Rational Expressions: Complex Rational Expressions*

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Abstract

<para>This module is from <link document="col10614">Elementary Algebra</link> by Denny Burzynski and Wade Ellis, Jr.</para> <para>A detailed study of arithmetic operations with rational expressions is presented in this chapter, beginning with the definition of a rational expression and then proceeding immediately to a discussion of the domain. The process of reducing a rational expression and illustrations of multiplying, dividing, adding, and subtracting rational expressions are also included. Since the operations of addition and subtraction can cause the most difficulty, they are given particular attention. We have tried to make the written explanation of the examples clearer by using a "freeze frame" approach, which walks the student through the operation step by step.</para> <para>The five-step method of solving applied problems is included in this chapter to show the problem-solving approach to number problems, work problems, and geometry problems. The chapter also illustrates simplification of complex rational expressions, using the combine-divide method and the LCD-multiply-divide method.</para> <para>Objectives of this module: be able to distinguish between simple and complex fractions, be able to simplify complex fractions using the combine-divide and the LCD-multiply-divide method.</para>

1 Overview

- Simple And Complex Fractions
- The Combine-Divide Method
- The LCD-Multiply-Divide Method

2 Simple And Complex Fractions

Simple Fraction

In Section here[1] we saw that a **simple fraction** was a fraction of the form $\frac{P}{Q}$, where P and Q are polynomials and $Q \neq 0$.

*Version 1.3: May 27, 2009 4:50 am -0500

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[1]"Rational Expressions: Rational Expressions" <http://cnx.org/content/m18878/latest/>

Complex Fraction

A **complex fraction** is a fraction in which the numerator or denominator, or both, is a fraction. The fractions

$$\frac{\frac{8}{15}}{\frac{2}{3}} \qquad \text{and} \qquad \frac{1-\frac{1}{x}}{1-\frac{1}{x^2}}$$

are examples of complex fractions, or more generally, complex rational expressions.

There are two methods for simplifying complex rational expressions: the combine-divide method and the LCD-multiply-divide method.

3 The Combine-Divide Method

1. If necessary, combine the terms of the numerator together.
2. If necessary, combine the terms of the denominator together.
3. Divide the numerator by the denominator.

4 Sample Set A

Simplify each complex rational expression.

Example 1

$$\frac{\frac{x^3}{8}}{\frac{x^5}{12}}$$

Steps 1 and 2 are not necessary so we proceed with step 3.

$$\frac{\frac{x^3}{8}}{\frac{x^5}{12}} = \frac{x^3}{8} \cdot \frac{12}{x^5} = \frac{\cancel{)x^3}}{\underset{2}{\cancel{)8}}} \cdot \frac{\overset{3}{\cancel{)12}}}{\cancel{x)^5}2} = \frac{3}{2x^2}$$

Example 2

$\frac{1-\frac{1}{x}}{1-\frac{1}{x^2}}$

Step 1: Combine the terms of the numerator: LCD $= x$.

$1-\frac{1}{x}=\frac{x}{x}-\frac{1}{x}=\frac{x-1}{x}$

Step 2: Combine the terms of the denominator: LCD $= x^2$

$1-\frac{1}{x^2}=\frac{x^2}{x^2}-\frac{1}{x^2}=\frac{x^2-1}{x^2}$

Step 3: Divide the numerator by the denominator.

$$\begin{aligned}\frac{\frac{x-1}{x}}{\frac{x^2-1}{x^2}} &= \frac{x-1}{x}\cdot\frac{x^2}{x^2-1}\\ &= \frac{\cancel{)x-1}}{\cancel{)x}}\,\frac{x^{\cancel{)2}}}{(x+1)\cancel{)(x-1)}}\\ &= \frac{x}{x+1}\end{aligned}$$

Thus,

$\frac{1-\frac{1}{x}}{1-\frac{1}{x^2}}=\frac{x}{x+1}$

Example 3

$\frac{2-\frac{13}{m}-\frac{7}{m^2}}{2+\frac{3}{m}+\frac{1}{m^2}}$

Step 1: Combine the terms of the numerator: LCD $= m^2$.

$2-\frac{13}{m}-\frac{7}{m^2}=\frac{2m^2}{m^2}-\frac{13m}{m^2}-\frac{7}{m^2}=\frac{2m^2-13m-7}{m^2}$

Step 2: Combine the terms of the denominator: LCD $= m^2$

$2+\frac{3}{m}+\frac{1}{m^2}=\frac{2m^2}{m^2}+\frac{3m}{m^2}+\frac{1}{m^2}=\frac{2m^2+3m+1}{m^2}$

Step 3: Divide the numerator by the denominator.

$$\begin{aligned}\frac{\frac{2m^2-13m-7}{m^2}}{\frac{2m^2+3m-1}{m^2}} &= \frac{2m^2-13m-7}{m^2}\cdot\frac{m^2}{2m^2+3m+1}\\ &= \frac{\cancel{)(2m+1)}(m-7)}{\cancel{)m^2}}\cdot\frac{\cancel{)m^2}}{\cancel{)(2m+1)}(m+1)}\\ &= \frac{m-7}{m+1}\end{aligned}$$

Thus,

$\frac{2-\frac{13}{m}-\frac{7}{m^2}}{2+\frac{3}{m}+\frac{1}{m^2}}=\frac{m-7}{m+1}$

5 Practice Set A

Use the combine-divide method to simplify each expression.

Exercise 1 *(Solution on p. 10.)*

$\frac{\frac{27x^2}{6}}{\frac{15x^3}{8}}$

Exercise 2 *(Solution on p. 10.)*

$\frac{3-\frac{1}{x}}{3+\frac{1}{x}}$

Exercise 3 *(Solution on p. 10.)*

$\frac{1+\frac{x}{y}}{x-\frac{y^2}{x}}$

Exercise 4 *(Solution on p. 10.)*

$\frac{m-3+\frac{2}{m}}{m-4+\frac{3}{m}}$

Exercise 5 *(Solution on p. 10.)*

$\frac{1+\frac{1}{x-1}}{1-\frac{1}{x-1}}$

6 The LCD-Multiply-Divide Method

1. Find the LCD of all the terms.
2. Multiply the numerator and denominator by the LCD.
3. Reduce if necessary.

7 Sample Set B

Simplify each complex fraction.

Example 4

$$\frac{1-\frac{4}{a^2}}{1+\frac{2}{a}}$$

Step 1: The LCD $= a^2$.

Step 2: Multiply both the numerator and denominator by a^2.

$$\begin{aligned}\frac{a^2\left(1-\frac{4}{a^2}\right)}{a^2\left(1+\frac{2}{a}\right)} &= \frac{a^2\cdot 1-a^2\cdot\frac{4}{a^2}}{a^2\cdot 1+a^2\cdot\frac{2}{a}}\\ &= \frac{a^2-4}{a^2+2a}\end{aligned}$$

Step 3: Reduce.

$$\begin{aligned}\frac{a^2-4}{a^2+2a} &= \frac{\cancel{)(a+2)}(a-2)}{a\cancel{)(a+2)}}\\ &= \frac{a-2}{a}\end{aligned}$$

Thus,

$$\frac{1-\frac{4}{a^2}}{1+\frac{2}{a}} = \frac{a-2}{a}$$

Example 5

$$\frac{1-\frac{5}{x}-\frac{6}{x^2}}{1+\frac{6}{x}+\frac{5}{x^2}}$$

Step 1 : The LCD is x^2.

Step 2 : Multiply the numerator and denominator by x^2.

$$\begin{aligned}\frac{x^2\left(1-\frac{5}{x}-\frac{6}{x^2}\right)}{x^2\left(1+\frac{6}{x}+\frac{5}{x^2}\right)} &= \frac{x^2\cdot 1-x^{)2}\cdot\frac{5}{)x}-\overline{)x^2}\cdot\frac{6}{)x^2}}{x^2\cdot 1+x^{)2}\cdot\frac{6}{)x}+\overline{)x^2}\cdot\frac{5}{)x^2}}\\ &= \frac{x^2-5x-6}{x^2+6x+5}\end{aligned}$$

Step 3 : Reduce.

$$\begin{aligned}\frac{x^2-5x-6}{x^2+6x+5} &= \frac{(x-6)(x+1)}{(x+5)(x+1)}\\ &= \frac{x-6}{x+5}\end{aligned}$$

Thus,

$$\frac{1-\frac{5}{x}-\frac{6}{x^2}}{1+\frac{6}{x}+\frac{5}{x^2}}=\frac{x-6}{x+5}$$

8 Practice Set B

The following problems are the same problems as the problems in Practice Set A. Simplify these expressions using the LCD-multiply-divide method. Compare the answers to the answers produced in Practice Set A.

Exercise 6 *(Solution on p. 10.)*

$$\frac{\frac{27x^2}{6}}{\frac{15x^3}{8}}$$

Exercise 7 *(Solution on p. 10.)*

$$\frac{3-\frac{1}{x}}{3+\frac{1}{x}}$$

Exercise 8 *(Solution on p. 10.)*

$$\frac{1+\frac{x}{y}}{x-\frac{y^2}{x}}$$

Exercise 9 *(Solution on p. 10.)*

$$\frac{m-3+\frac{2}{m}}{m-4+\frac{3}{m}}$$

Exercise 10 *(Solution on p. 10.)*

$$\frac{1+\frac{1}{x-1}}{1-\frac{1}{x-1}}$$

9 Exercises

For the following problems, simplify each complex rational expression.

Exercise 11 *(Solution on p. 10.)*

$$\frac{1+\frac{1}{4}}{1-\frac{1}{4}}$$

Exercise 12

$$\frac{1-\frac{1}{3}}{1+\frac{1}{3}}$$

Exercise 13 *(Solution on p. 10.)*

$\frac{1-\frac{1}{y}}{1+\frac{1}{y}}$

Exercise 14

$\frac{a+\frac{1}{x}}{a-\frac{1}{x}}$

Exercise 15 *(Solution on p. 10.)*

$\frac{\frac{a}{b}+\frac{c}{b}}{\frac{a}{b}-\frac{c}{b}}$

Exercise 16

$\frac{\frac{5}{m}+\frac{4}{m}}{\frac{5}{m}-\frac{4}{m}}$

Exercise 17 *(Solution on p. 10.)*

$\frac{3+\frac{1}{x}}{\frac{3x+1}{x^2}}$

Exercise 18

$\frac{1+\frac{x}{x+y}}{1-\frac{x}{x+y}}$

Exercise 19 *(Solution on p. 10.)*

$\frac{2+\frac{5}{a+1}}{2-\frac{5}{a+1}}$

Exercise 20

$\frac{1-\frac{1}{a-1}}{1+\frac{1}{a-1}}$

Exercise 21 *(Solution on p. 10.)*

$\frac{4-\frac{1}{m^2}}{2+\frac{1}{m}}$

Exercise 22

$\frac{9-\frac{1}{x^2}}{3-\frac{1}{x}}$

Exercise 23 *(Solution on p. 10.)*

$\frac{k-\frac{1}{k}}{\frac{k+1}{k}}$

Exercise 24

$\frac{\frac{m}{m+1}-1}{\frac{m+1}{2}}$

Exercise 25 *(Solution on p. 10.)*

$\frac{\frac{2xy}{2x-y}-y}{\frac{2x-y}{3}}$

Exercise 26

$\frac{\frac{1}{a+b}-\frac{1}{a-b}}{\frac{1}{a+b}+\frac{1}{a-b}}$

Exercise 27 *(Solution on p. 10.)*

$\frac{\frac{5}{x+3}-\frac{5}{x-3}}{\frac{5}{x+3}+\frac{5}{x-3}}$

Exercise 28

$\frac{2+\frac{1}{y+1}}{\frac{1}{y}+\frac{2}{3}}$

Exercise 29 *(Solution on p. 10.)*

$\frac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{x}+\frac{1}{y}}$

Exercise 30

$\frac{1+\frac{5}{x}+\frac{6}{x^2}}{1-\frac{1}{x}-\frac{12}{x^2}}$

Exercise 31 *(Solution on p. 10.)*

$\frac{1+\frac{1}{y}-\frac{2}{y^2}}{1+\frac{7}{y}+\frac{10}{y^2}}$

Exercise 32

$\frac{\frac{3n}{m}-2-\frac{m}{n}}{\frac{3n}{m}+4+\frac{m}{n}}$

Exercise 33 *(Solution on p. 10.)*

$\frac{x-\frac{4}{3x-1}}{1-\frac{2x-2}{3x-1}}$

Exercise 34

$\frac{\frac{y}{x+y}-\frac{x}{x-y}}{\frac{x}{x+y}+\frac{y}{x-y}}$

Exercise 35 *(Solution on p. 10.)*

$\frac{\frac{a}{a-2}-\frac{a}{a+2}}{\frac{2a}{a-2}+\frac{a^2}{a+2}}$

Exercise 36

$3-\frac{2}{1-\frac{1}{m+1}}$

Exercise 37 *(Solution on p. 10.)*

$\frac{x-\frac{1}{1-\frac{1}{x}}}{x+\frac{1}{1+\frac{1}{x}}}$

Exercise 38

In electricity theory, when two resistors of resistance R_1 and R_2 ohms are connected in parallel, the total resistance R is

$$R=\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$$

Write this complex fraction as a simple fraction.

Exercise 39 *(Solution on p. 10.)*

According to Einstein's theory of relativity, two velocities v_1 and v_2 are not added according to $v=v_1+v_2$, but rather by

$$v=\frac{v_1+v_2}{1+\frac{v_1v_2}{c^2}}$$

Write this complex fraction as a simple fraction.

Einstein's formula is really only applicale for velocities near the speed of light $(c=186,000\,\text{miles per second})$. At very much lower velocities, such as 500 miles per hour, the formula $v=v_1+v_2$ provides an extremely good approximation.

10 Exercises For Review

Exercise 40

(**here**[2]) Supply the missing word. Absolute value speaks to the question of how _____ and not "which way."

Exercise 41 *(Solution on p. 11.)*

(**here**[3]) Find the product. $(3x+4)^2$.

Exercise 42

(**here**[4]) Factor x^4-y^4.

[2] "Basic Operations with Real Numbers: Absolute Value" <http://cnx.org/content/m21876/latest/>
[3] "Algebraic Expressions and Equations: Special Binomial Products" <http://cnx.org/content/m21858/latest/>
[4] "Factoring Polynomials: Factoring Two Special Products" <http://cnx.org/content/m21903/latest/>

Exercise 43 ***(Solution on p. 11.)***

(here[5]) Solve the equation $\frac{3}{x-1} - \frac{5}{x+3} = 0$.

Exercise 44

(here[6]) One inlet pipe can fill a tank in 10 minutes. Another inlet pipe can fill the same tank in 4 minutes. How long does it take both pipes working together to fill the tank?

[5] "Rational Expressions: Rational Equations" <http://cnx.org/content/m21951/latest/>
[6] "Rational Expressions: Applications" <http://cnx.org/content/m21938/latest/>

Solutions to Exercises in this Module

Solution to Exercise (p. 4)

$\frac{12}{5x}$

Solution to Exercise (p. 4)

$\frac{3x-1}{3x+1}$

Solution to Exercise (p. 4)

$\frac{x}{y(x-y)}$

Solution to Exercise (p. 4)

$\frac{m-2}{m-3}$

Solution to Exercise (p. 4)

$\frac{x}{x-2}$

Solution to Exercise (p. 6)

$\frac{12}{5x}$

Solution to Exercise (p. 6)

$\frac{3x-1}{3x+1}$

Solution to Exercise (p. 6)

$\frac{x}{y(x-y)}$

Solution to Exercise (p. 6)

$\frac{m-2}{m-3}$

Solution to Exercise (p. 6)

$\frac{x}{x-2}$

Solution to Exercise (p. 6)

$\frac{5}{3}$

Solution to Exercise (p. 6)

$\frac{y-1}{y+1}$

Solution to Exercise (p. 7)

$\frac{a+c}{a-c}$

Solution to Exercise (p. 7)

x

Solution to Exercise (p. 7)

$\frac{2a+7}{2a-3}$

Solution to Exercise (p. 7)

$\frac{2m-1}{m}$

Solution to Exercise (p. 7)

$k-1$

Solution to Exercise (p. 7)

$\frac{3y^2}{(2x-y)^2}$

Solution to Exercise (p. 7)

$\frac{-3}{x}$

Solution to Exercise (p. 7)

$\frac{y-x}{xy}$

Solution to Exercise (p. 7)

$\frac{y-1}{y+5}$

Solution to Exercise (p. 8)

$3x-4$

Solution to Exercise (p. 8)

$\frac{4}{a^2+4}$

Solution to Exercise (p. 8)

$\frac{(x-2)(x+1)}{(x-1)(x+2)}$

Solution to Exercise (p. 8)

$\frac{c^2(V_1+V_2)}{c^2+V_1V_2}$

Solution to Exercise (p. 8)

$9x^2+24x+16$

Solution to Exercise (p. 8)

$x=7$