Roots, Radicals, and Square Root Equations: Square Root Equations with Applications*

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Abstract

This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. The distinction between the principal square root of the number x and the secondary square root of the number x is made by explanation and by example. The simplification of the radical expressions that both involve and do not involve fractions is shown in many detailed examples; this is followed by an explanation of how and why radicals are eliminated from the denominator of a radical expression. Real-life applications of radical equations have been included, such as problems involving daily output, daily sales, electronic resonance frequency, and kinetic energy. Objectives of this module: be able to recognize square root equations and extraneous solutions, be able to sove square root equations.

1 Overview

- Square Root Equations And Extraneous Solutions
- Method For Solving Square Root Equations

2 Square Root Equations And Extraneous Solutions

Square Root Equation
A **square root equation** is an equation that contains a variable under a square root sign. The fact that $\sqrt{x} \cdot \sqrt{x} = (\sqrt{x})^2 = x$suggests that we can solve a square root equation by squaring both sides of the equation.
Extraneous Solutions
Squaring both sides of an equation can, however, introduce extraneous solutions. Consider the equation

$x = -6$
The solution is -6. Square both sides.

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$x^2 = (-6)^2$
$x^2 = 36$

This equation has two solutions, -6 and $+6$. The $+6$ is an extraneous solution since it does not check in the original equation: $+6 \neq -6$.

3 Method For Solving Square Root Equations

Solving Square Root Equations

1. Isolate a radical. This means get a square root expression by itself on one side of the equal sign.
2. Square both sides of the equation.
3. Simplify the equation by combining like terms.
4. Repeat step 1 if radicals are still present.
5. Obtain potential solutions by solving the resulting non-square root equation.
6. Check each potential solution by substitution into the original equation.

4 Sample Set A

Solve each square root equation.

Example 1

	$\sqrt{x}$	$=$	$8.$	The radical is isolated Square both sides.
	$(\sqrt{x})^2$	$=$	8^2	
	x	$=$	64	Check this potential solution.
Check :	$\sqrt{64}$	$=$	8	Is this correct?
	8	$=$	8	Yes, this is correct.

64 is the solution.

Example 2

	$\sqrt{y-3}$	$=$	$4.$	The radical is isolated. Square both sides.
	$y-3$	$=$	16	Solve this nonradical equation.
	y	$=$	19	Check this potential solution.
Check :	$\sqrt{19-3}$	$=$	$\sqrt{16}$	Is this correct?
	$\sqrt{16}$	$=$	4	Is this correct?
	4	$=$	4	Yes, this is correct.

19 is the solution.

Example 3

$$\begin{array}{rrcll} & \sqrt{2m+3}-\sqrt{m-8} & = & 0. & \text{Isolate either radical.} \\ & \sqrt{2m+3} & = & \sqrt{m-8} & \text{Square both sides.} \\ & 2m+3 & = & m-8 & \text{Solve this nonradical equation.} \\ & m & = & -11 & \text{Check this potential solution.} \\ \textit{Check}: & \sqrt{2(-11)+3}-\sqrt{(-11)-8} & = & 0 & \text{Is this correct?} \\ & \sqrt{-22+3}-\sqrt{-19} & = & 0 & \text{Is this correct?} \end{array}$$

Since $\sqrt{-19}$ is not a real number, the potential solution of $m = -11$ does not check. This equation has no real solution.

Example 4

$\sqrt{4x-5}=-6.$ By inspection, this equation has no real solution.

The symbol, $\sqrt{\ }$, signifies the **positive** square root and not the negative square root.

5 Practice Set A

Solve each square root equation.

Exercise 1 *(Solution on p. 7.)*
$\sqrt{y}=14$

Exercise 2 *(Solution on p. 7.)*
$\sqrt{a-7}=5$

Exercise 3 *(Solution on p. 7.)*
$\sqrt{3a+8}-\sqrt{2a+5}=0$

Exercise 4 *(Solution on p. 7.)*
$\sqrt{m-4}=-11$

6 Exercises

For the following problems, solve the square root equations.

Exercise 5 *(Solution on p. 7.)*
$\sqrt{x}=5$

Exercise 6
$\sqrt{y}=7$

Exercise 7 *(Solution on p. 7.)*
$\sqrt{a}=10$

Exercise 8
$\sqrt{c}=12$

Exercise 9 *(Solution on p. 7.)*
$\sqrt{x} = -3$

Exercise 10
$\sqrt{y} = -6$

Exercise 11 *(Solution on p. 7.)*
$\sqrt{x} = 0$

Exercise 12
$\sqrt{x} = 1$

Exercise 13 *(Solution on p. 7.)*
$\sqrt{x+3} = 3$

Exercise 14
$\sqrt{y-5} = 5$

Exercise 15 *(Solution on p. 7.)*
$\sqrt{a+2} = 6$

Exercise 16
$\sqrt{y+7} = 9$

Exercise 17 *(Solution on p. 7.)*
$\sqrt{y-4} - 4 = 0$

Exercise 18
$\sqrt{x-10} - 10 = 0$

Exercise 19 *(Solution on p. 7.)*
$\sqrt{x-16} = 0$

Exercise 20
$\sqrt{y-25} = 0$

Exercise 21 *(Solution on p. 7.)*
$\sqrt{6m-4} = \sqrt{5m-1}$

Exercise 22
$\sqrt{5x+6} = \sqrt{3x+7}$

Exercise 23 *(Solution on p. 7.)*
$\sqrt{7a+6} = \sqrt{3a-18}$

Exercise 24
$\sqrt{4x+3} = \sqrt{x-9}$

Exercise 25 *(Solution on p. 7.)*
$\sqrt{10a-7} - \sqrt{2a+9} = 0$

Exercise 26
$\sqrt{12k-5} - \sqrt{9k+10} = 0$

Exercise 27 *(Solution on p. 7.)*
$\sqrt{x-6} - \sqrt{3x-8} = 0$

Exercise 28
$\sqrt{4a-5} - \sqrt{7a-20} = 0$

Exercise 29 *(Solution on p. 7.)*
$\sqrt{2m-6} = \sqrt{m-2}$

Exercise 30
$\sqrt{6r-11} = \sqrt{5r+3}$

Exercise 31 *(Solution on p. 7.)*

$\sqrt{3x+1} = \sqrt{2x-6}$

Exercise 32

$\sqrt{x-7} - \sqrt{5x+1} = 0$

Exercise 33 *(Solution on p. 7.)*

$\sqrt{2a+9} - \sqrt{a-4} = 0$

Exercise 34

At a certain electronics company, the daily output Q is related to the number of people A on the assembly line by $Q = 400 + 10\sqrt{A+125}$.
(a) Determine the daily output if there are 44 people on the assembly line.

(b) Determine how many people are needed on the assembly line if the daily output is to be 520.

Exercise 35 *(Solution on p. 7.)*

At a store, the daily number of sales S is approximately related to the number of employees E by $S = 100 + 15\sqrt{E+6}$
(a) Determine the approximate number of sales if there are 19 employees.

(b) Determine the number of employees the store would need to produce 310 sales.

Exercise 36

 The resonance frequency f in an electronic circuit containing inductance L and capacitance C in series is given by

$f = \frac{1}{2\pi\sqrt{LC}}$

(a) Determine the resonance frequency in an electronic circuit if the inductance is 4 and the capacitance is 0.0001. Use $\pi = 3.14$.

(b) Determine the inductance in an electric circuit if the resonance frequency is 7.12 and the capacitance is 0.0001. Use $\pi = 3.14$.

Exercise 37 *(Solution on p. 7.)*

If two magnetic poles of strength m and m' units are at a distance r centimeters (cm) apart, the force F of repulsion in air between them is given by

$F = \frac{mm'}{r^2}$

(a) Determine the force of repulsion if two magnetic poles of strengths 20 and 40 units are 5 cm apart in air.

(b) Determine how far apart are two magnetic poles of strengths 30 and 40 units if the force of repulsion in air between them is 0.0001.

Exercise 38

The velocity V in feet per second of outflow of a liquid from an orifice is given by $V = 8\sqrt{h}$, where h is the height in feet of the liquid above the opening.
(a) Determine the velocity of outflow of a liquid from an orifice that is 9 feet below the top surface of a liquid (V is in feet/sec).

(b) Determine how high a liquid is above an orifice if the velocity of outflow is 81 feet/second.

Exercise 39 *(Solution on p. 7.)*

The period T in seconds of a simple pendulum of length L in feet is given by $T = 2\pi\sqrt{\frac{L}{32}}$.

(a) Determine the period of a simple pendulum that is 2 feet long. Use $\pi = 3.14$.

(b) Determine the length in feet of a simple pendulum whose period is 10.8772 seconds. Use $\pi = 3.14$.

Exercise 40

The kinetic energy KE in foot pounds of a body of mass m in slugs moving with a velocity v in feet/sec is given by

$KE = \frac{1}{2}mv^2$

(a) Determine the kinetic energy of a 2-slug body moving with a velocity of 4 ft/sec.

(b) Determine the velocity in feet/sec of a 4-slug body if its kinetic energy is 50 foot pounds.

7 Exercises For Review

Exercise 41 *(Solution on p. 7.)*

(**here**[1]) Write $\frac{x^{10}y^3(x+7)^4}{x^{-2}y^3(x+7)^{-1}}$ so that only positive exponents appear.

Exercise 42

(**here**[2]) Classify $x + 4 = x + 7$ as an identity, a contradiction, or a conditional equation.

Exercise 43 *(Solution on p. 7.)*

(**here**[3]) Supply the missing words. In the coordinate plane, lines with _____ slope rise and lines with _____ slope fall.

Exercise 44

(**here**[4]) Simplify $\sqrt{(x+3)^4(x-2)^6}$.

Exercise 45 *(Solution on p. 7.)*

(**here**[5]) Simplify $\left(3+\sqrt{5}\right)\left(4-\sqrt{5}\right)$.

[1] "Basic Operations with Real Numbers: Negative Exponents" <http://cnx.org/content/m21882/latest/>
[2] "Solving Linear Equations and Inequalities: Solving Equations" <http://cnx.org/content/m18876/latest/>
[3] "Graphing Linear Equations and Inequalities: The Slope-Intercept Form of a Line" <http://cnx.org/content/m22014/latest/>
[4] "Roots, Radicals, and Square Root Equations: Simplifying Square Root Expressions" <http://cnx.org/content/m21973/latest/>
[5] "Roots, Radicals, and Square Root Equations: Addition and Subtraction of Square Root Expressions" <http://cnx.org/content/m21957/latest/>

Solutions to Exercises in this Module

Solution to Exercise (p. 3)
$y = 196$
Solution to Exercise (p. 3)
$a = 32$
Solution to Exercise (p. 3)
$a = -3$ is extraneous, no real solution
Solution to Exercise (p. 3)
no real solution
Solution to Exercise (p. 3)
$x = 25$
Solution to Exercise (p. 3)
$a = 100$
Solution to Exercise (p. 3)
no solution
Solution to Exercise (p. 4)
$x = 0$
Solution to Exercise (p. 4)
$x = 6$
Solution to Exercise (p. 4)
$a = 34$
Solution to Exercise (p. 4)
$y = 20$
Solution to Exercise (p. 4)
$x = 16$
Solution to Exercise (p. 4)
$m = 3$
Solution to Exercise (p. 4)
no solution
Solution to Exercise (p. 4)
$a = 2$
Solution to Exercise (p. 4)
no solution
Solution to Exercise (p. 4)
$m = 4$
Solution to Exercise (p. 4)
no solution
Solution to Exercise (p. 5)
no solution
Solution to Exercise (p. 5)
$(a)\, S = 175;\ (b)\, E = 190$
Solution to Exercise (p. 5)
$(a)\, F = 32\ (b)\, r = 8\text{cm}$
Solution to Exercise (p. 6)
$(a)\, T = 1.57\,\text{sec}\ (b)\, L = 95.99\,\text{cm}$
Solution to Exercise (p. 6)
$x^{12}(x+7)^5$
Solution to Exercise (p. 6)
positive; negative

Solution to Exercise (p. 6)
$7 + \sqrt{5}$