Quadratic Equations: Solving Quadratic Equations by Factoring*

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Abstract

This module is from Elementary Algebra</link> by Denny Burzynski and Wade Ellis, Jr. Methods of solving quadratic equations as well as the logic underlying each method are discussed. Factoring, extraction of roots, completing the square, and the quadratic formula are carefully developed. The zero-factor property of real numbers is reintroduced. The chapter also includes graphs of quadratic equations based on the standard parabola, y = x^2, and applied problems from the areas of manufacturing, population, physics, geometry, mathematics (numbers and volumes), and astronomy, which are solved using the five-step method. Objectives of this module: be able to solve quadratic equations by factoring.

1 Overview

- Factoring Method
- Solving Mentally After Factoring

2 Factoring Method

To solve quadratic equations by factoring, we must make use of the zero-factor property.

1. Set the equation equal to zero, that is, get all the nonzero terms on one side of the equal sign and 0 on the other.

   $ax^2 + bx + c = 0$
2. Factor the quadratic expression.

   $() () = 0$
3. By the zero-factor property, at least one of the factors must be zero, so, set each of the factors equal to 0 and solve for the variable.

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3 Sample Set A

Solve the following quadratic equations. (We will show the check for problem 1.)

Example 1

$$\begin{aligned} x^2 - 7x + 12 &= 0. && \text{The equation is already set equal to 0. Factor.} \\ (x-3)(x-4) &= 0 && \text{Set each factor equal to 0.} \\ x - 3 &= 0 \quad \text{or} \quad x - 4 = 0 \\ x &= 3 \quad \text{or} \quad x = 4 \end{aligned}$$

$$\begin{aligned} Check: \text{ If } x = 3,\ x^2 - 7x &+ 12 = 0 \\ 3^2 - 7 \cdot 3 &+ 12 = 0 \quad \text{Is this correct?} \\ 9 - 21 &+ 12 = 0 \quad \text{Is this correct?} \\ 0 &= 0 \quad \text{Yes, this is correct.} \\ Check: \text{ If } x = 4,\ x^2 - 7x &+ 12 = 0 \\ 4^2 - 7 \cdot 4 &+ 12 = 0 \quad \text{Is this correct?} \\ 16 - 28 &+ 12 = 0 \quad \text{Is this correct?} \\ 0 &= 0 \quad \text{Yes, this is correct.} \end{aligned}$$

Thus, the solutions to this equation are $x = 3,\ 4$.

Example 2

$$\begin{aligned} x^2 &= 25. && \text{Set the equation equal to 0.} \\ x^2 - 25 &= 0 && \text{Factor.} \\ (x+5)(x-5) &= 0 && \text{Set each factor equal to 0.} \\ x + 5 = 0 \quad &\text{or} \quad x - 5 = 0 \\ x = -5 \quad &\text{or} \quad x = 5 \end{aligned}$$

Thus, the solutions to this equation are $x = 5, -5$.

Example 3

$$\begin{aligned} x^2 &= 2x. && \text{Set the equation equal to 0.} \\ x^2 - 2x &= 0 && \text{Factor.} \\ x(x-2) \quad & && \text{Set each factor equal to 0.} \\ x = 0 \quad &\text{or} \quad x - 2 = 0 \\ & \quad\ \ x = 2 \end{aligned}$$

Thus, the solutions to this equation are $x = 0,\ 2$.

Example 4

$$\begin{array}{lcll} 2x^2+7x-15 & = & 0. & \text{Factor.} \\ (2x-3)(x+5) & = & 0 & \text{Set each factor equal to 0.} \\ 2x-3=0 & \text{or} & x+5=0 & \\ 2x=3 & \text{or} & x=-5 & \\ x=\frac{3}{2} & & & \end{array}$$

Thus, the solutions to this equation are $x=\frac{3}{2},-5$.

Example 5
$63x^2=13x+6$

$$\begin{array}{lcl} 63x^2-13x-6 & = & 0 \\ (9x+2)(7x-3) & = & 0 \\ 9x+2=0 & \text{or} & 7x-3=0 \\ 9x=-2 & \text{or} & 7x=3 \\ x=\frac{-2}{9} & \text{or} & x=\frac{3}{7} \end{array}$$

Thus, the solutions to this equation are $x=\frac{-2}{9},\frac{3}{7}$.

4 Practice Set A

Solve the following equations, if possible.

Exercise 1 *(Solution on p. 9.)*
$(x-7)(x+4)=0$

Exercise 2 *(Solution on p. 9.)*
$(2x+5)(5x-7)=0$

Exercise 3 *(Solution on p. 9.)*
$x^2+2x-24=0$

Exercise 4 *(Solution on p. 9.)*
$6x^2+13x-5=0$

Exercise 5 *(Solution on p. 9.)*
$5y^2+2y=3$

Exercise 6 *(Solution on p. 9.)*
$m(2m-11)=0$

Exercise 7 *(Solution on p. 9.)*
$6p^2=-(5p+1)$

Exercise 8 *(Solution on p. 9.)*
$r^2-49=0$

5 Solving Mentally After Factoring

Let's consider problems 4 and 5 of Sample Set A in more detail. Let's look particularly at the factorizations $(2x-3)(x+5)=0$ and $(9x+2)(7x-3)=0$. The next step is to set each factor equal to zero and solve. We can solve mentally if we understand how to solve linear equations: we transpose the constant from the variable term and then divide by the coefficient of the variable.

6 Sample Set B

Example 6
Solve the following equation mentally.
$(2x-3)(x+5)=0$

$2x-3$	$=$	0	Mentally add 3 to both sides. The constant changes sign.
$2x$	$=$	3	Divide by 2, the coefficient of x. The 2 divides the constant 3 into $\frac{3}{2}$. The coefficient becomes the denominator.
x	$=$	$\frac{3}{2}$	
$x+5$	$=$	0	Mentally subtract 5 from both sides. The constant changes sign.
x	$=$	-5	Divide by the coefficient of x, 1. The coefficient becomes the denominator.
$x=\frac{-5}{1}$	$=$	-5	
x	$=$	-5	

Now, we can immediately write the solution to the equation after factoring by looking at each factor, changing the sign of the constant, then dividing by the coefficient.

7 Practice Set B

Exercise 9 *(Solution on p. 9.)*
Solve $(9x+2)(7x-3)=0$ using this mental method.

8 Exercises

For the following problems, solve the equations, if possible.

Exercise 10 *(Solution on p. 9.)*
$(x+1)(x+3)=0$

Exercise 11
$(x+4)(x+9)=0$

Exercise 12 *(Solution on p. 9.)*
$(x-5)(x-1)=0$

Exercise 13
$(x-6)(x-3)=0$

Exercise 14 *(Solution on p. 9.)*
$(x-4)(x+2)=0$

Exercise 15
$(x+6)(x-1)=0$

Exercise 16 *(Solution on p. 9.)*
$(2x+1)(x-7)=0$

Exercise 17
$(3x+2)(x-1)=0$

Exercise 18 *(Solution on p. 9.)*
$(4x+3)(3x-2)=0$

Exercise 19
$(5x-1)(4x+7)=0$

Exercise 20 *(Solution on p. 9.)*
$(6x+5)(9x-4)=0$

Exercise 21
$(3a+1)(3a-1)=0$

Exercise 22 *(Solution on p. 9.)*
$x(x+4)=0$

Exercise 23
$y(y-5)=0$

Exercise 24 *(Solution on p. 9.)*
$y(3y-4)=0$

Exercise 25
$b(4b+5)=0$

Exercise 26 *(Solution on p. 9.)*
$x(2x+1)(2x+8)=0$

Exercise 27
$y(5y+2)(2y-1)=0$

Exercise 28 *(Solution on p. 9.)*
$(x-8)^2=0$

Exercise 29
$(x-2)^2=0$

Exercise 30 *(Solution on p. 9.)*
$(b+7)^2=0$

Exercise 31
$(a+1)^2=0$

Exercise 32 *(Solution on p. 9.)*
$x(x-4)^2=0$

Exercise 33
$y(y+9)^2=0$

Exercise 34 *(Solution on p. 9.)*
$y(y-7)^2=0$

Exercise 35
$y(y+5)^2=0$

Exercise 36 *(Solution on p. 9.)*
$x^2-4=0$

Exercise 37
$x^2+9=0$

Exercise 38 *(Solution on p. 9.)*
$x^2 + 36 = 0$

Exercise 39
$x^2 - 25 = 0$

Exercise 40 *(Solution on p. 9.)*
$a^2 - 100 = 0$

Exercise 41
$a^2 - 81 = 0$

Exercise 42 *(Solution on p. 10.)*
$b^2 - 49 = 0$

Exercise 43
$y^2 - 1 = 0$

Exercise 44 *(Solution on p. 10.)*
$3a^2 - 75 = 0$

Exercise 45
$5b^2 - 20 = 0$

Exercise 46 *(Solution on p. 10.)*
$y^3 - y = 0$

Exercise 47
$a^2 = 9$

Exercise 48 *(Solution on p. 10.)*
$b^2 = 4$

Exercise 49
$b^2 = 1$

Exercise 50 *(Solution on p. 10.)*
$a^2 = 36$

Exercise 51
$3a^2 = 12$

Exercise 52 *(Solution on p. 10.)*
$-2x^2 = -4$

Exercise 53
$-2a^2 = -50$

Exercise 54 *(Solution on p. 10.)*
$-7b^2 = -63$

Exercise 55
$-2x^2 = -32$

Exercise 56 *(Solution on p. 10.)*
$3b^2 = 48$

Exercise 57
$a^2 - 8a + 16 = 0$

Exercise 58 *(Solution on p. 10.)*
$y^2 + 10y + 25 = 0$

Exercise 59
$y^2 + 9y + 16 = 0$

Exercise 60 *(Solution on p. 10.)*
$x^2 - 2x - 1 = 0$

Exercise 61
$a^2 + 6a + 9 = 0$

Exercise 62 *(Solution on p. 10.)*
$a^2 + 4a + 4 = 0$

Exercise 63
$x^2 + 12x = -36$

Exercise 64 *(Solution on p. 10.)*
$b^2 - 14b = -49$

Exercise 65
$3a^2 + 18a + 27 = 0$

Exercise 66 *(Solution on p. 10.)*
$2m^3 + 4m^2 + 2m = 0$

Exercise 67
$3mn^2 - 36mn + 36m = 0$

Exercise 68 *(Solution on p. 10.)*
$a^2 + 2a - 3 = 0$

Exercise 69
$a^2 + 3a - 10 = 0$

Exercise 70 *(Solution on p. 10.)*
$x^2 + 9x + 14 = 0$

Exercise 71
$x^2 - 7x + 12 = 3$

Exercise 72 *(Solution on p. 10.)*
$b^2 + 12b + 27 = 0$

Exercise 73
$b^2 - 3b + 2 = 0$

Exercise 74 *(Solution on p. 10.)*
$x^2 - 13x = -42$

Exercise 75
$a^3 = -8a^2 - 15a$

Exercise 76 *(Solution on p. 10.)*
$6a^2 + 13a + 5 = 0$

Exercise 77
$6x^2 - 4x - 2 = 0$

Exercise 78 *(Solution on p. 10.)*
$12a^2 + 15a + 3 = 0$

Exercise 79
$18b^2 + 24b + 6 = 0$

Exercise 80 *(Solution on p. 10.)*
$12a^2 + 24a + 12 = 0$

Exercise 81
$4x^2 - 4x = -1$

Exercise 82 *(Solution on p. 10.)*
$2x^2 = x + 15$

Exercise 83
$4a^2 = 4a + 3$

Exercise 84 *(Solution on p. 10.)*
$4y^2 = -4y - 2$

Exercise 85
$9y^2 = 9y + 18$

9 Exercises For Review

Exercise 86 *(Solution on p. 10.)*
(**here**[1]) Simplify $\left(x^4y^3\right)^2\left(xy^2\right)^4$.

Exercise 87
(**here**[2]) Write $\left(x^{-2}y^3w^4\right)^{-2}$ so that only positive exponents appear.

Exercise 88 *(Solution on p. 10.)*
(**here**[3]) Find the sum: $\frac{x}{x^2-x-2} + \frac{1}{x^2-3x+2}$.

Exercise 89
(**here**[4]) Simplify $\frac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{a}-\frac{1}{b}}$.

Exercise 90 *(Solution on p. 10.)*
(**here**[5]) Solve $(x+4)(3x+1) = 0$.

[1] "Basic Properties of Real Numbers: The Power Rules for Exponents" <http://cnx.org/content/m21897/latest/>
[2] "Basic Operations with Real Numbers: Negative Exponents" <http://cnx.org/content/m21882/latest/>
[3] "Rational Expressions: Adding and Subtracting Rational Expressions" <http://cnx.org/content/m21936/latest/>
[4] "Rational Expressions: Complex Rational Expressions" <http://cnx.org/content/m21944/latest/>
[5] "Quadratic Equations: Solving Quadratic Equations" <http://cnx.org/content/m18880/latest/>

Solutions to Exercises in this Module

Solution to Exercise (p. 3)
$x = 7,\ -4$
Solution to Exercise (p. 3)
$x = \frac{-5}{2}, \frac{7}{5}$
Solution to Exercise (p. 3)
$x = 4,\ -6$
Solution to Exercise (p. 3)
$x = \frac{1}{3}, \frac{-5}{2}$
Solution to Exercise (p. 3)
$y = \frac{3}{5}, -1$
Solution to Exercise (p. 3)
$m = 0, \frac{11}{2}$
Solution to Exercise (p. 3)
$p = \frac{-1}{3}, \frac{-1}{2}$
Solution to Exercise (p. 3)
$r = 7, -7$
Solution to Exercise (p. 4)
$x = -\frac{2}{9}, \frac{3}{7}$
Solution to Exercise (p. 4)
$x = -1,\ -3$
Solution to Exercise (p. 4)
$x = 1,\ 5$
Solution to Exercise (p. 4)
$x = -2,\ 4$
Solution to Exercise (p. 5)
$x = -\frac{1}{2},\ 7$
Solution to Exercise (p. 5)
$x = -\frac{3}{4},\ \frac{2}{3}$
Solution to Exercise (p. 5)
$x = -\frac{5}{6},\ \frac{4}{9}$
Solution to Exercise (p. 5)
$x = -4,\ 0$
Solution to Exercise (p. 5)
$y = 0,\ \frac{4}{3}$
Solution to Exercise (p. 5)
$x = -4,\ -\frac{1}{2},\ 0$
Solution to Exercise (p. 5)
$x = 8$
Solution to Exercise (p. 5)
$b = -7$
Solution to Exercise (p. 5)
$x = 0,\ 4$
Solution to Exercise (p. 5)
$y = 0,\ 7$
Solution to Exercise (p. 5)
$x = -2,\ 2$
Solution to Exercise (p. 5)
no solution

Solution to Exercise (p. 6)
$a = -10,\ 10$
Solution to Exercise (p. 6)
$b = 7,\ -7$
Solution to Exercise (p. 6)
$a = 5,\ -5$
Solution to Exercise (p. 6)
$y = 0,\ 1,\ -1$
Solution to Exercise (p. 6)
$b = 2,\ -2$
Solution to Exercise (p. 6)
$a = 6,\ -6$
Solution to Exercise (p. 6)
$x = \sqrt{2},\ -\sqrt{2}$
Solution to Exercise (p. 6)
$b = 3,\ -3$
Solution to Exercise (p. 6)
$b = 4,\ -4$
Solution to Exercise (p. 6)
$y = -5$
Solution to Exercise (p. 6)
no solution
Solution to Exercise (p. 7)
$a = -2$
Solution to Exercise (p. 7)
$b = 7$
Solution to Exercise (p. 7)
$m = 0,\ -1$
Solution to Exercise (p. 7)
$a = -3,\ 1$
Solution to Exercise (p. 7)
$x = -7,\ -2$
Solution to Exercise (p. 7)
$b = -9,\ -3$
Solution to Exercise (p. 7)
$x = 6,\ 7$
Solution to Exercise (p. 7)
$a = -\frac{5}{3},\ -\frac{1}{2}$
Solution to Exercise (p. 7)
$a = -\frac{1}{4},\ -1$
Solution to Exercise (p. 7)
$a = -1$
Solution to Exercise (p. 7)
$x = -\frac{5}{2},\ 3$
Solution to Exercise (p. 8)
no solution
Solution to Exercise (p. 8)
$x^{12}y^{14}$
Solution to Exercise (p. 8)
$\frac{x^2+1}{(x+1)(x-1)(x-2)}$

Solution to Exercise (p. 8)

$x = -4, \ \frac{-1}{3}$