Quadratic Equations: Solving Quadratic Equations Using the Method of Extraction of Roots[*]

Wade Ellis

Denny Burzynski

This work is produced by OpenStax-CNX and licensed under the
Creative Commons Attribution License 3.0[†]

Abstract

This module is from Elementary Algebra</link> by Denny Burzynski and Wade Ellis, Jr. Methods of solving quadratic equations as well as the logic underlying each method are discussed. Factoring, extraction of roots, completing the square, and the quadratic formula are carefully developed. The zero-factor property of real numbers is reintroduced. The chapter also includes graphs of quadratic equations based on the standard parabola, y = x^2, and applied problems from the areas of manufacturing, population, physics, geometry, mathematics (numbers and volumes), and astronomy, which are solved using the five-step method. Objectives of this module: be able to solve quadratic equations using the method of extraction of roots, be able to determine the nature of the solutions to a quadratic equation.

1 Overview

- The Method Of Extraction Of Roots
- The Nature Of Solutions

2 The Method Of Extraction Of Roots

Extraction of Roots

Quadratic equations of the form $x^2 - K = 0$ can be solved by **the method of extraction of roots** by rewriting it in the form $x^2 = K$.

To solve $x^2 = K$, we are required to find some number, x, that when squared produces K. This number, x, must be a square root of K. If K is greater than zero, we know that it possesses two square roots, $\sqrt{K}$ and $-\sqrt{K}$. We also know that

$$\left(\sqrt{K}\right)^2 = \left(\sqrt{K}\right)\left(\sqrt{K}\right) = K \quad \text{and} \quad \left(-\sqrt{K}\right)^2 = \left(-\sqrt{K}\right)\left(-\sqrt{K}\right) = K$$

We now have two replacements for x that produce true statements when substituted into the equation. Thus, $x = \sqrt{K}$ and $x = -\sqrt{K}$ are both solutions to $x^2 = K$. We use the notation $x = \pm\sqrt{K}$ to denote both the principal and the secondary square roots.

3 The Nature of Solutions

Solutions of $x^2 = K$
For quadratic equations of the form $x^2 = K$,

1. If K is greater than or equal to zero, the solutions are $\pm\sqrt{K}$.
2. If K is negative, no real number solutions exist.
3. If K is zero, the only solution is 0.

4 Sample Set A

Solve each of the following quadratic equations using the method of extraction of roots.

Example 1

$$
\begin{aligned}
x^2 - 49 &= 0. \qquad \text{Rewrite.}\\
x^2 &= 49\\
x &= \pm\sqrt{49}\\
x &= \pm 7
\end{aligned}
$$

$Check:$ $\quad (7)^2 = 49$ $\quad$ Is this correct? $\quad$ $(-7)^2 = 49$ $\quad$ Is this correct

$\qquad\qquad\qquad 49 = 49$ $\quad$ Yes, this is correct. $\quad$ $49 = 49$ $\quad$ Yes, this is correct.

Example 2

$$
\begin{aligned}
25a^2 &= 36\\
a^2 &= \frac{36}{25}\\
a &= \pm\sqrt{\frac{36}{25}}\\
a &= \pm\frac{6}{5}
\end{aligned}
$$

$Check:$ $\quad 25\left(\frac{6}{5}\right)^2 = 36$ Is this correct? $\qquad 25\left(\frac{-6}{5}\right)^2 = 36$ Is this correct?

$\qquad\qquad 25\left(\frac{36}{25}\right)^2 = 36$ Is this correct? $\qquad 25\left(\frac{36}{25}\right) = 36$ Is this correct?

$\qquad\qquad\qquad\qquad 36 = 36$ Yes, this is correct. $\qquad\qquad 36 = 36$ Yes, this is correct.

Example 3

$$
\begin{aligned}
4m^2 - 32 &= 0\\
4m^2 &= 32\\
m^2 &= \frac{32}{4}\\
m^2 &= 8\\
m &= \pm\sqrt{8}\\
m &= \pm 2\sqrt{2}
\end{aligned}
$$

$$
\begin{array}{rcll}
Check: \quad 4\left(2\sqrt{2}\right)^2 & = & 32 & \text{Is this correct?} \\
4\left[2^2\left(\sqrt{2}\right)^2\right] & = & 32 & \text{Is this correct?} \\
4\left[4\cdot 2\right] & = & 32 & \text{Is this correct?} \\
4\cdot 8 & = & 32 & \text{Is this correct?} \\
32 & = & 32 & \text{Yes, this is correct.}
\end{array}
\qquad
\begin{array}{rcll}
4\left(-2\sqrt{2}\right)^2 & = & 32 & \text{Is this correct?} \\
4\left[(-2)^2\left(\sqrt{2}\right)^2\right] & = & 32 & \text{Is this correct?} \\
4\left[4\cdot 2\right] & = & 32 & \text{Is this correct?} \\
4\cdot 8 & = & 32 & \text{Is this correct?} \\
32 & = & 32 & \text{Yes, this is correct.}
\end{array}
$$

Example 4

Solve $5x^2 - 15y^2z^7 = 0$ for x.

$$
\begin{array}{rcll}
5x^2 & = & 15y^2z^7 & \text{Divide both sides by 5.} \\
x^2 & = & 3y^2z^7 & \\
x & = & \pm\sqrt{3y^2z^7} & \\
x & = & \pm yz^3\sqrt{3z} &
\end{array}
$$

Example 5

⌨ Calculator problem. Solve $14a^2 - 235 = 0$. Round to the nearest hundredth.

$$
\begin{array}{rcll}
14a^2 - 235 & = & 0. & \text{Rewrite.} \\
14a^2 & = & 235 & \text{Divide both sides by 14.} \\
a^2 & = & \frac{235}{14} &
\end{array}
$$

On the Calculator

Type	235
Press	$\boxed{\div}$
Type	14
Press	$\boxed{=}$
Press	$\sqrt{}$

Display reads: 4.0970373

Rounding to the nearest hundredth produces 4.10. We must be sure to insert the $\pm$ symbol. $a \approx \pm 4.10$

Example 6

$$
\begin{array}{rcl}
k^2 & = & -64 \\
k & = & \pm\sqrt{-64}
\end{array}
$$

The radicand is negative so no real number solutions exist.

5 Practice Set A

Solve each of the following quadratic equations using the method of extraction of roots.

Exercise 1   (*Solution on p. 9.*)
$x^2 - 144 = 0$

Exercise 2   (*Solution on p. 9.*)
$9y^2 - 121 = 0$

Exercise 3   (*Solution on p. 9.*)
$6a^2 = 108$

Exercise 4 *(Solution on p. 9.)*
  Solve $4n^2 = 24m^2p^8$ for n.

Exercise 5 *(Solution on p. 9.)*
  Solve $5p^2q^2 = 45p^2$ for q.

Exercise 6 *(Solution on p. 9.)*

  Solve $16m^2 - 2206 = 0$. Round to the nearest hundredth.

Exercise 7
  $h^2 = -100$

6 Sample Set B

Solve each of the following quadratic equations using the method of extraction of roots.

Example 7

$$\begin{aligned}
(x+2)^2 &= 81 \\
x+2 &= \pm\sqrt{81} \\
x+2 &= \pm 9 \qquad\qquad\qquad \text{Subtract } 2 \text{ from both sides.} \\
x &= -2 \pm 9 \\
x &= -2 + 9 \quad \text{and} \quad x = -2 - 9 \\
x &= 7 \qquad\qquad\qquad\quad x = -11
\end{aligned}$$

Example 8

$$\begin{aligned}
(a+3)^2 &= 5 \\
a+3 &= \pm\sqrt{5} \qquad\quad \text{Subtract } 3 \text{ from both sides.} \\
a &= -3 \pm \sqrt{5}
\end{aligned}$$

7 Practice Set B

Solve each of the following quadratic equations using the method of extraction of roots.

Exercise 8 *(Solution on p. 9.)*
  $(a+6)^2 = 64$

Exercise 9 *(Solution on p. 9.)*
  $(m-4)^2 = 15$

Exercise 10 *(Solution on p. 9.)*
  $(y-7)^2 = 49$

Exercise 11 *(Solution on p. 9.)*
  $(k-1)^2 = 12$

Exercise 12 *(Solution on p. 9.)*
  $(x-11)^2 = 0$

8 Exercises

For the following problems, solve each of the quadratic equations using the method of extraction of roots.

Exercise 13   *(Solution on p. 9.)*
$$x^2 = 36$$

Exercise 14
$$x^2 = 49$$

Exercise 15   *(Solution on p. 9.)*
$$a^2 = 9$$

Exercise 16
$$a^2 = 4$$

Exercise 17   *(Solution on p. 9.)*
$$b^2 = 1$$

Exercise 18
$$a^2 = 1$$

Exercise 19   *(Solution on p. 9.)*
$$x^2 = 25$$

Exercise 20
$$x^2 = 81$$

Exercise 21   *(Solution on p. 9.)*
$$a^2 = 5$$

Exercise 22
$$a^2 = 10$$

Exercise 23   *(Solution on p. 9.)*
$$b^2 = 12$$

Exercise 24
$$b^2 = 6$$

Exercise 25   *(Solution on p. 9.)*
$$y^2 = 3$$

Exercise 26
$$y^2 = 7$$

Exercise 27   *(Solution on p. 9.)*
$$a^2 - 8 = 0$$

Exercise 28
$$a^2 - 3 = 0$$

Exercise 29   *(Solution on p. 9.)*
$$a^2 - 5 = 0$$

Exercise 30
$$y^2 - 1 = 0$$

Exercise 31   *(Solution on p. 9.)*
$$x^2 - 10 = 0$$

Exercise 32
$$x^2 - 11 = 0$$

Exercise 33   *(Solution on p. 9.)*
$$3x^2 - 27 = 0$$

Exercise 34
$5b^2 - 5 = 0$

Exercise 35 *(Solution on p. 9.)*
$2x^2 = 50$

Exercise 36
$4a^2 = 40$

Exercise 37 *(Solution on p. 9.)*
$2x^2 = 24$

For the following problems, solve for the indicated variable.

Exercise 38
$x^2 = 4a^2$, for x

Exercise 39 *(Solution on p. 9.)*
$x^2 = 9b^2$, for x

Exercise 40
$a^2 = 25c^2$, for a

Exercise 41 *(Solution on p. 10.)*
$k^2 = m^2n^2$, for k

Exercise 42
$k^2 = p^2q^2r^2$, for k

Exercise 43 *(Solution on p. 10.)*
$2y^2 = 2a^2n^2$, for y

Exercise 44
$9y^2 = 27x^2z^4$, for y

Exercise 45 *(Solution on p. 10.)*
$x^2 - z^2 = 0$, for x

Exercise 46
$x^2 - z^2 = 0$, for z

Exercise 47 *(Solution on p. 10.)*
$5a^2 - 10b^2 = 0$, for a

For the following problems, solve each of the quadratic equations using the method of extraction of roots.

Exercise 48
$(x - 1)^2 = 4$

Exercise 49 *(Solution on p. 10.)*
$(x - 2)^2 = 9$

Exercise 50
$(x - 3)^2 = 25$

Exercise 51 *(Solution on p. 10.)*
$(a - 5)^2 = 36$

Exercise 52
$(a + 3)^2 = 49$

Exercise 53 *(Solution on p. 10.)*
$(a + 9)^2 = 1$

Exercise 54
$(a - 6)^2 = 3$

Exercise 55   *(Solution on p. 10.)*
$(x + 4)^2 = 5$

Exercise 56
$(b + 6)^2 = 7$

Exercise 57   *(Solution on p. 10.)*
$(x + 1)^2 = a$, for x

Exercise 58
$(y + 5)^2 = b$, for y

Exercise 59   *(Solution on p. 10.)*
$(y + 2)^2 = a^2$, for y

Exercise 60
$(x + 10)^2 = c^2$, for x

Exercise 61   *(Solution on p. 10.)*
$(x - a)^2 = b^2$, for x

Exercise 62
$(x + c)^2 = a^2$, for x

8.1 ▦ Calculator Problems

For the following problems, round each result to the nearest hundredth.

Exercise 63   *(Solution on p. 10.)*
$8a^2 - 168 = 0$

Exercise 64
$6m^2 - 5 = 0$

Exercise 65   *(Solution on p. 10.)*
$0.03y^2 = 1.6$

Exercise 66
$0.048x^2 = 2.01$

Exercise 67   *(Solution on p. 10.)*
$1.001x^2 - 0.999 = 0$

9 Exercises For Review

Exercise 68
(here[1]) Graph the linear inequality $3(x + 2) < 2(3x + 4)$.

Exercise 69   *(Solution on p. 10.)*
(here[2]) Solve the fractional equation $\frac{x-1}{x+4} = \frac{x+3}{x-1}$.

---

[1]"Graphing Linear Equations and Inequalities: Graphing Linear Equations and Inequalities in One Variable"
<http://cnx.org/content/m18877/latest/>
[2]"Rational Expressions: Rational Equations" <http://cnx.org/content/m21951/latest/>

Exercise 70
(here[3]) Find the product: $\sqrt{32x^3y^5}\sqrt{2x^3y^3}$.

Exercise 71 *(Solution on p. 10.)*
(here[4]) Solve $x^2 - 4x = 0$.

Exercise 72
(here[5]) Solve $y^2 - 8y = -12$.

---

[3]"Roots, Radicals, and Square Root Equations: Multiplication of Square Root Expressions"
<http://cnx.org/content/m21975/latest/>
[4]"Quadratic Equations: Solving Quadratic Equations by Factoring" <http://cnx.org/content/m21933/latest/>
[5]"Quadratic Equations: Solving Quadratic Equations by Factoring" <http://cnx.org/content/m21933/latest/>

Solutions to Exercises in this Module

Solution to Exercise (p. 3)
$x = \pm 12$
Solution to Exercise (p. 3)
$y = \pm \frac{11}{3}$
Solution to Exercise (p. 3)
$a = \pm 3\sqrt{2}$
Solution to Exercise (p. 3)
$n = \pm m p^4 \sqrt{6}$
Solution to Exercise (p. 4)
$q = \pm 3$
Solution to Exercise (p. 4)
$m = \pm 11.74$
Solution to Exercise (p. 4)
$a = 2, -14$
Solution to Exercise (p. 4)
$m = 4 \pm \sqrt{15}$
Solution to Exercise (p. 4)
$y = 0, 14$
Solution to Exercise (p. 4)
$k = 1 \pm 2\sqrt{3}$
Solution to Exercise (p. 4)
$x = 11$
Solution to Exercise (p. 5)
$x = \pm 6$
Solution to Exercise (p. 5)
$a = \pm 3$
Solution to Exercise (p. 5)
$b = \pm 1$
Solution to Exercise (p. 5)
$x = \pm 5$
Solution to Exercise (p. 5)
$a = \pm \sqrt{5}$
Solution to Exercise (p. 5)
$b = \pm 2\sqrt{3}$
Solution to Exercise (p. 5)
$y = \pm \sqrt{3}$
Solution to Exercise (p. 5)
$a = \pm 2\sqrt{2}$
Solution to Exercise (p. 5)
$a = \pm \sqrt{5}$
Solution to Exercise (p. 5)
$x = \pm \sqrt{10}$
Solution to Exercise (p. 5)
$x = \pm 3$
Solution to Exercise (p. 6)
$x = \pm 5$
Solution to Exercise (p. 6)
$x = \pm 2\sqrt{3}$

Solution to Exercise (p. 6)
$x = \pm 3b$
Solution to Exercise (p. 6)
$k = \pm mn$
Solution to Exercise (p. 6)
$y = \pm an$
Solution to Exercise (p. 6)
$x = \pm z$
Solution to Exercise (p. 6)
$a = b\sqrt{2}, -b\sqrt{2}$
Solution to Exercise (p. 6)
$x = 5, -1$
Solution to Exercise (p. 6)
$x = 11, -1$
Solution to Exercise (p. 6)
$a = -8, -10$
Solution to Exercise (p. 6)
$a = -4 \pm \sqrt{5}$
Solution to Exercise (p. 7)
$x = -1 \pm \sqrt{a}$
Solution to Exercise (p. 7)
$y = -2 \pm a$
Solution to Exercise (p. 7)
$x = a \pm b$
Solution to Exercise (p. 7)
$a = \pm 4.58$
Solution to Exercise (p. 7)
$y = \pm 7.30$
Solution to Exercise (p. 7)
$x = \pm 1.00$
Solution to Exercise (p. 7)
$x = \frac{-11}{9}$
Solution to Exercise (p. 8)
$x = 0, 4$