Factoring Polynomials: Factoring a Monomial from a Polynomial*

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Abstract

This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. Factoring is an essential skill for success in algebra and higher level mathematics courses. Therefore, we have taken great care in developing the student's understanding of the factorization process. The technique is consistently illustrated by displaying an empty set of parentheses and describing the thought process used to discover the terms that are to be placed inside the parentheses. The factoring scheme for special products is presented with both verbal and symbolic descriptions, since not all students can interpret symbolic descriptions alone. Two techniques, the standard "trial and error" method, and the "collect and discard" method (a method similar to the "ac" method), are presented for factoring trinomials with leading coefficients different from 1. Objectives of this module: be able to factor a monomial from a polynomial.

1 Overview

• The Factorization Process

2 The Factorization Process

We introduce the process of factoring a monomial from a polynomial by examining a problem: Suppose that $12x^2 + 20x$ is the product and one of the factors is 4x. To find the other factor we could set up the problem this way:

$$4x \cdot ($$
 $) = 12x^2 + 20x$

Since the product $12x^2 + 20x$ consists of two terms, the expression multiplying 4x must consist of two terms, since, by the distributive property

$$4x \cdot () = 12x^2 + 20x$$

Now we see that this problem is simply an extension of finding the factors of a monomial.

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$$1st term: 4x () = 12x^{2} 2nd term: 4x () = 20x () = $\frac{12x^{2}}{4x}$ () = $\frac{20x}{4x}$
 () = 3x () = 5$$

Thus, $4x \cdot (3x+5) = 12x^2 + 20x$.

Usually, these divisions can be done mentally and the terms of the factor filled in directly.

3 Sample Set A

Example 1

The product is $3x^7 - 2x^6 + 4x^5 - 3x^4$ and one factor is x^4 . Find the other factor.

We have the problem: x^4 times "what expression" yields $3x^7 - 2x^6 + 4x^5 - 3x^4$? Mathematically, $x^4 \cdot () = 3x^7 - 2x^6 + 4x^5 - 3x^4$

Since there are four terms in the product, there must be four terms inside the parentheses. To find each of the four terms, we'll divide (mentally) each term of the product by x^4 . The resulting quotient will be the necessary term of the factor.

Therefore, the other factor is $3x^3 - 2x^2 + 4x - 3$.

This result can be checked by applying the distributive property.

$$x^4 \cdot (3x^3 - 2x^2 + 4x - 3) = 3x^7 - 2x^6 + 4x^5 - 3x^4$$
 Is this correct?

$$x^4 \cdot (3x^3 - 2x^2 + 4x - 3) \stackrel{?}{=} 3x^7 - 2x^6 + 4x^5 - 3x^4$$

$$3x^{4+3} - 2x^{4+2} + 4x^{4+1} = 3x^7 - 2x^6 + 4x^5 - 3x^4$$
 Is this correct?

$$3x^7 - 2x^6 + 4x^5 - 3x^4 = 3x^7 - 2x^6 + 4x^5 - 3x^4$$
 Yes, this is correct.

Thus,

$$x^4 \cdot (3x^3 - 2x^2 + 4x - 3) = 3x^7 - 2x^6 + 4x^5 - 3x^4$$

Again, if the divisions can be performed mentally, the process can proceed very quickly.

Example 2

The product is $10x^3y^6 + 15x^3y^4 - 5x^2y^4$ and a factor is $5x^2y^4$. Find the other factor. $5x^2y^4 \cdot ($ $) = 10x^3y^6 + 15x^3y^4 - 5x^2y^4$

Since there are three terms in the product, there must be three terms inside the parentheses. To find each of these three terms, we'll divide each term of the product by $5x^2y^4$.

$$\begin{array}{lll} 1st\ term: & \frac{10x^3y^6}{5x^2y^4} & = & 2xy^2 & \text{Place the } 2xy^2 \text{ into the 1st position in the } \Big(& \Big) \\ 2nd\ term: & \frac{15x^3y^4}{5x^2y^4} & = & 3x & \text{Place the } 3x \text{ into the 2nd position in the } \Big(& \Big) \\ 3rd\ term: & \frac{-5x^2y^4}{5x^2y^4} & = & -1 & \text{Place the } -1 \text{ into the 3rd position in the } \Big(& \Big) \\ . \end{array}$$

The other factor is $2xy^2 + 3x - 1$, and

$$5x^2y^4 \cdot (2xy^2 + 3x - 1) = 10x^3y^6 + 15x^3y^4 - 5x^2y^4$$

Example 3

The product is $-4a^2 - b^3 + 2c$ and a factor is -1. Find the other factor.

$$-1$$
 () = $-4a^2 - b^3 + 2c$

Since there are three terms in the product, there must be three terms inside the parentheses. We will divide (mentally) each term of the product by -1.

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 $\begin{array}{ll} 1st\,term: & \frac{-4a^2}{-1} &= 4a^2 & \operatorname{Place}\,4a^2 \operatorname{into}\operatorname{the}\operatorname{1st}\operatorname{position}\operatorname{inside}\operatorname{the}\left(\right). \\ 2nd\,term: & \frac{-b^3}{-1} &= b^3 & \operatorname{Place}\,b^3\operatorname{into}\operatorname{the}\operatorname{2nd}\operatorname{position}\operatorname{inside}\operatorname{the}\left(\right). \\ 3rd\,term: & \frac{2c}{-1} &= -2c & \operatorname{Place}-2c\operatorname{into}\operatorname{the}\operatorname{3rd}\operatorname{position}\operatorname{inside}\operatorname{the}\left(\right). \\ \text{The other factor is } 4a^2+b^3-2c, \text{ and } \\ -1\left(4a^2+b^3-2c\right) &= -4a^2-b^3+2c \\ \text{Without writing the } -1, \text{ we get} \\ -\left(4a^2+b^3-2c\right) &= -4a^2-b^3+2c \end{array}$

Example 4

The product is $-3a^2b^5 - 15a^3b^2 + 9a^2b^2$ and a factor is $-3a^2b^2$. Find the other factor. $-3a^2b^2$ () = $-3a^2b^5 - 15a^3b^2 + 9a^2b^2$

Mentally dividing each term of the original trinomial by $-3a^2b^2$, we get $b^3 + 5a - 3$ as the other factor, and

 $-3a^{2}b^{2}(b^{3} + 5a - 3) = -3a^{2}b^{5} - 15a^{3}b^{2} + 9a^{2}b^{2}$

4 Practice Set A

Exercise 1 (Solution on p. 7.)

The product is $3x^2 - 6x$ and a factor is 3x. Find the other factor.

Exercise 2 (Solution on p. 7.)

The product is $5y^4 + 10y^3 - 15y^2$ and a factor is $5y^2$. Find the other factor.

Exercise 3 (Solution on p. 7.)

The product is $4x^5y^3 - 8x^4y^4 + 16x^3y^5 + 24xy^7$ and a factor is $4xy^3$. Find the other factor.

Exercise 4 (Solution on p. 7.)

The product is $-25a^4 - 35a^2 + 5$ and a factor is -5. Find the other factor.

Exercise 5 (Solution on p. 7.)

The product is $-a^2 + b^2$ and a factor is -1. Find the other factor.

5 Exercises

For the following problems, the first quantity represents the product and the second quantity a factor. Find the other factor.

Exercise 6 (Solution on p. 7.)

4x + 10, 2

Exercise 7

6y + 18, 3

Exercise 8 (Solution on p. 7.) 5x + 25, 5

Exercise 9

16a + 64, 8

Exercise 10 (Solution on p. 7.)

 $3a^2 + 9a$, 3a

Exercise 11

 $14b^2 + 16b$, 2b

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Exercise 12
                                                                           (Solution on p. 7.)
 21x^2 + 28x,
              7x
Exercise 13
 45y^2 + 50y, 5y
Exercise 14
                                                                           (Solution on p. 7.)
 18a^2 - 4a, 2a
Exercise 15
 20a^2 - 12a,
Exercise 16
                                                                           (Solution on p. 7.)
 7x^2 - 14x, 7x
Exercise 17
 6y^2 - 24y, 6y
Exercise 18
                                                                           (Solution on p. 7.)
 8a^2 + 4a, 4a
Exercise 19
 26b^2 + 13b, 13b
Exercise 20
                                                                           (Solution on p. 7.)
 9x^2 + 6x + 18, 6
Exercise 21
 12b^2 + 16b + 20, 4
Exercise 22
                                                                           (Solution on p. 7.)
 21x^2 + 7x - 14, 7
Exercise 23
 35x^2 + 40x - 5, 5
Exercise 24
                                                                           (Solution on p. 7.)
 14y^2 - 28y + 14, 14
Exercise 25
 36a^2 - 16a + 12, 4
Exercise 26
                                                                           (Solution on p. 7.)
 4y^2 - 10y - 12, 2
Exercise 27
 6b^2 - 6b - 3, 3
Exercise 28
                                                                           (Solution on p. 7.)
 18x^3 + 20x,
Exercise 29
 40y^3 + 24y, \quad 4y
Exercise 30
                                                                           (Solution on p. 7.)
 16x^3 - 12x^2, 4x^2
Exercise 31
 11x^3 - 11x + 11, 11
Exercise 32
                                                                           (Solution on p. 7.)
 10a^3 + 12a^2 + 16a + 8, \quad 2
Exercise 33
 14b^3 + 16b^2 + 26b + 30, 2
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(Solution on p. 7.)

Exercise 34

6 Exercises for Review

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Exercise 56
                                                                                      (Solution on p. 8.)
( here<sup>1</sup>) How many 4y^2's are there in 24x^2y^3?
Exercise 57
( here<sup>2</sup>) Find the product. (2y-3)^2.
Exercise 58
                                                                                      (Solution on p. 8.)
( here<sup>3</sup>) Solve 2(2a-1)-a=7.
Exercise 59
( here<sup>4</sup>) Given that 3m^2n is a factor of 12m^3n^4, find the other factor.
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^{1&}quot;Algebraic Expressions and Equations: Algebraic Expressions" http://cnx.org/content/m18875/latest/ 2"Algebraic Expressions and Equations: Special Binomial Products" http://cnx.org/content/m21858/latest/

³"Solving Linear Equations and Inequalities: Further Techniques in Equation Solving"

 $< \\ http://cnx.org/content/m21992/latest/> \\ \\ ^{4} \\ "Factoring Polynomials: Finding the factors of a Monomial"} < \\ http://cnx.org/content/m18870/latest/> \\ \\ \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+) \\ (+)$

Solutions to Exercises in this Module

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Solution to Exercise (p. 3)
x-2
Solution to Exercise (p. 3)
y^2 + 2y - 3
Solution to Exercise (p. 3)
x^4 - 2x^3y + 4x^2y^2 + 6y^4
Solution to Exercise (p. 3)
5a^4 + 7a^2 - 1
Solution to Exercise (p. 3)
a^2 - b^2
Solution to Exercise (p. 3)
2x + 5
Solution to Exercise (p. 3)
Solution to Exercise (p. 3)
Solution to Exercise (p. 3)
3x + 4
Solution to Exercise (p. 4)
Solution to Exercise (p. 4)
Solution to Exercise (p. 4)
2a + 1
Solution to Exercise (p. 4)
\frac{3}{2}x^2 + x + 3
Solution to Exercise (p. 4)
3x^2 + x - 2
Solution to Exercise (p. 4)
y^2 - 2y + 1
Solution to Exercise (p. 4)
2y^2 - 5y - 6
Solution to Exercise (p. 4)
9x^2 + 10
Solution to Exercise (p. 4)
4x - 3
Solution to Exercise (p. 4)
5a^3 + 6a^2 + 8a + 4
Solution to Exercise (p. 4)
2a^3 - a^2 - 3a + 4
Solution to Exercise (p. 5)
x^5 + 4x^3 - 4
Solution to Exercise (p. 5)
2x - 7
Solution to Exercise (p. 5)
5a + 2b
Solution to Exercise (p. 5)
4x^4y^4 + x + 1
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Solution to Exercise (p. 5) $8a^2bc^6 + 7ab^2c^5 - 6b^3c^4 - 1$

Solution to Exercise (p. 5) -a-2

Solution to Exercise (p. 5) -2x-3

Solution to Exercise (p. 5) -a-b

Solution to Exercise (p. 5) -a+b-c

Solution to Exercise (p. 5) a+b+c

Solution to Exercise (p. 6) $6x^2y$

Solution to Exercise (p. 6) a = 3