Discrete Time Impulse Response*

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Abstract

This module explains what is and how to use the Impulse Response of LTI systems.

1 Introduction

The output of a discrete time LTI system is completely determined by the input and the system's response to a unit impulse.

System Output

$$x[n] \rightarrow \boxed{\mathrm{H}} \rightarrow y[n]$$

Figure 1: We can determine the system's output, y[n], if we know the system's impulse response, h[n], and the input, x[n].

The output for a unit impulse input is called the impulse response.

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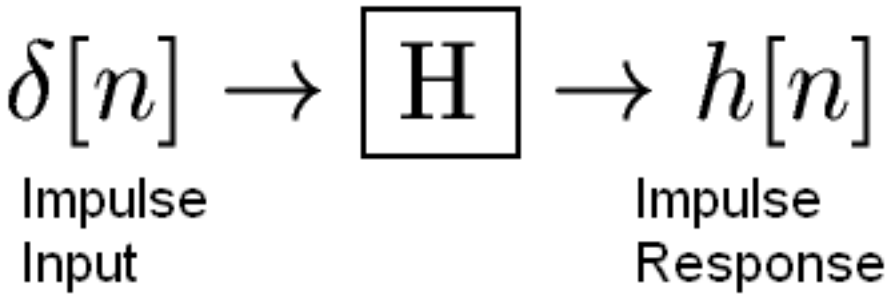
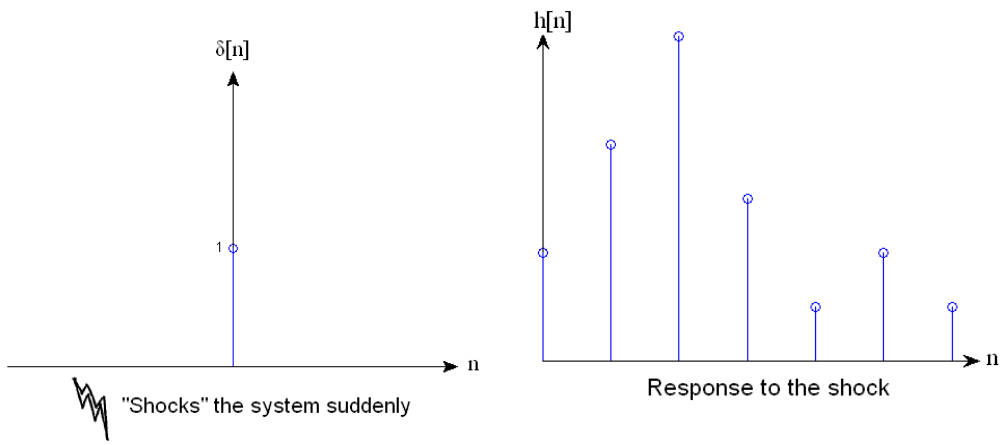


Figure 2



(a) (b)

Figure 3

1.1 Example Impulses

Since we are considering discrete time signals and systems, an ideal impulse is easy to simulate on a computer or some other digital device. It is simply a signal that is 1 at the point n = 0, and 0 everywhere else.

2 LTI Systems and Impulse Responses

2.1 Finding System Outputs

By the sifting property of impulses, any signal can be decomposed in terms of an infinite sum of shifted, scaled impulses.

$$\begin{aligned} \mathrm{x}[\mathrm{n}] &= \sum_{k=-\infty}^{\infty} \mathrm{x}[\mathrm{k}]\delta_k[\mathrm{n}] \\ &= \sum_{k=-\infty}^{\infty} \mathrm{x}[\mathrm{k}]\delta[\mathrm{n}-\mathrm{k}] \end{aligned} \tag{1}$$

The function $\delta_k[\mathrm{n}] = \delta[\mathrm{n} - \mathrm{k}]$ peaks up where $n = k$.

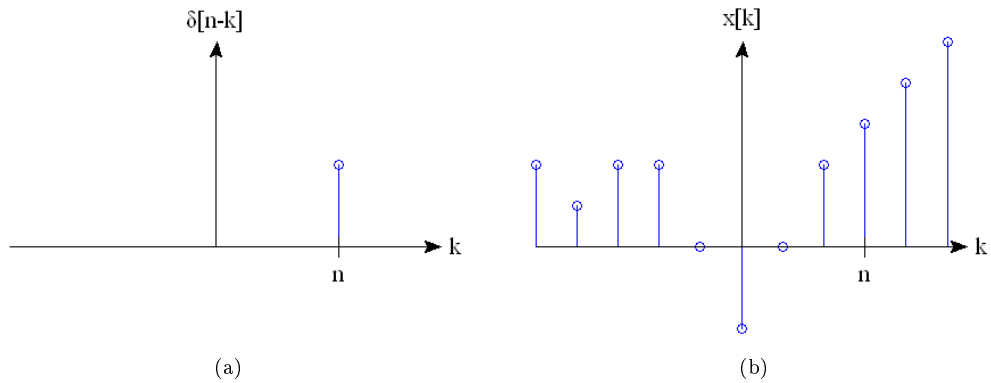


Figure 4

Since we know the response of the system to an impulse and any signal can be decomposed into impulses, all we need to do to find the response of the system to any signal is to decompose the signal into impulses, calculate the system's output for every impulse and add the outputs back together. This is the process known as **Convolution**. Since we are in **Discrete Time**, this is the **Discrete Time Convolution Sum**.

2.2 Finding Impulse Responses

Theory:

a. Solve the system's **Difference Equation** for y[n] with f[n] = δ[n]
b. Use the **Z-Transform**

Practice:

a. Apply an **impulse** input signal to the system and record the output
b. Use **Fourier methods**

We will assume that h[n] is given for now.

The goal is now to comput the output y[n] given the impulse response h[n] and the input x[n].

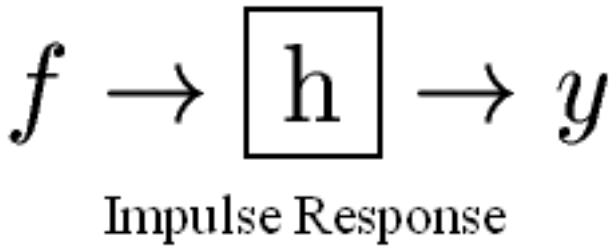


Figure 5

3 Impulse Response Summary

When a system is "shocked" by a delta function, it produces an output known as its impulse response. For an LTI system, the impulse response completely determines the output of the system given any arbitrary input. The output can be found using discrete time convolution.