Discrete Time Convolution and the DTFT*

Stephen Kruzick
Dan Calderon

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Abstract

This module describes the relationship between discrete convolution and the DTFT.

1 Introduction

This module discusses convolution of discrete signals in the time and frequency domains.

2 The Discrete-Time Convolution

2.1 Discrete Time Fourier Transform

The DTFT transforms an infinite-length discrete signal in the time domain into an finite-length (or 2π-periodic) continuous signal in the frequency domain.

DTFT

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-(j\omega n)} \tag{1}$$

Inverse DTFT

$$x(n) = \frac{1}{2\pi} \int_0^{2\pi} X(\omega) e^{j\omega n} d\omega \tag{2}$$

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2.2 Demonstration

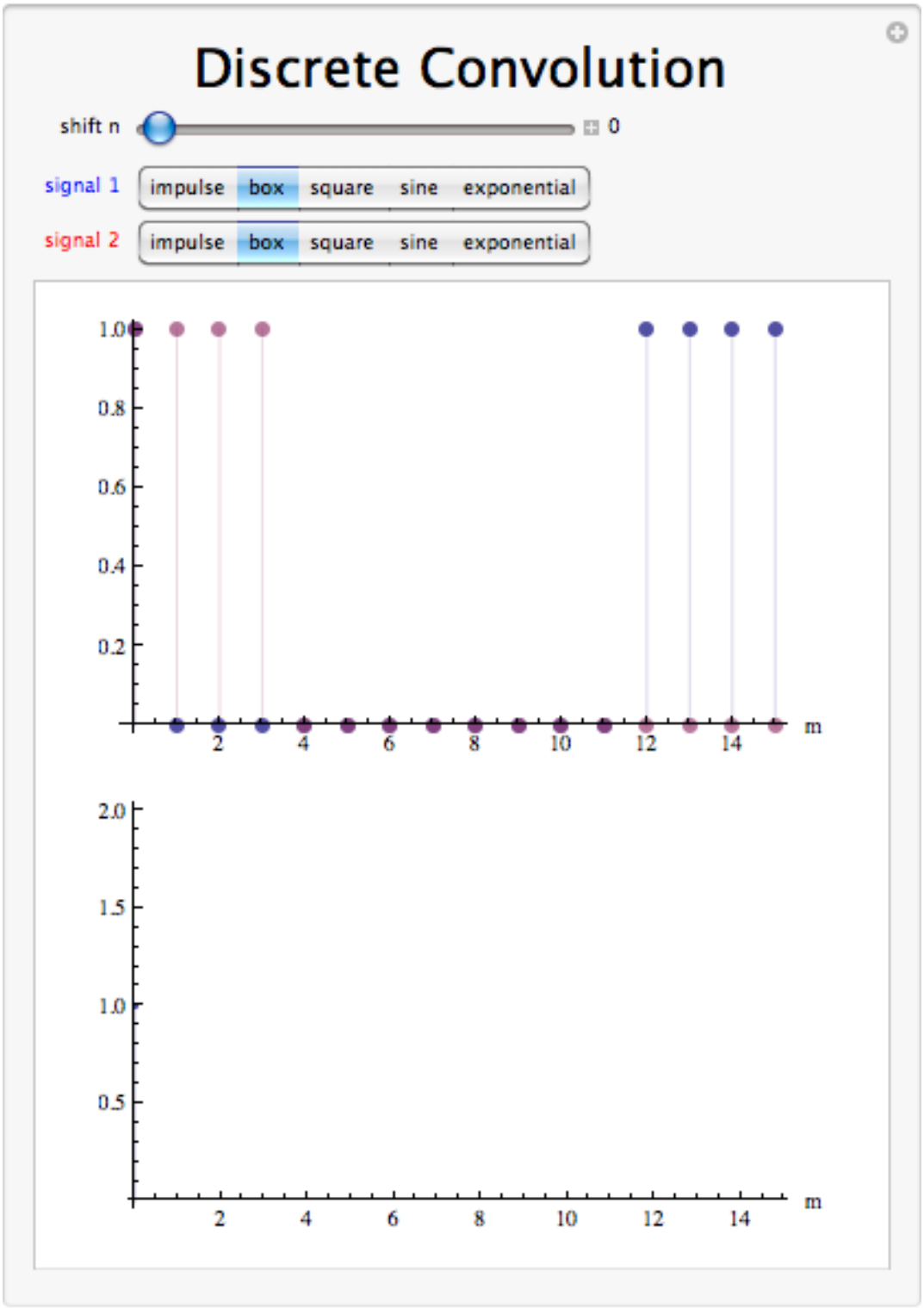

Figure 1: Interact (when online) with a Mathematica CDF demonstrating the Discrete Convolution. To Download, right-click and save as .cdf.

2.3 Convolution Sum

As mentioned above, the convolution sum provides a concise, mathematical way to express the output of an LTI system based on an arbitrary discrete-time input signal and the system's impulse response. The convolution sum is expressed as

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] \tag{3}$$

As with continuous-time, convolution is represented by the symbol *, and can be written as

$$y[n] = x[n] * h[n] \tag{4}$$

Convolution is commutative. For more information on the characteristics of convolution, read about the Properties of Convolution[1].

2.4 Convolution Theorem

Let f and g be two functions with convolution $f * g$.. Let F be the Fourier transform operator. Then

$$F(f * g) = F(f) \cdot F(g) \tag{5}$$

$$F(f \cdot g) = F(f) * F(g) \tag{6}$$

By applying the inverse Fourier transform F^{-1}, we can write:

$$f * g = F^{-1}(F(f) \cdot F(g)) \tag{7}$$

3 Conclusion

The Fourier transform of a convolution is the pointwise product of Fourier transforms. In other words, convolution in one domain (e.g., time domain) corresponds to point-wise multiplication in the other domain (e.g., frequency domain).

[1] "Properties of Continuous Time Convolution" <http://cnx.org/content/m10088/latest/>