Graphing Linear Equations and Inequalities: Plotting Points in the Plane*

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Abstract

This module is from Elementary Algebra by Denny Burzynski and Wade Ellis, Jr. In this chapter the student is shown how graphs provide information that is not always evident from the equation alone. The chapter begins by establishing the relationship between the variables in an equation, the number of coordinate axes necessary to construct its graph, and the spatial dimension of both the coordinate system and the graph. Interpretation of graphs is also emphasized throughout the chapter, beginning with the plotting of points. The slope formula is fully developed, progressing from verbal phrases to mathematical expressions. The expressions are then formed into an equation by explicitly stating that a ratio is a comparison of two quantities of the same type (e.g., distance, weight, or money). This approach benefits students who take future courses that use graphs to display information. The student is shown how to graph lines using the intercept method, the table method, and the slope-intercept method, as well as how to distinguish, by inspection, oblique and horizontal/vertical lines. Objectives of this module: be familiar with the plane, know what is meant by the coordinates of a point, be able to plot points in the plane.

1 Overview

- The Plane
- Coordinates of a Point
- Plotting Points

2 The Plane

Ordered Pairs

We are now interested in studying graphs of linear equations in two variables. We know that solutions to equations in two variables consist of a pair of values, one value for each variable. We have called these pairs of values **ordered pairs.** Since we have a pair of values to graph, we must have a pair of axes (number lines) upon which the values can be located.

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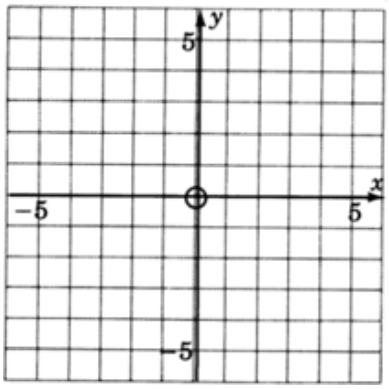
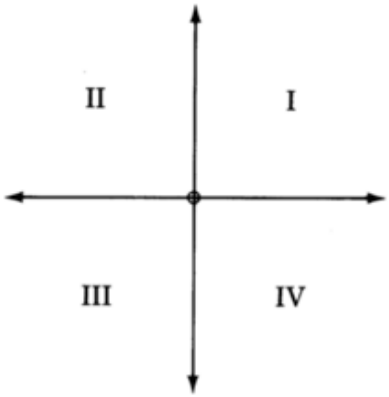
Origin
We draw the axes so they are perpendicular to each other and so that they intersect each other at their 0's. This point is called the **origin.**



Rectangular Coordinate System
These two lines form what is called a **rectangular coordinate system.** They also determine a plane.
xy**-plane**
A **plane** is a flat surface, and a result from geometry states that through any two intersecting lines (the axes) exactly one plane (flat surface) may be passed. If we are dealing with a linear equation in the two variables x and y, we sometimes say we are graphing the equation using a rectangular coordinate system, or that we are graphing the equation in the xy-plane.
Quadrant
Notice that the two intersecting coordinate axes divide the plane into four equal regions. Since there are four regions, we call each one a **quadrant** and number them counterclockwise using Roman numerals.



Recall that when we first studied the number line we observed the following:

For each real number there exists a unique point on the number line, and for each point on the number line we can associate a unique real number.

We have a similar situation for the plane.

For each ordered pair (a, b), there exists a unique point in the plane, and to each point in the plane we can associate a unique ordered pair (a, b) of real numbers.

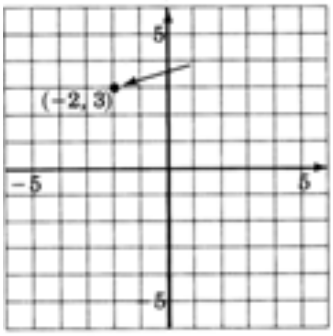
3 Coordinates of a Point

Coordinates of a Point
The numbers in an ordered pair that are associated with a particular point are called the **coordinates of**

the point. The **first number** in the ordered pair expresses the point's horizontal distance and direction (left or right) from the origin. The **second number** expresses the point's vertical distance and direction (up or down) from the origin.

The Coordinates Determine Distance and Direction

A **positive number** means a direction to the **right or up**. A **negativenumber** means a direction to the **left or down**.



This point is located 2 units to the left of the origin and 3 units up from the origin.

4 Plotting Points

Since points and ordered pairs are so closely related, the two terms are sometimes used interchangeably. The following two phrases have the same meaning:

1. Plot the point (a, b).
2. Plot the ordered pair (a, b).

Plotting a Point

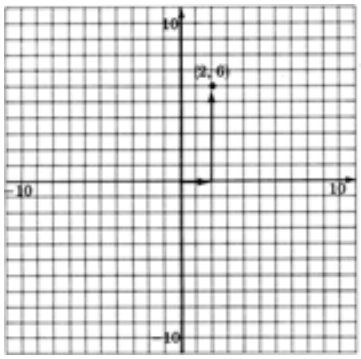
Both phrases mean: Locate, in the plane, the point associated with the ordered pair (a, b) and draw a mark at that position.

5 Sample Set A

Example 1

Plot the ordered pair (2, 6).

We begin at the origin. The first number in the ordered pair, 2, tells us we move 2 units to the right (+2 means 2 units to the right) The second number in the ordered pair, 6, tells us we move 6 units up (+6 means 6 units up).



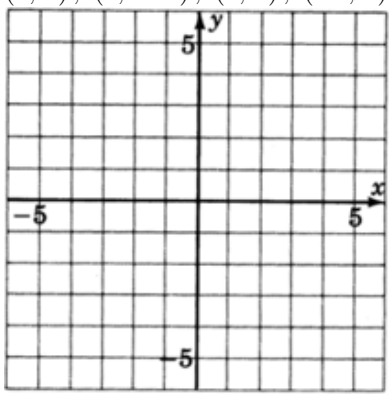
It is sometimes helpful to read (2, 6) as "if $x = 2$, then $y = 6$".

6 Practice Set A

Exercise 1 ***(Solution on p. 11.)***

Plot the ordered pairs.

$(1, 3)$, $(4, -5)$, $(0, 1)$, $(-4, 0)$.

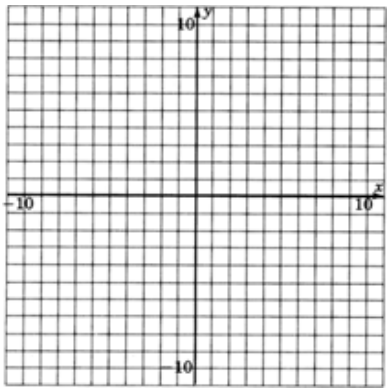


7 Exercises

Exercise 2 ***(Solution on p. 11.)***

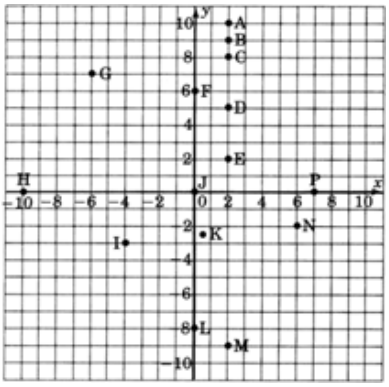
Plot the following ordered pairs. (Do not draw the arrows as in Practice Set A.)

$(8, 2)$, $(10, -3)$, $(-3, 10)$, $(0, 5)$, $(5, 0)$, $(0, 0)$, $\left(-7, -\frac{3}{2}\right)$.



Exercise 3

As accurately as possible, state the coordinates of the points that have been plotted on the following graph.



Exercise 4 ***(Solution on p. 11.)***

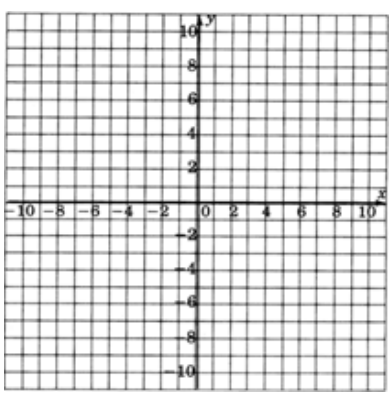
Using ordered pair notation, what are the coordinates of the origin?

Exercise 5

We know that solutions to linear equations in two variables can be expressed as ordered pairs. Hence, the solutions can be represented as points in the plane. Consider the linear equation $y = 2x - 1$. Find at least ten solutions to this equation by choosing x-values between -4 and 5 and computing the corresponding y-values. Plot these solutions on the coordinate system below. Fill in the table to help you keep track of the ordered pairs.

x												
y												

Table 1



Keeping in mind that there are infinitely many ordered pair solutions to $y = 2x - 1$, speculate on the geometric structure of the graph of *all* the solutions. Complete the following statement:

The name of the type of geometric structure of the graph of all the solutions to the linear equation $y = 2x - 1$ seems to be _____ .

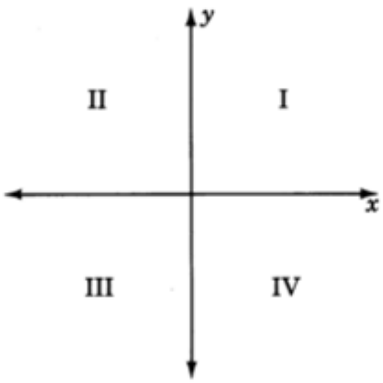
Where does this figure cross the y-axis? Does this number appear in the equation $y = 2x - 1$?

Place your pencil at any point on the figure (you may have to connect the dots to see the figure clearly). Move your pencil exactly *one* unit to the right (horizontally). To get back onto the figure, you must move your pencil either up or down a particular number of units. How many units

must you move vertically to get back onto the figure, and do you see this number in the equation $y = 2x - 1$?

Exercise 6 ***(Solution on p. 11.)***

Consider the xy-plane.



Complete the table by writing the appropriate inequalities.

I	II	III	IV
$x > 0$	$x < 0$	x	x
$y > 0$	y	y	y

Table 2

In the following problems, the graphs of points are called **scatter diagrams** and are frequently used by statisticians to determine if there is a relationship between the two variables under consideration. The first component of the ordered pair is called the **input variable** and the second component is called the **output variable**. Construct the scatter diagrams. Determine if there appears to be a relationship between the two variables under consideration by making the following observations: A relationship may exist if

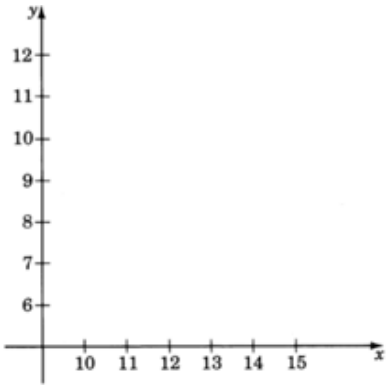
a. as one variable increases, the other variable increases
b. as one variable increases, the other variable decreases

Exercise 7

A psychologist, studying the effects of a placebo on assembly line workers at a particular industrial site, noted the time it took to assemble a certain item before the subject was given the placebo, x, and the time it took to assemble a similar item after the subject was given the placebo, y. The psychologist's data are

x	y
10	8
12	9
11	9
10	7
14	11
15	12
13	10

Table 3

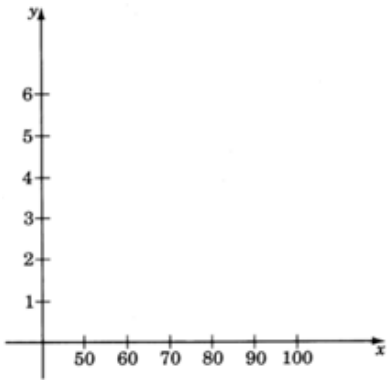


Exercise 8 ***(Solution on p. 11.)***

The following data were obtained in an engineer's study of the relationship between the amount of pressure used to form a piece of machinery, x, and the number of defective pieces of machinery produced, y.

x	y
50	0
60	1
65	2
70	3
80	4
70	5
90	5
100	5

Table 4

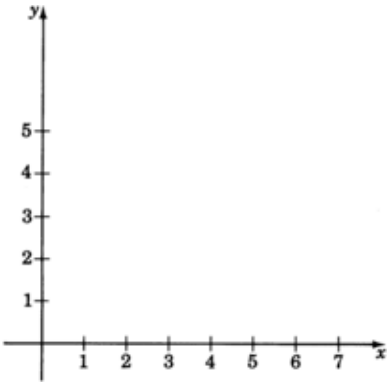


Exercise 9

The following data represent the number of work days missed per year, x, by the employees of an insurance company and the number of minutes they arrive late from lunch, y.

x	y
1	3
6	4
2	2
2	3
3	1
1	4
4	4
6	3
5	2
6	1

Table 5



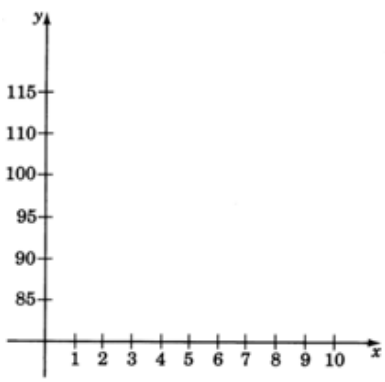
Exercise 10 ***(Solution on p. 12.)***

A manufacturer of dental equipment has the following data on the unit cost (in dollars), y, of a

particular item and the number of units, x, manufactured for each order.

x	y
1	85
3	92
5	99
3	91
4	100
1	87
6	105
8	111
8	114

Table 6



8 Exercises for Review

Exercise 11

(**here**[1]) Simplify $\left(\frac{18x^5y^6}{9x^2y^4}\right)^5$.

Exercise 12 ***(Solution on p. 12.)***

(**here**[2]) Supply the missing word. An _____ is a statement that two algebraic expressions are equal.

Exercise 13

(**here**[3]) Simplify the expression $5xy\left(xy-2x+3y\right)-2xy\left(3xy-4x\right)-15xy^2$.

[1] "Basic Properties of Real Numbers: The Power Rules for Exponents" <http://cnx.org/content/m21897/latest/>
[2] "Algebraic Expressions and Equations: Equations" <http://cnx.org/content/m21850/latest/>
[3] "Algebraic Expressions and Equations: Classification of Expressions and Equations" <http://cnx.org/content/m21848/latest/>

Exercise 14 ***(Solution on p. 12.)***

(**here**[4]) Identify the equation $x + 2 = x + 1$ as an identity, a contradiction, or a conditional equation.

Exercise 15

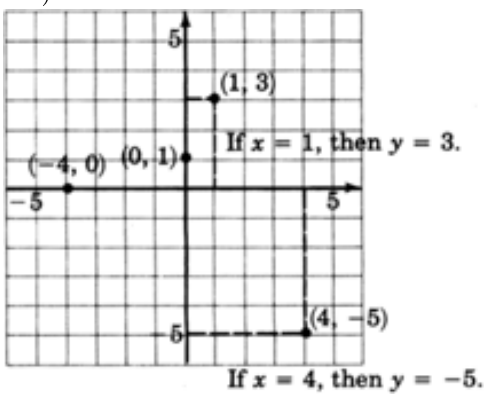
(**here**[5]) Supply the missing phrase. A system of axes constructed for graphing an equation is called a _____ .

[4] "Solving Linear Equations and Inequalities: Solving Equations" <http://cnx.org/content/m18876/latest/>

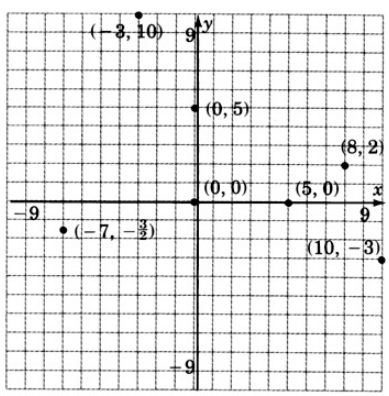
[5] "Graphing Linear Equations and Inequalities: Graphing Linear Equations and Inequalities in One Variable" <http://cnx.org/content/m18877/latest/>

Solutions to Exercises in this Module

Solution to Exercise (p. 4)
(Notice that the dotted lines on the graph are only for illustration and should not be included when plotting points.)



Solution to Exercise (p. 4)

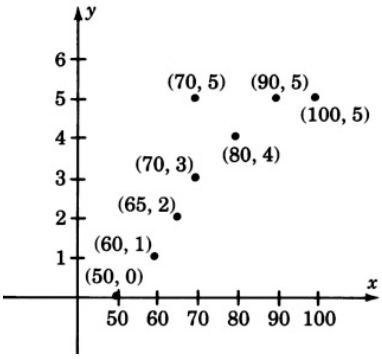


Solution to Exercise (p. 5)
Coordinates of the origin are $(0, 0)$.
Solution to Exercise (p. 6)

I	II	III	IV
$x > 0$	$x < 0$	$x < 0$	$x > 0$
$y > 0$	$y > 0$	$y < 0$	$y < 0$

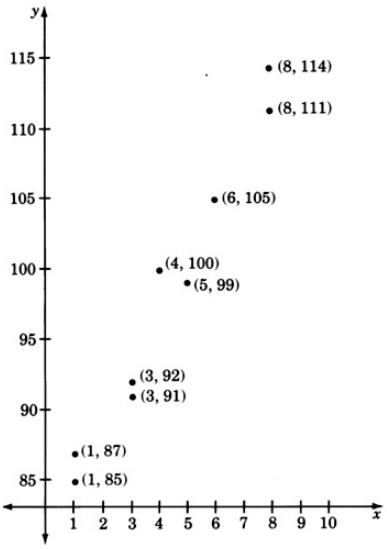
Table 7

Solution to Exercise (p. 7)
Yes, there does appear to be a relation.



Solution to Exercise (p. 8)

Yes, there does appear to be a relation.



Solution to Exercise (p. 9)

equation

Solution to Exercise (p. 10)

contradiction