# Factoring Polynomials: Factoring a Monomial from a Polynomial\*

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#### Abstract

This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. Factoring is an essential skill for success in algebra and higher level mathematics courses. Therefore, we have taken great care in developing the student's understanding of the factorization process. The technique is consistently illustrated by displaying an empty set of parentheses and describing the thought process used to discover the terms that are to be placed inside the parentheses. The factoring scheme for special products is presented with both verbal and symbolic descriptions, since not all students can interpret symbolic descriptions alone. Two techniques, the standard "trial and error" method, and the "collect and discard" method (a method similar to the "ac" method), are presented for factoring trinomials with leading coefficients different from 1. Objectives of this module: be able to factor a monomial from a polynomial.

## **1** Overview

• The Factorization Process

## 2 The Factorization Process

We introduce the process of factoring a monomial from a polynomial by examining a problem: Suppose that  $12x^2 + 20x$  is the product and one of the factors is 4x. To find the other factor we could set up the problem this way:

 $4x \cdot ($   $) = 12x^2 + 20x$ 

Since the product  $12x^2 + 20x$  consists of two terms, the expression multiplying 4x must consist of two terms, since, by the distributive property

$$4x \cdot ( ) = 12x^2 + 20x$$

Now we see that this problem is simply an extension of finding the factors of a monomial.

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$$1st term: 4x ( ) = 12x^{2} \qquad 2nd term: 4x ( ) = 20x \\ ( ) = \frac{12x^{2}}{4x} \qquad ( ) = \frac{20x}{4x} \\ ( ) = 3x \qquad ( ) = 5$$

Thus,  $4x \cdot (3x+5) = 12x^2 + 20x$ .

Usually, these divisions can be done mentally and the terms of the factor filled in directly.

#### 3 Sample Set A

#### Example 1

The product is  $3x^7 - 2x^6 + 4x^5 - 3x^4$  and one factor is  $x^4$ . Find the other factor.

We have the problem:  $x^4$  times "what expression" yields  $3x^7 - 2x^6 + 4x^5 - 3x^4$ ? Mathematically,  $x^4 \cdot ( ) = 3x^7 - 2x^6 + 4x^5 - 3x^4$ 

Since there are four terms in the product, there must be four terms inside the parentheses. To find each of the four terms, we'll divide (mentally) each term of the product by  $x^4$ . The resulting quotient will be the necessary term of the factor.

1stterm:	$\frac{3x'}{x^4}$	=	$3x^{7-4} = 3x^3$	Place $3x^3$ into the 1st position in the (	).
2ndterm:	$\frac{-2x^6}{x^4}$	=	$-2x^{2}$	Place $-2x^2$ into the 2nd position in the (	)
3rdterm:	$\frac{4x^5}{x^4}$	=	4x	Place $4x$ into the 3rd position in the $\begin{pmatrix} & & \\ & & \end{pmatrix}$	).
4thterm:	$\frac{-3x^4}{x^4}$	=	-3	Place $-3$ into the 4th position in the (	).
Therefore, th	e other	fac	tor is $3x^3 - 2x$	$x^2 + 4x - 3$	

Therefore, the other factor is  $5x^2 - 2x^2 + 4x - 5$ .

This result can be checked by applying the distributive property.

 $x^{4} \cdot (3x^{3} - 2x^{2} + 4x - 3) = 3x^{7} - 2x^{6} + 4x^{5} - 3x^{4}$  Is this correct?

$$x^{4} \cdot (3x^{3} - 2x^{2} + 4x - 3) \stackrel{?}{=} 3x^{7} - 2x^{6} + 4x^{5} - 3x^{4}$$
  

$$3x^{4+3} - 2x^{4+2} + 4x^{4+1} = 3x^{7} - 2x^{6} + 4x^{5} - 3x^{4}$$
 Is this correct?  

$$3x^{7} - 2x^{6} + 4x^{5} - 3x^{4} = 3x^{7} - 2x^{6} + 4x^{5} - 3x^{4}$$
 Yes, this is correct.

 $x^{4} \cdot (3x^{3} - 2x^{2} + 4x - 3) = 3x^{7} - 2x^{6} + 4x^{5} - 3x^{4}$ 

Again, if the divisions can be performed mentally, the process can proceed very quickly.

#### Example 2

The product is  $10x^3y^6 + 15x^3y^4 - 5x^2y^4$  and a factor is  $5x^2y^4$ . Find the other factor.  $5x^2y^4 \cdot () = 10x^3y^6 + 15x^3y^4 - 5x^2y^4$ 

Since there are three terms in the product, there must be three terms inside the parentheses. To find each of these three terms, we'll divide each term of the product by  $5x^2y^4$ .

$$\begin{array}{rcl} 1st \ term: & \frac{10x^3y^6}{5x^2y^4} &=& 2xy^2 & \mbox{Place the } 2xy^2 \ \mbox{into the 1st position in the} \left( \begin{array}{c} \end{array} \right) \\ 2nd \ term: & \frac{15x^3y^4}{5x^2y^4} &=& 3x & \mbox{Place the } 3x \ \mbox{into the 2nd position in the} \left( \begin{array}{c} \end{array} \right) \\ 3rd \ term: & \frac{-5x^2y^4}{5x^2y^4} &=& -1 & \mbox{Place the } -1 \ \mbox{into the 3rd position in the} \left( \begin{array}{c} \end{array} \right) \\ \mbox{The other factor is } 2xy^2 + 3x - 1, \ \mbox{and} \\ 5x^2y^4 \cdot \left(2xy^2 + 3x - 1\right) = 10x^3y^6 + 15x^3y^4 - 5x^2y^4 \end{array}$$

#### Example 3

The product is  $-4a^2 - b^3 + 2c$  and a factor is -1. Find the other factor.

$$-1() = -4a^2 - b^3 + 2c$$

Since there are three terms in the product, there must be three terms inside the parentheses. We will divide (mentally) each term of the product by -1.

 $\begin{array}{rcl} 1st\,term: & \frac{-4a^2}{-1} &=& 4a^2 & \operatorname{Place} 4a^2 \operatorname{into} \operatorname{the} 1\operatorname{st} \operatorname{position} \operatorname{inside} \operatorname{the} \left( \begin{array}{c} \end{array} \right).\\ 2nd\,term: & \frac{-b^3}{-1} &=& b^3 & \operatorname{Place} b^3 \operatorname{into} \operatorname{the} 2\operatorname{nd} \operatorname{position} \operatorname{inside} \operatorname{the} \left( \begin{array}{c} \end{array} \right).\\ 3rd\,term: & \frac{2c}{-1} &=& -2c & \operatorname{Place} -2c \operatorname{into} \operatorname{the} 3\operatorname{rd} \operatorname{position} \operatorname{inside} \operatorname{the} \left( \begin{array}{c} \end{array} \right).\\ \end{array} \right).\\ \text{The other factor is } 4a^2 + b^3 - 2c, \operatorname{and} \\ -1\left(4a^2 + b^3 - 2c\right) = -4a^2 - b^3 + 2c\\ \text{Without writing the} -1, \operatorname{we get} \\ -\left(4a^2 + b^3 - 2c\right) = -4a^2 - b^3 + 2c \end{array}$ 

#### Example 4

The product is  $-3a^2b^5 - 15a^3b^2 + 9a^2b^2$  and a factor is  $-3a^2b^2$ . Find the other factor.  $-3a^2b^2( ) = -3a^2b^5 - 15a^3b^2 + 9a^2b^2$ 

Mentally dividing each term of the original trinomial by  $-3a^2b^2$ , we get  $b^3 + 5a - 3$  as the other factor, and

 $-3a^{2}b^{2}(b^{3}+5a-3) = -3a^{2}b^{5}-15a^{3}b^{2}+9a^{2}b^{2}$ 

## 4 Practice Set A

Exercise 1	(Solution on p.	7.)
The product is $3x^2 - 6x$ and a factor is $3x$ . Find the other factor.		·
Exercise 2	(Solution on p.	7.)
The product is $5y^4 + 10y^3 - 15y^2$ and a factor is $5y^2$ . Find the other factor.		
Exercise 3	(Solution on p.	7.)
The product is $4x^5y^3 - 8x^4y^4 + 16x^3y^5 + 24xy^7$ and a factor is $4xy^3$ . Find t	he other factor.	
Exercise 4	(Solution on p.	7.)
The product is $-25a^4 - 35a^2 + 5$ and a factor is $-5$ . Find the other factor.		
Exercise 5	(Solution on p.	7.)
The product is $-a^2 + b^2$ and a factor is $-1$ . Find the other factor.		

## 5 Exercises

For the following problems, the first quantity represents the product and the second quantity a factor. Find the other factor.

Exercise 6 4x + 10, 2	(Solution on p. 7.)
Exercise 7 6y + 18, 3	
Exercise 8 $5x + 25, 5$	(Solution on p. 7.)
<b>Exercise 9</b> $16a + 64, 8$	
Exercise 10 $3a^2 + 9a$ , $3a$	(Solution on p. 7.)
<b>Exercise 11</b> $14b^2 + 16b, 2b$	

Exercise 12

Exercise 13  $45y^2 + 50y, 5y$ Exercise 14

Exercise 16

 $21x^2 + 28x$ ,

 $18a^2 - 4a, 2a$ Exercise 15  $20a^2 - 12a$ ,

 $7x^2 - 14x$ , 7xExercise 17  $6y^2 - 24y, \quad 6y$ Exercise 18

 $8a^2 + 4a, 4a$ Exercise 19  $26b^2 + 13b$ , 13b

 $9x^2 + 6x + 18$ , 6

 $12b^2 + 16b + 20, 4$ 

 $21x^2 + 7x - 14$ , 7

 $35x^2 + 40x - 5$ , 5

 $14y^2 - 28y + 14$ , 14

 $36a^2 - 16a + 12, 4$ 

 $4y^2 - 10y - 12$ , 2

Exercise 20

Exercise 21

Exercise 22

Exercise 23

Exercise 24

Exercise 25

Exercise 26

Exercise 27  $6b^2 - 6b - 3$ , 3 Exercise 28

 $18x^3 + 20x$ ,

Exercise 29  $40y^3 + 24y, \quad 4y$ 

Exercise 30

Exercise 31

Exercise 32

 $16x^3 - 12x^2$ ,  $4x^2$ 

 $11x^3 - 11x + 11, \quad 11$ 

7x

4a

(Solution	on p.	7.)
(Solution	on p.	7.)

(Solution on p. 7.)

# $10a^3 + 12a^2 + 16a + 8, 2$ Exercise 33

 $14b^3 + 16b^2 + 26b + 30, 2$ 

2x

Exercise 34 (Solution on p. 7.)  $8a^3 - 4a^2 - 12a + 16, 4$ Exercise 35  $25x^3 - 30x^2 + 15x - 10, 5$ Exercise 36 (Solution on p. 7.)  $4x^6 + 16x^4 - 16x, \quad 4x$ Exercise 37  $9a^5 + 6a^5 - 18a^4 + 24a^2$ ,  $3a^2$ Exercise 38 (Solution on p. 7.)  $10x^3 - 35x^2$ ,  $5x^2$ Exercise 39  $12x^3y^5 + 20x^3y^2$ ,  $4x^3y^2$ Exercise 40 (Solution on p. 7.)  $10a^4b^3 + 4a^3b^4$ ,  $2a^3b^3$ Exercise 41  $8a^3b^6c^8 + 12a^2b^5c^6 - 16a^2b^7c^5, \quad 4a^2b^5c^5$ Exercise 42 (Solution on p. 7.)  $4x^5y^4 + x^2 + x$ , x Exercise 43  $14a^5b^2 - 3a^4b^4 + 7a^3$ ,  $a^3$ Exercise 44 (Solution on p. 7.)  $64a^5b^3c^{11} + 56a^4b^4c^{10} - 48a^3b^5c^9 - 8a^3b^2c^5, \quad 8a^3b^2c^5$ Exercise 45  $3h^3b^2 - 2h^6b^3 - 9h^2b + hb$ , hbExercise 46 (Solution on p. 8.) 5a + 10, -5Exercise 47 6b + 8, -2Exercise 48 (Solution on p. 8.)  $8x^2 + 12x, -4x$ Exercise 49  $20a^2b^2 - 10a^2, \quad -10a^2$ Exercise 50 (Solution on p. 8.) a+b, -1Exercise 51 x+y, -1Exercise 52 (Solution on p. 8.) a-b+c, -1Exercise 53 2x + 4y - z, -1Exercise 54 (Solution on p. 8.) -a-b-c, -1Exercise 55  $x^2 - x + 1, -1$ 

## 6 Exercises for Review

Exercise 56 ( here<sup>1</sup>) How many  $4y^2$ 's are there in  $24x^2y^3$ ? Exercise 57 ( here<sup>2</sup>) Find the product.  $(2y-3)^2$ . Exercise 58 ( here<sup>3</sup>) Solve 2(2a-1) - a = 7. Exercise 59 (here<sup>4</sup>) Given that  $3m^2n$  is a factor of  $12m^3n^4$ , find the other factor.

(Solution on p. 8.)

(Solution on p. 8.)

<sup>&</sup>lt;sup>1</sup>"Algebraic Expressions and Equations: Algebraic Expressions" <a href="http://cnx.org/content/m18875/latest/">http://cnx.org/content/m18875/latest/</a> <sup>2</sup>"Algebraic Expressions and Equations: Special Binomial Products" <a href="http://cnx.org/content/m21858/latest/">http://cnx.org/content/m21858/latest/</a> <sup>3</sup>"Solving Linear Equations and Inequalities: Further Techniques in Equation Solving'

<sup>&</sup>lt;http://cnx.org/content/m21992/latest/>
4"Factoring Polynomials: Finding the factors of a Monomial" <http://cnx.org/content/m18870/latest/>

# Solutions to Exercises in this Module

Solution to Exercise (p. 3) x - 2Solution to Exercise (p. 3)  $y^2 + 2y - 3$ Solution to Exercise (p. 3)  $x^4 - 2x^3y + 4x^2y^2 + 6y^4$ Solution to Exercise (p. 3)  $5a^4 + 7a^2 - 1$ Solution to Exercise (p. 3)  $a^2 - b^2$ Solution to Exercise (p. 3) 2x + 5Solution to Exercise (p. 3) x+5Solution to Exercise (p. 3) a+3Solution to Exercise (p. 3) 3x + 4Solution to Exercise (p. 4) 9a - 2Solution to Exercise (p. 4) x-2Solution to Exercise (p. 4) 2a + 1Solution to Exercise (p. 4)  $\frac{3}{2}x^2 + x + 3$ Solution to Exercise (p. 4)  $3x^2 + x - 2$ Solution to Exercise (p. 4)  $y^2 - 2y + 1$ Solution to Exercise (p. 4)  $2y^2 - 5y - 6$ Solution to Exercise (p. 4)  $9x^2 + 10$ Solution to Exercise (p. 4) 4x - 3Solution to Exercise (p. 4)  $5a^3 + 6a^2 + 8a + 4$ Solution to Exercise (p. 4)  $2a^3 - a^2 - 3a + 4$ Solution to Exercise (p. 5)  $x^5 + 4x^3 - 4$ Solution to Exercise (p. 5) 2x - 7Solution to Exercise (p. 5) 5a + 2bSolution to Exercise (p. 5)  $4x^4y^4 + x + 1$ 

Solution to Exercise (p. 5)  $8a^2bc^6 + 7ab^2c^5 - 6b^3c^4 - 1$ Solution to Exercise (p. 5) -a - 2Solution to Exercise (p. 5) -2x - 3Solution to Exercise (p. 5) -a-bSolution to Exercise (p. 5) -a+b-cSolution to Exercise (p. 5) a + b + cSolution to Exercise (p. 6)  $6x^2y$ Solution to Exercise (p. 6) a = 3