RATIONAL EXPRESSIONS: BUILDING RATIONAL EXPRESSIONS AND THE LCD^*

Wade Ellis

Denny Burzynski

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Abstract

<para>This module is from <link document="col10614">Elementary Algebra</link> by Denny Burzynski and Wade Ellis, Jr.</para> <para>A detailed study of arithmetic operations with rational expressions is presented in this chapter, beginning with the definition of a rational expression and then proceeding immediately to a discussion of the domain. The process of reducing a rational expression and illustrations of multiplying, dividing, adding, and subtracting rational expressions are also included. Since the operations of addition and subtraction can cause the most difficulty, they are given particular attention. We have tried to make the written explanation of the examples clearer by using a "freeze frame" approach, which walks the student through the operation step by step.</para> <para>The five-step method of solving applied problems is included in this chapter to show the problem-solving approach to number problems, work problems, and geometry problems. The chapter also illustrates simplification of complex rational expressions, using the combine-divide method and the LCD-multiply-divide method.</para> <para>Objectives of this module: understand and be able to use the process of building rational expressions and know why it is often necessary to build them, be able to find the LCD

1 Overview

- The Process
- The Reason For Building Rational Expressions
- The Least Common Denominator (LCD)

2 The Process

Recall, from Section here¹, the equality property of fractions.

Equality Property Of Fractions

If $\frac{a}{b} = \frac{c}{d}$, then ad = bc.

Using the fact that $1 = \frac{b}{b}, b \neq 0$, and that 1 is the multiplicative identity, it follows that if $\frac{P}{Q}$ is a rational expression, then

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 $^{^{-1}&}quot;Rational \ Expressions: \ Rational \ Expressions" \ < http://cnx.org/content/m18878/latest/> \\$

 $\frac{P}{Q} \cdot \frac{b}{b} = \frac{Pb}{Qb}, \quad b \neq 0$ This equation asserts that a rational expression can be transformed into an equivalent rational expression by multiplying both the numerator and denominator by the same nonzero number.

Process of Building Rational Expressions

This process is known as the process of **building rational expressions** and it is exactly the opposite of reducing rational expressions. The process is shown in these examples:

Example 1

 $\frac{3}{4}$ can be built to $\frac{12}{16}$ since

 $\frac{3}{4} \cdot 1 = \frac{3}{4} \cdot \frac{4}{4} = \frac{3 \cdot 4}{4 \cdot 4} = \frac{12}{16}$ Example 2 $\frac{-4}{5}$ can be built to $\frac{-8}{10}$ since $\frac{-4}{5} \cdot 1 = \frac{-4}{5} \cdot \frac{2}{2} = \frac{-4 \cdot 2}{5 \cdot 2} = \frac{-8}{10}$ $\mathbf{Example} \ \mathbf{3}$ $\frac{3}{7}$ can be built to $\frac{3xy}{7xy}$ since $\frac{3}{7} \cdot 1 = \frac{3}{7} \cdot \frac{xy}{xy} = \frac{3xy}{7xy}$

Example 4

 $\frac{4a}{3b}$ can be built to $\frac{4a^2(a+1)}{3ab(a+1)}$ since

 $\frac{4a}{3b} \cdot 1 = \frac{4a}{3b} \cdot \frac{a(a+1)}{a(a+1)} = \frac{4a^2(a+1)}{3ab(a+1)}$

Suppose we're given a rational expression $\frac{P}{Q}$ and wish to build it into a rational expression with denominator Qb^2 , that is,

 $\frac{P}{Q} \rightarrow \frac{?}{Ob^2}$

Since we changed the denominator, we must certainly change the numerator in the same way. To determine how to change the numerator we need to know how the denominator was changed. Since one rational expression is built into another equivalent expression by multiplication by 1, the first denominator must have been multiplied by some quantity. Observation of

$$\frac{P}{Q} \rightarrow \frac{?}{Qb^2}$$

tells us that Q was multiplied by b^2 . Hence, we must multiply the numerator P by b^2 . Thus,

 $\frac{P}{Q} = \frac{Pb^2}{Qb^2}$ Quite often a simple comparison of the original denominator with the new denominator will tell us the factor being used. However, there will be times when the factor is unclear by simple observation. We need a method for finding the factor.

Observe the following examples; then try to speculate on the method.

Example 5 $\frac{3}{4} = \frac{?}{20}$.

The original denominator 4 was multiplied by 5 to yield 20. What arithmetic process will yield 5 using 4 and 20?

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$$\frac{s}{10} = \frac{1}{10y}$$

The original denominator 10 was multiplied by y to yield 10y.

Example 7
$$\frac{-6xy}{2a^3b} = \frac{?}{16a^5b^3}$$

The original denominator $2a^3b$ was multiplied by $8a^2b^2$ to yield $16a^5b^3$.

Example 8 $\frac{5ax}{(a+1)^2} = \frac{?}{4(a+1)^2(a-2)}.$

The original denominator $(a + 1)^2$ was multiplied by 4(a - 2) to yield $4(a + 1)^2(a - 2)$.

To determine the quantity that the original denominator was multiplied by to yield the new denominator, we ask, "What did I multiply the original denominator by to get the new denominator?" We find this factor by dividing the original denominator into the new denominator.

It is precisely this quantity that we multiply the numerator by to build the rational expression.

3 Sample Set A

Determine \mathbf{N} in each of the following problems.

Example 9

$\frac{8}{3} = \frac{N}{15}.$	${ m The original denominator is 3 and the new}$
	${\rm denominato}{\rm is}15.{\rm Divide}{\rm the}{\rm original}$
	${ m denominatorintothenewdenominatorand}$
	$\mathrm{multiply}\mathrm{the}\mathrm{numerator}8\mathrm{by}\mathrm{this}\mathrm{result}.$
	$15 \div 3 = 5$
	Then, $8 \cdot 5 = 40$. So,
$\frac{8}{2} = \frac{40}{15}$ and $N = 40$.	

 $\frac{3}{3} = \frac{1}{15}$ and $1^{v} = 40$. Check by reducing $\frac{40}{15}$.

Example 10

Example 11

$$\frac{-6a}{a+2} = \frac{N}{(a+2)(a-7)}.$$
 The new denominator divided by the original denominator is

$$\frac{(a+2)(a-7)}{a+2} = a - 7$$
Multiply $-6a$ by $a - 7$.
 $-6a (a - 7) = -6a^2 + 42a$

$$\frac{-6a}{a+2} = \frac{-6a^2 + 42a}{(a+2)(a-7)} \text{ and } N = -6a^2 + 42a.$$

Example 12

$$\frac{-3(a-1)}{a-4} = \frac{N}{a^2-16}.$$
 The new denominator divided by the original denominator is

$$\frac{a^2-16}{a-4} = \frac{(a+4)\overline{)(a-4)}}{\overline{)a-4}}$$

$$= a+4$$
Multiply $-3(a-1)$ by $a+4$.
 $-3(a-1)(a+4) = -3(a^2+3a-4)$
 $= -3a^2-9a+12$

$$\frac{-3(a-1)}{a-4} = \frac{-3a^2-9a+12}{a^2-16}$$
 and $N = -3a^2-9a+12$

Example 13

7x	=	$\frac{N}{x^2y^3}$.	Write $7x \operatorname{as} \frac{7x}{1}$.
			Now we can see clearly that the original denominator
$\frac{7x}{1}$	=	$\frac{N}{x^2y^3}$	1 was multiplied by x^2y^3 . We need to multiply the
			numerator $7x$ by x^2y^3 .
7x	=	$\frac{7x \cdot x^2 y^3}{x^2 y^3}$	
7x	=	$\frac{7x^3y^3}{x^2y^3}$ and $N = 7x^3y^3$.	

Example 14

$$\frac{5x}{x+3} = \frac{5x^2 - 20x}{N}.$$
The same process works in this case. Divide the original numerator $5x$ into the new numerator $5x^2 - 20x$.

$$\frac{5x^2 - 20x}{5x} = \frac{\overline{)5x}(x-4)}{\overline{)5x}}$$

$$= x - 4$$

Multiply the denominator by x - 4.

$$(x+3)(x-4)$$

 $\frac{5x}{x+3} = \frac{5x^2-20}{(x+3)(x-4)}$ and $N = 5x^2 - 20$.

Example 15

$$\frac{4x}{3-x} = \frac{N}{x-3}.$$
The two denominators have nearly the same terms; each has
the opposite sign. Factor - 1 from the original denominator.
$$3 - x = -1 (-3 + x)$$
$$= - (x - 3)$$
$$\frac{4x}{3-x} = \frac{4x}{-(x-3)} = \frac{-4x}{x-3} \text{ and } N = -4x.$$

It is important to note that we **factored**-1 from the original denominator. We **did not** multiply it by -1. Had we multiplied only the denominator by -1 we would have had to multiply the numerator by -1 also.

4 Practice Set A

Determine N .	
Exercise 1	(Solution on p. 14.)
$\frac{3}{8} = \frac{N}{48}$	
$\mathbf{Exercise}_{N}^{2}$	(Solution on p. 14.)
$\frac{9a}{5b} = \frac{18}{35b^2x^3}$	
Exercise 3	(Solution on p. 14.)
$\frac{-2y}{y-1} = \frac{1}{y^2-1}$	
Exercise 4 N	(Solution on p. 14.)
$\frac{a+1}{a-5} = \frac{1}{a^2-3a-10}$	
Exercise 5 $\Lambda = \frac{N}{N}$	(Solution on p. 14.)
$4a = \frac{6a^{3}(a-1)}{6a^{3}(a-1)}$	
Exercise 6 N	(Solution on p. 14.)
$-2x = \frac{1}{8x^3y^3z^5}$	
$\begin{array}{ccc} \textbf{Exercise 7} \\ 6ab \ _ & N \end{array}$	(Solution on p. 14.)
$\frac{\overline{b+3}}{\overline{b+3}} = \frac{\overline{b^2+6b+9}}{\overline{b+3}}$	
Exercise 8 $3m 3m^2 - 18m$	(Solution on p. 14.)
$\frac{1}{m+5} = \frac{1}{N}$	
Exercise 9 $-2r^2$ $-2r^3+8r^2$	(Solution on p. 14.)
$\frac{1}{r-3} = \frac{1}{N}$	
Exercise 10 $-8ab^2$ N	(Solution on p. 14.)
$\overline{a-4} = \overline{4-a}$	

5 The Reason For Building Rational Expressions

Building Rational Expressions

Normally, when we write a rational expression, we write it in reduced form. The reason for building rational expressions is to make addition and subtraction of rational expressions convenient (simpler).

To add or subtract two or more rational expressions they must have the same denominator.

Building rational expressions allows us to transform fractions into fractions with the same denominators (which we can then add or subtract). The most convenient new denominator is the **least common denominator** (LCD) of the given fractions.

6 The Least Common Denominator (LCD)

In arithmetic, the **least common denominator** is the smallest (least) quantity that each of the given denominators will divide into without a remainder. For algebraic expressions, the LCD is the polynomial of **least degree** divisible by each denominator. Some examples are shown below.

Example 16 $\frac{3}{4}, \frac{1}{6}, \frac{5}{12}$.

The LCD is 12 since 12 is the smallest number that 4, 6, and 12 will divide into without a remainder.

Example 17 $\frac{1}{3}, \frac{5}{6}, \frac{5}{8}, \frac{7}{12}$.

The LCD is 24 since 24 is the smallest number that 3, 6, 8, and 12 will divide into without a remainder.

Example 18 $\frac{2}{r}, \frac{3}{r^2}$.

The LCD is x^2 since x^2 is the smallest quantity that x and x^2 will divide into without a remainder.

Example 19

 $\frac{5a}{6a^2b}, \frac{3a}{8ab^3}.$

The LCD is $24a^2b^3$ since $24a^2b^3$ is the smallest quantity that $6a^2b$ and $8ab^3$ will divide into without a remainder.

Example 20

 $\frac{2y}{y-6}, \quad \frac{4y^2}{(y-6)^3}, \quad \frac{y}{y-1}.$

The LCD is $(y-6)^3 (y-1)$ since $(y-6)^3 \cdot (y-1)$ is the smallest quantity that $y-6, (y-6)^3$ and y-1 will divide into without a remainder.

We'll now propose and demonstrate a method for obtaining the LCD.

Method for Obtaining the LCD

- 1. Factor each denominator. Use exponents for repeated factors. It is usually not necessary to factor numerical quantities.
- 2. Write down each **different** factor that appears. If a factor appears more than once, use only the factor with the highest exponent.
- 3. The LCD is the product of the factors written in step 2.

7 Sample Set B

Find the LCD.

Example 21

 $\frac{1}{x}, \frac{3}{x^3}, \frac{2}{4y}$

- 1. The denominators are already factored.
- 2. Note that x appears as x and x^3 . Use only the x with the higher exponent, x^3 . The term 4y appears, so we must also use 4y.
- 3. The LCD is $4x^3y$.

Example 22

$$\frac{5}{(x-1)^2}, \ \frac{2x}{(x-1)(x-4)}, \ \frac{-5x}{x^2-3x+2}$$

1. Only the third denominator needs to be factored.

 $x^{2} - 3x + 2 = (x - 2)(x - 1)$

- Now the three denominators are $(x-1)^2$, (x-1)(x-4), and (x-2)(x-1). 2. Note that x-1 appears as $(x-1)^2$, x-1, and x-1. Use only the x-1 with the highest exponent, $(x-1)^2$. Also appearing are x-4 and x-2.
- 3. The LCD is $(x-1)^2 (x-4) (x-2)$.

Example 23

$$\frac{-1}{6a^4}, \ \frac{3}{4a^3b}, \ \frac{1}{3a^3(b+5)}$$

- 1. The denominators are already factored.
- 2. We can see that the LCD of the numbers 6, 4, and 3 is 12. We also need a^4 , b, and b + 5.
- 3. The LCD is $12a^4b(b+5)$.

Example 24

 $\frac{9}{x}, \frac{4}{8y}$

- 1. The denominators are already factored.
- 2. x, 8y.
- 3. The LCD is 8xy.

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8 Practice Set B

Find the LCD.
Exercise 11 (Solution on p. 14.)

$$\frac{3}{x^2}, \frac{4}{x^5}, \frac{-6}{xy}$$
 (Solution on p. 14.)
 $\frac{x+1}{x-4}, \frac{x-7}{(x-4)^2}, \frac{-6}{x+1}$ (Solution on p. 14.)
 $\frac{x+1}{x-4}, \frac{x-7}{(x-4)^2}, \frac{-6}{x+1}$ (Solution on p. 14.)
 $\frac{2}{m-6}, \frac{-5m}{(m+1)^2(m-2)}, \frac{12m^2}{(m-2)^3(m-6)}$ (Solution on p. 14.)
 $\frac{1}{x^2-1}, \frac{2}{x^2-2x-3}, \frac{-3x}{x^2-6x+9}$ (Solution on p. 14.)
 $\frac{3}{4y^2-8y}, \frac{8}{y^2-4y+4}, \frac{10y-1}{3y^3-6y^2}$ (Solution on p. 14.)

9 Sample Set C

Change the given rational expressions into rational expressions having the same denominator.

Example 25

$\frac{3}{x^2}, \frac{4}{x}.$	The LCD, by inspection, is x^2 . Rewrite each expression
	with x^2 as the new denominator.
$\overline{x^2}, \overline{x^2}$	Determine the numerators. In $\frac{3}{x^2}$, the denominator was not
	changed so we need not $change the numerator.$
	In the second fraction, the original denominator was x .
$\frac{3}{x^2}, \frac{1}{x^2}$	We can see that x must be multiplied by x to build it to x^2 .
	So we must also multiply the numerator 4 by x. Thus, $4 \cdot x = 4x$.
$\frac{3}{x^2}, \frac{4x}{x^2}$	

Example 26

$\frac{4b}{b-1}, \frac{-2b}{b+3}.$	By inspection, the LCD is $(b-1)(b+3)$.
	Rewrite each fraction with new denominator $(b-1)(b+3)$.
	The denominator of the first rational expression has been multiplied
$\overline{(b-1)(b+3)}, \ \overline{(b-1)(b+3)}$	by $b + 3$, so the numerator $4b$ must be multiplied by $b + 3$.
	$4b(b+3) = 4b^2 + 12b$
$4b^2 + 12b$	The denominator of the second rational expression has been multiplied
$\frac{1}{(b-1)(b+3)}, \ \overline{(b-1)(b+3)}$	by $b - 1$, so the numerator $-2b$ must be multiplied by $b - 1$.
	$-2b(b-1) = -2b^2 + 2b$
$\frac{4b^2+12b}{(b-1)(b+3)}, \frac{-2b^2+2b}{(b-1)(b+3)}$	

Example 27

 $\frac{6x}{x^2 - 8x + 15}, \quad \frac{-2x^2}{x^2 - 7x + 12}.$ $\frac{6x}{(x-3)(x-5)}, \quad \frac{-2x^2}{(x-3)(x-4)}$ $\overline{(x-3)(x-5)(x-4)}, \quad \overline{(x-3)(x-5)(x-4)}$ $\frac{6x^2 - 24x}{(x-3)(x-5)(x-4)}, \quad \overline{(x-3)(x-5)(x-4)}$

We first find the LCD. Factor.

The LCD is (x - 3) (x - 5) (x - 4). Rewrite each of these fractions with new denominator (x - 3) (x - 5) (x - 4). By comparing the denominator of the first fraction with the LCD we see that we must multiply the numerator 6x by x - 4. $6x (x - 4) = 6x^2 - 24x$ By comparing the denominator of the second fraction with the LCD,

 $\frac{6x^2 - 24x}{(x-3)(x-5)(x-4)}, \quad \frac{-2x^3 + 10x^2}{(x-3)(x-5)(x-4)}$

These examples have been done step-by-step and include explanations. This makes the process seem fairly long. In practice, however, the process is much quicker.

Example 28

$$\frac{\frac{6ab}{a^2 - 5a + 4}}{\frac{6ab}{(a-1)(a-4)}}, \ \frac{\frac{a+b}{(a-4)^2}}{\frac{6ab(a-4)}{(a-1)(a-4)^2}}, \ \frac{(a+b)(a-1)}{(a-1)(a-4)^2}$$
 LCD = $(a-1)(a-4)^2$.

Example 29

$$\frac{x+1}{x^3+3x^2}, \frac{2x}{x^3-4x}, \frac{x-4}{x^2-4x+4}$$

$$\frac{x+1}{x^2(x+3)}, \frac{2x}{x(x+2)(x-2)}, \frac{x-4}{(x-2)^2} \quad \text{LCD} = x^2 (x+3) (x+2) (x-2)^2.$$

$$\frac{(x+1)(x+2)(x-2)^2}{x^2(x+3)(x+2)(x-2)^2}, \frac{2x^2(x+3)(x-2)}{x^2(x+3)(x+2)(x-2)^2}, \frac{x^2(x+3)(x+2)(x-4)}{x^2(x+3)(x+2)(x-2)^2}$$

10 Practice Set C

Change the given rational expressions into rational expressions with the same denominators.

Exercise 16	(Solution on p. 14.)
$\frac{\overline{x^3}, \overline{x^5}}{\overline{x^5}}$ Exercise 17	(Solution on p. 14.)
$\frac{\overline{x+6}}{\overline{x-1}}, \frac{\overline{x-1}}{\overline{x-1}}$ Exercise 18	(Solution on p. 14.)
$\frac{\overline{b^{-3}}}{\overline{b^2-b}}, \frac{\overline{b^2-1}}{\overline{b^2-1}}$ Exercise 19	(Solution on p. 14.)
$\frac{8}{x^2-x-6}, \frac{-1}{x^2+x-2}$ Exercise 20	(Solution on p. 14.)
$\frac{10x}{x^2+8x+16}, \frac{5x}{x^2-16}$	· _ /

Exercise 21 (Solution on p. 14.)
$$\frac{-2ab^2}{a^3-6a^2}, \frac{6b}{a^4-2a^3}, \frac{-2a}{a^2-4a+4}$$

11 Exercises

For the following problems, replace N with the proper quantity.

the following problems, replace N with the proper quantity.	
Exercise 22 $\frac{3}{x} = \frac{N}{x^3}$	(Solution on p. 14.)
Exercise 23	
$\frac{4}{a} = \frac{N}{a^2}$	
Exercise 24	(Solution on p. 14.)
$\frac{-2}{r} = \frac{N}{ru}$	
Evercise 25	
$\frac{-7}{m} = \frac{N}{ms}$	
Exercise 26	(Solution on p. 14.)
$\frac{6a}{5} = \frac{N}{10b}$	
Exercise 27	
$\frac{a}{2} = \frac{N}{12}$	
3z = 12z	(G-1-+
Exercise 26 r^2 N	(Solution on p. 15.)
$\frac{x}{4y^2} = \frac{1}{20y^4}$	
Exercise 29 $\frac{b^3}{6a} = \frac{N}{18a^5}$	
Exercise 30	(Solution on p. 15.)
$\frac{-4a}{5} = \frac{N}{15^3}$	(Solution on p. 10)
$5x^2y$ $15x^3y^3$ Exercise 31	
$-10z \ N$	
$\frac{1}{7a^{3}b} = \frac{1}{21a^{4}b^{5}}$	
Exercise 32	(Solution on p. 15.)
$\frac{8x^2y}{5a^3} = \frac{N}{25a^3x^2}$	
Exercise 33	
$\frac{2}{a^2} = \frac{N}{a^2(a-1)}$	
Exercise 34	(Solution on p. 15.)
$\frac{5}{x^3} = \frac{N}{x^3(x-2)}$	
Exercise 35	
$\frac{2a}{b^2} = \frac{N}{b^3 - b}$	
Exercise 36	(Solution on p. 15.)
$\frac{4x}{a} = \frac{N}{a^4 - 4a^2}$	
Exercise 37	
$\frac{6b^3}{5a} = \frac{N}{10a^2 - 30a}$	
Exercise 38	(Solution on p. 15.)
$\frac{4x}{3b} = \frac{N}{3b^5 - 15b}$	
Exercise 39	
$\frac{2m}{m-1} = \frac{N}{(m-1)(m+2)}$	
$\frac{m-1}{(m-1)(m+2)}$	(Solution on a 15)
3s = N	(Solution on p. 15.)
$\frac{1}{s+12} - \frac{1}{(s+12)(s-7)}$	

Exercise 41 $\frac{a+1}{2} = \frac{N}{2N(n+1)}$			
$\begin{array}{c} a-3 & (a-3)(a-4) \\ \mathbf{Exercise} \ 42 \\ a+2 & N \end{array}$	(Solution o	n p.	15.)
$\frac{a+2}{(a-2)(a-4)} = \frac{1}{(a-2)(a-4)}$			
$\frac{b+7}{b-6} = \frac{N}{(b-6)(b+6)}$			
Exercise 44	(Solution o	n p.	15.)
$\frac{3m}{2m+1} = \frac{1}{(2m+1)(m-2)}$			
$\frac{4}{a+6} = \frac{N}{a^2+5a-6}$			
Exercise 46	(Solution o	n p.	15.)
$\frac{b}{b-2} = \frac{b}{b^2-6b+8}$			
$\frac{3b}{b-3} = \frac{N}{b^2 - 11b + 24}$			
Exercise 48 $-2x - N$	(Solution o	n p.	15.)
$\frac{1}{x-7} = \frac{1}{x^2-4x-21}$ Exercise 49			
$\frac{-6m}{m+6} = \frac{N}{m^2 + 10m + 24}$			
Exercise 50 $\frac{4y}{2} = \frac{N}{2}$	(Solution o	n p.	15.)
$y_{+1} - y^2_{+9y+8}$ Exercise 51			
$\frac{x+2}{x-2} = \frac{N}{x^2-4}$			
Exercise 52 $\frac{y-3}{z-2} = \frac{N}{z-2}$	(Solution o	n p.	15.)
$\begin{array}{c} y_{+3} & y_{-9} \\ \textbf{Exercise 53} \\ \textbf{53} \end{array}$			
$\frac{a+5}{a-5} = \frac{N}{a^2-25}$	(C-1-+;		15)
$\frac{z-4}{z+4} = \frac{N}{z^2-16}$	(Solution o	on p.	15.)
Exercise 55 N			
$\frac{1}{2a+1} = \frac{1}{2a^2-5a-3}$	(Solution o	nn	15)
$\frac{1}{3b-1} = \frac{N}{3b^2 + 11b - 4}$	(Solution o	п <i>р</i> .	10.)
Exercise 57 $\frac{a+2}{2} - \frac{N}{2}$			
$2a-1 - 2a^2+9a-5$ Exercise 58	(Solution o	np.	15.)
$\frac{-3}{4x+3} = \frac{N}{4x^2 - 13x - 12}$		-	
Exercise 59 $\frac{b+2}{b+2} = \frac{N}{a^{12}}$			
	(Solution o	n p.	15.)
$\frac{x-1}{4x-5} = \frac{N}{12x^2 - 11x-5}$			
Exercise of $\frac{3}{x+2} = \frac{3x-21}{N}$			
Exercise 62	(Solution o	n p.	15.)
$\frac{4}{y+6} = \frac{4y+6}{N}$			

Exercise 63

$$\frac{-6}{a-1} = \frac{-6a-18}{N}$$
Exercise 64
(Solution on p. 15.)

$$\frac{-8a}{a+3} = \frac{-8a^2 - 40a}{N}$$
Exercise 65

$$\frac{y+1}{y-8} = \frac{y^2 - 2y - 3}{N}$$
Exercise 66
(Solution on p. 15.)

$$\frac{x-4}{x+9} = \frac{x^2 + x - 20}{N}$$
Exercise 67

$$\frac{3x}{2-x} = \frac{N}{x-2}$$
Exercise 68
(Solution on p. 15.)

$$\frac{7a}{5-a} = \frac{N}{a-5}$$
Exercise 69

$$\frac{-m+1}{3-m} = \frac{N}{m-3}$$
Exercise 70
(Solution on p. 15.)

For the following problems, convert the given rational expressions to rational expressions having the same denominators.

Exercise $\frac{2}{a}, \frac{3}{a^4}$	71		
Exercise $\frac{5}{b^2}, \frac{4}{b^3}$	72	(Solution on p.	15.)
Exercise $\frac{8}{z}, \frac{3}{4z^3}$	73		
Exercise $\frac{9}{x^2}, \frac{1}{4x}$	74	(Solution on p.	15.)
Exercise $\frac{2}{a+3}, \frac{4}{a+1}$	75		
Exercise $\frac{2}{x+5}, \frac{4}{x-5}$	76	(Solution on p.	15.)
Exercise $\frac{1}{x-7}, \frac{4}{x-1}$	77		
Exercise $\frac{10}{u+2}, \frac{1}{u+8}$	78	(Solution on p.	16.)
Exercise $\frac{4}{a^2}, \frac{a}{a+4}$	79		
Exercise $\frac{-3}{b^2}, \frac{b^2}{b+5}$	80	(Solution on p.	16.)
Exercise $\frac{-6}{b-1}, \frac{5b}{4b}$	81		
Exercise $\frac{10a}{a-6}, \frac{2}{a^2-4}$	82	(Solution on p.	16.)
Exercise $\frac{4}{x^2+2x}, \frac{4}{x^2}$	$83 \\ \frac{1}{2-4}$		

Exercise 84 $\frac{x+1}{x^2-x-6}, \frac{x+4}{x^2+x-2}$	(Solution on p. 1	16.)
Exercise 85		
$\frac{x-5}{x^2-9x+20}, \frac{4}{x^2-3x-10}$		
Exercise 86	(Solution on p. 1	16.)
$\frac{-4}{b^2+5b-6}, \frac{b+6}{b^2-1}$		
Exercise 87		
$rac{b+2}{b^2+6b+8}, rac{b-1}{b^2+8b+12}$		
Exercise 88	(Solution on p. 1	16.)
$\frac{x+7}{x^2-2x-3}, \frac{x+3}{x^2-6x-7}$		
Exercise 89		
$\frac{2}{a^2+a}, \frac{a+3}{a^2-1}$		
Exercise 90	(Solution on p. 1	16.)
$\frac{x-2}{x^2+7x+6}, \frac{2x}{x^2+4x-12}$,
Exercise 91		
$\frac{x-2}{2x^2+5x-3}, \frac{x-1}{5x^2+16x+3}$		
Exercise 92	(Solution on p. 1	16.)
$\frac{2}{x-5}, \frac{-3}{5-x}$		
Exercise 93		
$\frac{4}{a-6}, \frac{-5}{6-a}$		
Exercise 94	(Solution on p. 1	16.)
$\frac{6}{2-x}, \frac{5}{x-2}$,
Exercise 95		
$\frac{k}{5-k}, \frac{3k}{k-5}$		
Exercise 96	(Solution on p. 1	16.)
$\frac{2m}{m-8}, \frac{7}{8-m}$	· -	

12 Excercises For Review

Exercise 97 (here²) Factor $m^2x^3 + mx^2 + mx$. Exercise 98 (here³) Factor $y^2 - 10y + 21$. Exercise 99 (here⁴) Write the equation of the line that passes through the points (1, 1) and (4, -2). Express the equation in slope-intercept form. Exercise 100 (Solution on p. 16.)

(here⁵) Reduce $\frac{y^2 - y - 6}{y - 3}$. Exercise 101 (here⁶) Find the quotient: $\frac{x^2 - 6x + 9}{x^2 - x - 6} \div \frac{x^2 + 2x - 15}{x^2 + 2x}$. 13

 $^{^{2}&}quot;Factoring \ Polynomials: \ The \ Greatest \ Common \ Factor" \ < http://cnx.org/content/m21913/latest/> \\$

³"Factoring Polynomials: Factoring Trinomials with Leading Coefficient 1" < http://cnx.org/content/m21904/latest/>

⁴"Graphing Linear Equations and Inequalities: Finding the Equation of a Line" http://cnx.org/content/m21998/latest/ ⁵"Rational Expressions: Reducing Rational Expressions" http://cnx.org/content/m21998/latest/

 $[\]label{eq:actional} {}^{6} "Rational Expressions" < http://cnx.org/content/m21964/latest/>$

Solutions to Exercises in this Module

Solution to Exercise (p. 5) N = 18Solution to Exercise (p. 5) $N = 63abx^3$ Solution to Exercise (p. 5) $N = -2y^2 - 2y$ Solution to Exercise (p. 5) $N = a^2 + 9a + 14$ Solution to Exercise (p. 5) $N = 24a^4 (a - 1)$ Solution to Exercise (p. 5) $N = -16x^4y^3z^5$ Solution to Exercise (p. 5) $N = 6ab^2 + 18ab$ Solution to Exercise (p. 5) $N = m^2 - m - 30$ Solution to Exercise (p. 5) $N = r^2 - 7r + 12$ Solution to Exercise (p. 5) $N = 8ab^2$ Solution to Exercise (p. 8) x^5y Solution to Exercise (p. 8) $(x-4)^2(x+1)$ Solution to Exercise (p. 8) $(m-6)(m+1)^2(m-2)^3$ Solution to Exercise (p. 8) $(x+1)(x-1)(x-3)^{2}$ Solution to Exercise (p. 8) $12y^2(y-2)^2$ Solution to Exercise (p. 9) $\frac{4x^2}{x^5}, \frac{7}{x^5}$ Solution to Exercise (p. 9) $\frac{2x(x-1)}{(x+6)(x-1)}, \frac{x(x+6)}{(x+6)(x-1)}$ Solution to Exercise (p. 9) $\tfrac{-3(b+1)}{b(b-1)(b+1)},\,\tfrac{4b^2}{b(b-1)(b+1)}$ Solution to Exercise (p. 9) $\frac{8(x-1)}{(x-3)(x+2)(x-1)}, \frac{-1(x-3)}{(x-3)(x+2)(x-1)}$ Solution to Exercise (p. 9) $\frac{10x(x-4)}{(x+4)^2(x-4)}, \frac{5x(x+4)}{(x+4)^2(x-4)}$ Solution to Exercise (p. 9) $\frac{-2a^2b^2(a-2)^2}{a^3(a-6)(a-2)^2}, \frac{6b(a-6)(a-2)}{a^3(a-6)(a-2)^2}, \frac{-2a^4(a-6)}{a^3(a-6)(a-2)^2}$ Solution to Exercise (p. 10) $3x^2$ Solution to Exercise (p. 10) -2y

Solution to Exercise (p. 10) 12abSolution to Exercise (p. 10) $5x^2y^2$ Solution to Exercise (p. 10) $-12axy^2$ Solution to Exercise (p. 10) $40x^{4}y$ Solution to Exercise (p. 10) 5(x-2)Solution to Exercise (p. 10) 4ax(a+2)(a-2)Solution to Exercise (p. 10) $4x(b^4-5)$ Solution to Exercise (p. 10) 3s(s-7)Solution to Exercise (p. 11) (a+2)(a-4)Solution to Exercise (p. 11) 5m(m-2)Solution to Exercise (p. 11) 9(b-4)Solution to Exercise (p. 11) -2x(x+3)Solution to Exercise (p. 11) 4y(y+8)Solution to Exercise (p. 11) $(y-3)^2$ Solution to Exercise (p. 11) $(z-4)^2$ Solution to Exercise (p. 11) b+4Solution to Exercise (p. 11) -3(x-4)Solution to Exercise (p. 11) (x-1)(3x+1)Solution to Exercise (p. 11) (y+6)(y+2)Solution to Exercise (p. 12) (a+3)(a+5)Solution to Exercise (p. 12) (x+9)(x+5)Solution to Exercise (p. 12) -7aSolution to Exercise (p. 12) -k - 6Solution to Exercise (p. 12) $\frac{5b}{b^3}, \frac{4}{b^3}$ Solution to Exercise (p. 12) $\frac{36}{4x^2}, \frac{x}{4x^2}$

Solution to Exercise (p. 12) $\frac{2(x-5)}{(x+5)(x-5)}, \frac{4(x+5)}{(x+5)(x-5)}$ Solution to Exercise (p. 12) $\tfrac{10(y+8)}{(y+2)(y+8)},\,\tfrac{y+2}{(y+2)(y+8)}$ Solution to Exercise (p. 12) $\frac{-3(b+5)}{b^2(b+5)}, \frac{b^4}{b^2(b+5)}$ Solution to Exercise (p. 12) $\frac{10a^2}{a(a-6)}, \frac{2}{a(a-6)}$ Solution to Exercise (p. 12) $\frac{(x+1)(x-1)}{(x-1)(x+2)(x-3)}, \frac{(x+4)(x-3)}{(x-1)(x+2)(x-3)}$ Solution to Exercise (p. 13) $\frac{-4(b+1)}{(b+1)(b-1)(b+6)}, \frac{(b+6)^2}{(b+1)(b-1)(b+6)}$ Solution to Exercise (p. 13) $\frac{(x+7)(x-7)}{(x+1)(x-3)(x-7)}, \frac{(x+3)(x-3)}{(x+1)(x-3)(x-7)}$ Solution to Exercise (p. 13) $\frac{(x-2)^2}{(x+1)(x-2)(x+6)}, \frac{2x(x+1)}{(x+1)(x-2)(x+6)}$ Solution to Exercise (p. 13) $\frac{2}{x-5}, \frac{3}{x-5}$ Solution to Exercise (p. 13) $\frac{\frac{-6}{x-2}, \frac{5}{x-2}}{\text{Solution to Exercise (p. 13)}}$ $\frac{2m}{m-8}, \frac{-7}{m-8}$ Solution to Exercise (p. 13) (y-7)(y-3)Solution to Exercise (p. 13) y+2