

IRRATIONAL NUMBERS AND ROUNDING OFF*

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1 Introduction

You have seen that repeating decimals may take a lot of paper and ink to write out. Not only is that impossible, but writing numbers out to many decimal places or a *high accuracy* is very inconvenient and rarely gives practical answers. For this reason we often estimate the number to a certain number of decimal places or to a given number of *significant figures*, which is even better.

2 Irrational Numbers

Irrational numbers are numbers that cannot be written as a fraction with the numerator and denominator as integers. This means that any number that is *not* a terminating decimal number or a repeating decimal number is irrational. Examples of irrational numbers are:

$$\sqrt{2}, \sqrt{3}, \sqrt[3]{4}, \pi, \frac{1+\sqrt{5}}{2} \approx 1,618\,033\,989 \quad (1)$$

TIP: When irrational numbers are written in decimal form, they go on forever and there is no repeated pattern of digits.

If you are asked to identify whether a number is rational or irrational, first write the number in decimal form. If the number is terminated then it is rational. If it goes on forever, then look for a repeated pattern of digits. If there is no repeated pattern, then the number is irrational.

When you write irrational numbers in decimal form, you may (if you have a lot of time and paper!) continue writing them for many, many decimal places. However, this is not convenient and it is often necessary to round off.

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2.1 Investigation : Irrational Numbers

Which of the following cannot be written as a rational number?

Remember: A rational number is a fraction with numerator and denominator as integers. Terminating decimal numbers or repeating decimal numbers are rational.

1. $\pi = 3, 14159265358979323846264338327950288419716939937510\dots$
2. 1,4
3. 1,618 033 989 ...
4. 100

3 Rounding Off

Rounding off or approximating a decimal number to a given number of decimal places is the quickest way to approximate a number. For example, if you wanted to round-off 2,6525272 to three decimal places then you would first count three places after the decimal.

$$2,652|5272 \quad (2)$$

All numbers to the right of | are ignored after you determine whether the number in the third decimal place must be rounded up or rounded down. You *round up* the final digit if the first digit after the | was greater or equal to 5 and *round down* (leave the digit alone) otherwise. In the case that the first digit before the | is 9 *and* the | you need to round up the 9 becomes a 0 and the second digit before the | is rounded up.

So, since the first digit after the | is a 5, we must round up the digit in the third decimal place to a 3 and the final answer of 2,6525272 rounded to three decimal places is

$$2,653 \quad (3)$$

Exercise 1: Rounding-Off

(Solution on p. 4.)

Round-off the following numbers to the indicated number of decimal places:

1. $\frac{120}{99} = 1,21212121\bar{2}$ to 3 decimal places
2. $\pi = 3,141592654\dots$ to 4 decimal places
3. $\sqrt{3} = 1,7320508\dots$ to 4 decimal places

4 Summary

- Irrational numbers are numbers that cannot be written as a fraction with the numerator and denominator as integers.
- For convenience irrational numbers are often rounded off to a specified number of decimal places

5 End of Chapter Exercises

1. Write the following rational numbers to 2 decimal places:
 - a. $\frac{1}{2}$
 - b. 1
 - c. $0,1111\bar{1}$
 - d. $0,9999\bar{1}$

Click here for the solution¹

2. Write the following irrational numbers to 2 decimal places:

- 3,141592654...
- 1,618033989...
- 1,41421356...
- 2,71828182845904523536...

Click here for the solution²

3. Use your calculator and write the following irrational numbers to 3 decimal places:

- $\sqrt{2}$
- $\sqrt{3}$
- $\sqrt{5}$
- $\sqrt{6}$

Click here for the solution³

4. Use your calculator (where necessary) and write the following irrational numbers to 5 decimal places:

- $\sqrt{8}$
- $\sqrt{768}$
- $\sqrt{100}$
- $\sqrt{0,49}$
- $\sqrt{0,0016}$
- $\sqrt{0,25}$
- $\sqrt{36}$
- $\sqrt{1960}$
- $\sqrt{0,0036}$
- $-8\sqrt{0,04}$
- $5\sqrt{80}$

Click here for the solution⁴

5. Write the following irrational numbers to 3 decimal places and then write them as a rational number to get an approximation to the irrational number. For example, $\sqrt{3} = 1,73205\dots$. To 3 decimal places, $\sqrt{3} = 1,732$. $1,732 = 1\frac{732}{1000} = 1\frac{183}{250}$. Therefore, $\sqrt{3}$ is approximately $1\frac{183}{250}$.

- 3,141592654...
- 1,618033989...
- 1,41421356...
- 2,71828182845904523536...

Click here for the solution⁵

¹See the file at <<http://cnx.org/content/m31341/latest/http://www.fhsst.org/11N>>

²See the file at <<http://cnx.org/content/m31341/latest/http://www.fhsst.org/11R>>

³See the file at <<http://cnx.org/content/m31341/latest/http://www.fhsst.org/11n>>

⁴See the file at <<http://cnx.org/content/m31341/latest/http://www.fhsst.org/11Q>>

⁵See the file at <<http://cnx.org/content/m31341/latest/http://www.fhsst.org/11U>>

Solutions to Exercises in this Module

Solution to Exercise (p. 2)

- Step 1. a. $\frac{120}{99} = 1,212\overline{121212}$
b. $\pi = 3,1415\overline{92654}$
c. $\sqrt{3} = 1,7320\overline{508}$
- Step 2. a. The last digit of $\frac{120}{99} = 1,212\overline{121212}$ must be rounded-down.
b. The last digit of $\pi = 3,1415\overline{92654}$ must be rounded-up.
c. The last digit of $\sqrt{3} = 1,7320\overline{508}$ must be rounded-up.
- Step 3. a. $\frac{120}{99} = 1,212$ rounded to 3 decimal places
b. $\pi = 3,1416$ rounded to 4 decimal places
c. $\sqrt{3} = 1,7321$ rounded to 4 decimal places