\right)(1+3ab)$
Solution to Exercise (p. 4)
$y\left(5y^2z+3xw\right)\left(x^2-2z\right)$
Solution to Exercise (p. 4)
$\left(m^6n^7p^3-8qt^2\right)\left(5m^4n^{10}-p\right)$
Solution to Exercise (p. 4)
6×10^{-3}
Solution to Exercise (p. 4)