Factoring Polynomials: Summary of Key Concepts*

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Abstract

This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. Factoring is an essential skill for success in algebra and higher level mathematics courses. Therefore, we have taken great care in developing the student's understanding of the factorization process. The technique is consistently illustrated by displaying an empty set of parentheses and describing the thought process used to discover the terms that are to be placed inside the parentheses. The factoring scheme for special products is presented with both verbal and symbolic descriptions, since not all students can interpret symbolic descriptions alone. Two techniques, the standard "trial and error" method, and the "collect and discard" method (a method similar to the "ac" method), are presented for factoring trinomials with leading coefficients different from 1. This module provides a summary of the key concepts in the chapter "Factoring Polynomials".

1 Summary of Key Concepts

Factoring (here[1])
Factoring is the process of determining the factors of some product. Factoring is the reverse of multiplication.
Greatest Common Factor (here[2])
The **greatest common factor** of a polynomial is the factor that is common to every term of the polynomial and also is such that

1. The numerical coefficient is the largest number common to each term.
2. The variables possess the largest exponents that are common to all the variables.

Factoring a Monomial from a Polynomial (here[3])
If A is the greatest common factor of $Ax + Ay$, then

$$Ax + Ay = A(x + y)$$

Factoring by Grouping (here[4])
We are alerted to the idea of **factoring by grouping** when the polynomial we are considering

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[1]"Factoring Polynomials: Finding the factors of a Monomial" <http://cnx.org/content/m18870/latest/>
[2]"Factoring Polynomials: The Greatest Common Factor" <http://cnx.org/content/m21913/latest/>
[3]"Factoring Polynomials: The Greatest Common Factor" <http://cnx.org/content/m21913/latest/>
[4]"Factoring Polynomials: Factoring by Grouping" <http://cnx.org/content/m21901/latest/>

1. Has no factor common to all terms.
2. Has an even number of terms.

$$\begin{array}{ccccc} \underset{\substack{\texttt{[U+23B5]} \\ A \text{ is common}}}{Ax+Ay} & + & \underset{\substack{\texttt{[U+23B5]} \\ B \text{ is common}}}{Bx+By} & = & \underset{\substack{\texttt{[U+23B5]} \\ x+y \text{ is common}}}{A(x+y)+B(x+y)} \\ & & & = & (x+y)(A+B) \end{array}$$

$$\underbrace{Ax+Ay}_{A \text{ is common}} + \underbrace{Bx+By}_{B \text{ is common}} = A\underbrace{(x+y)}_{x+y \text{ is common}} + B\underbrace{(x+y)}_{x+y \text{ is common}}$$

Special products (here[5])

$$\begin{aligned} a^2-b^2 &= (a+b)(a-b) \\ a^2+2ab+b^2 &= (a+b)^2 \\ a^2-2ab+b^2 &= (a-b)^2 \end{aligned}$$

Fundamental Rule of Factoring (here[6])

1. Factor out all common monomials first.
2. Factor completely.

Factoring Trinomials (here[7], here[8])

One method of factoring a trinomial is to list all the factor pairs of both of the first and last terms and then choose the combination that when multiplied and then added produces the middle term.

[5] "Factoring Polynomials: Factoring Two Special Products" <http://cnx.org/content/m21903/latest/>
[6] "Factoring Polynomials: Factoring Two Special Products" <http://cnx.org/content/m21903/latest/>
[7] "Factoring Polynomials: Factoring Trinomials with Leading Coefficient 1" <http://cnx.org/content/m21904/latest/>
[8] "Factoring Polynomials: Factoring Trinomials with Leading Coefficient Other Than 1" <http://cnx.org/content/m21912/latest/>