Continuous Time Aperiodic Signals*

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Abstract

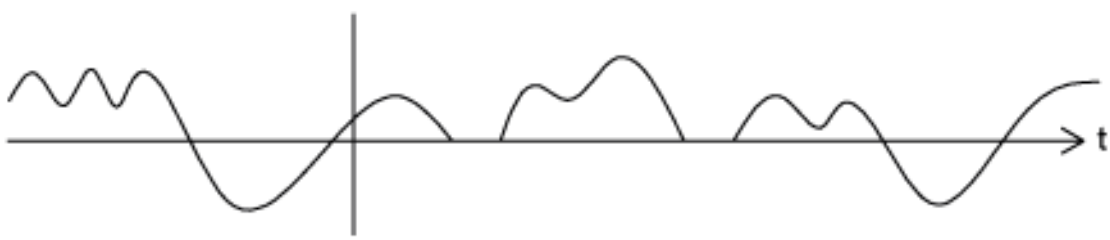
This module describes continuous time aperiodic signals.

1 Introduction

This module describes the type of signals acted on by the Continuous Time Fourier Transform.

2 Relevant Spaces

The Continuous-Time Fourier Transform maps infinite-length (a-periodic), continuous-time signals in L^2 to infinite-length, discrete-frequency signals in l^2.



3 Periodic and Aperiodic Signals

When a function repeats itself exactly after some given period, or cycle, we say it's **periodic**. A **periodic function** can be mathematically defined as:

$$f(t) = f(t + mT) \, \forall m : (m \in \mathbb{Z}) \tag{1}$$

where $T > 0$ represents the **fundamental period** of the signal, which is the smallest positive value of T for the signal to repeat. Because of this, you may also see a signal referred to as a T-periodic signal. Any function that satisfies this equation is said to be **periodic** with period T.

An **aperiodic** CT function $f(t)$ does not repeat for **any**$T \in \mathbb{R}$; *i.e.* there exists no T such that this equation (1) holds.

Suppose we have such an aperiodic function $f(t)$. We can construct a periodic extension of $f(t)$ called $f_{To}(t)$, where $f(t)$ is repeated every T_0 seconds. If we take the limit as $T_0 \to \infty$, we obtain a precise model of

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an aperiodic signal for which all rules that govern periodic signals can be applied, including Fourier Analysis (with an important modification). For more detail on this distinction, see the module on the **Continuous Time Fourier Transform**.

4 Aperiodic Signal Demonstration

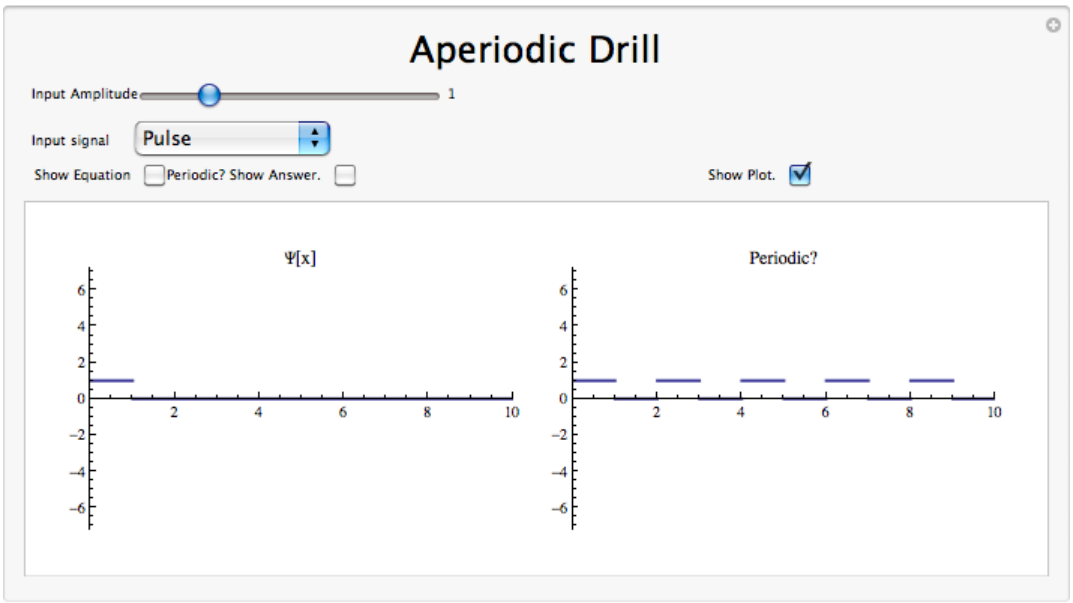

Figure 1: Interact (when online) with a Mathematica CDF demonstrating Periodic versus Aperiodic Signals.To download, right-click and save as .cdf.

5 Conclusion

Any aperiodic signal can be defined by an infinite sum of periodic functions, a useful definition that makes it possible to use Fourier Analysis on it by assuming all frequencies are present in the signal.