Bisection Method*

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Abstract

Bisection Method

Calculating the root of an equation

$$f(x) = 0 \tag{1}$$

is a widely used problem in engineering and applied mathematics. In this chapter we will explore some simple numerical methods for solving this equation.

The function $f(x)$ will usually have atleast one continuous derivative, and often we will have some estimate of the root that is being computed. By using this information, most of the numerical methods compute a sequence of increasingly improved roots for (1). These methods are called iteration methods.

1 Rate of convergence

Here, the speed at which a convergent sequence approaches its limit is called the rate of convergence. This concept is of practical importance if we deal with a sequence of successive approximations for an iterative method, as then typically fewer iterations are needed to yield a useful approximation if the rate of convergence is higher.

1.1 Convergence of A Sequence

A sequence $< x_n >$ is said to be converges to α if for every $\varepsilon > 0$ there is an integer $m > 0$ such that if $n > m$ then $|x_n - \alpha| < \varepsilon$. The number α is called the limit of the sequence and we sometimes write $x_n \to c$.

1.2 Convergence speed for iterative methods

Let $< x_n >$ be sequence of successive approximations of a root $x = \alpha$ of the equation $f(x) = 0$. Then the sequence $< x_n >$ is said to be converges to α with order q if

$$\lim_{n\to\infty} \frac{|x_{n+1} - \alpha|}{|x_n - \alpha|^q} = \mu \tag{2}$$

where $\mu > 0$. Constant μ is called the rate of converegence.

Particularly, if $|x_{n+1} - \alpha| = \mu|x_n - \alpha|,\ n \geq 0,\ \ 0 < \mu < 1$, then converegence is called linear or of first order. Convergence with order 2 is called quadratic convergence, and convergence with order 3 is called cubic convergence.

*Version 1.1: May 31, 2009 1:45 am -0500

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2 Bisection Method

Suppose $f(x)$ is continuous on an interval $[a, b]$, such that

$$f(a).f(b) < 0 \tag{3}$$

Then $f(x)$ changes sign on $[a, b]$, and $f(x) = 0$ has at least one root on the interval. Bisection method repeatedly halve the interval $[a, b]$, keeping the half on which $f(x)$ chages sign. It guaranteed to converge to a root.

More prececisly, Suppose that we are given an interval $[a, b]$ satisfying (3) and an error tolerance $\varepsilon > 0$. Then the bisection method is consists of the following steps:

[B1.] Compute $c = (a + b)/2$

[B2.] If $b - c \leq \varepsilon$, then accept c as the root and stop the procedure.

[B3.] If $f(a).f(c) \leq 0$, then set $b = c$ else, set $a = c$. Go to step B1.