Multiplication and Division of Whole Numbers: Concepts of Division of Whole Numbers[*]

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Abstract

This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to divide whole numbers. By the end of the module students should be able to understand the process of division, understand division of a nonzero number into zero, understand why division by zero is undefined, and use a calculator to divide one whole number by another.

1 Section Overview

- Division
- Division into Zero (Zero As a Dividend: $\frac{0}{a}$, $a \neq 0$)
- Division by Zero (Zero As a Divisor: $\frac{0}{a}$, $a \neq 0$)
- Division by and into Zero (Zero As a Dividend and Divisor: $\frac{0}{0}$)
- Calculators

2 Division

Division is a description of repeated subtraction.

In the process of division, the concern is how many times one number is contained in another number. For example, we might be interested in how many 5's are contained in 15. The word *times* is significant because it implies a relationship between division and multiplication.

There are several notations used to indicate division. Suppose Q records the number of times 5 is contained in 15. We can indicate this by writing

$$\underbrace{5 \overline{)15}}_{5 \text{ into } 15} \qquad \underbrace{\frac{15}{5} = Q}_{15 \text{ divided by } 5}$$

$$\underbrace{15/5 = Q}_{\text{15 divided by 5}} \qquad \underbrace{15 \div 5 = Q}_{\text{15 divided by 5}}$$

Each of these division notations describes the *same* number, represented here by the symbol Q. Each notation also converts to the same multiplication form. It is $15 = 5 \times Q$

In division,

Dividend

the number being divided into is called the **dividend**.

Divisor

the number dividing into the dividend is the **divisor**.

Quotient

the result of the division is called the **quotient**.

$$\text{divisor}\overline{)\text{dividend}}^{\text{quotient}}$$

$\frac{\text{dividend}}{\text{divisor}} = \text{quotient}$

$\text{dividend}/\text{divisor} = \text{quotient} \quad \text{dividend} \div \text{divisor} = \text{quotient}$

2.1 Sample Set A

Find the following quotients using multiplication facts.

Example 1

$18 \div 6$

Since $6 \times 3 = 18$,

$18 \div 6 = 3$

Notice also that

$$\begin{array}{r} 18 \\ -6 \\ \hline 12 \\ -6 \\ \hline 6 \\ -6 \\ \hline 0 \end{array} \Big\} \text{Repeated subtraction}$$

Thus, 6 is contained in 18 three times.

Example 2

$\frac{24}{3}$

Since $3 \times 8 = 24$,

$\frac{24}{3} = 8$

Notice also that 3 could be subtracted exactly 8 times from 24. This implies that 3 is contained in 24 eight times.

Example 3

$\frac{36}{6}$

Since $6 \times 6 = 36$,

$\frac{36}{6} = 6$

Thus, there are 6 sixes in 36.

Example 4

$9\overline{)72}$

Since $9 \times 8 = 72$,

$$8$$
$$9)\overline{72}$$

Thus, there are 8 nines in 72.

2.2 Practice Set A

Use multiplication facts to determine the following quotients.

Exercise 1   *(Solution on p. 9.)*

$32 \div 8$

Exercise 2   *(Solution on p. 9.)*

$18 \div 9$

Exercise 3   *(Solution on p. 9.)*

$\frac{25}{5}$

Exercise 4   *(Solution on p. 9.)*

$\frac{48}{8}$

Exercise 5   *(Solution on p. 9.)*

$\frac{28}{7}$

Exercise 6   *(Solution on p. 9.)*

$4)\overline{36}$

3 Division into Zero (Zero as a Dividend: $\frac{0}{a}$, $a \neq 0$)

Let's look at what happens when the dividend (the number being divided into) is zero, and the divisor (the number doing the dividing) is any whole number except zero. The question is

What number, if any, is $\frac{0}{\text{any nonzero whole number}}$?

Let's represent this unknown quotient by Q. Then,

$\frac{0}{\text{any nonzero whole number}} = Q$

Converting this division problem to its corresponding multiplication problem, we get

$0 = Q \times (\text{any nonzero whole number})$

From our knowledge of multiplication, we can understand that if the product of two whole numbers is zero, then one or both of the whole numbers must be zero. Since any nonzero whole number is certainly not zero, Q must represent zero. Then,

$\frac{0}{\text{any nonzero whole number}} = 0$

Zero Divided By Any Nonzero Whole Number Is Zero

Zero divided any nonzero whole number is zero.

4 Division by Zero (Zero as a Divisor: $\frac{a}{0}$, $a \neq 0$)

Now we ask,

What number, if any, is $\frac{\text{any nonzero whole number}}{0}$?

Letting Q represent a possible quotient, we get

$\frac{\text{any nonzero whole number}}{0} = Q$

Converting to the corresponding multiplication form, we have

$(\text{any nonzero whole number}) = Q \times 0$

Since $Q \times 0 = 0$, $(\text{any nonzero whole number}) = 0$. But this is absurd. This would mean that $6 = 0$, or $37 = 0$. A nonzero whole number *cannot* equal 0! Thus,

$\frac{\text{any nonzero whole number}}{0} \, does \, not$ name a number

Division by Zero is Undefined
Division by zero does not name a number. It is, therefore, undefined.

5 Division by and Into Zero (Zero as a Dividend and Divisor:$\frac{0}{0}$)

We are now curious about zero divided by zero $\left(\frac{0}{0}\right)$. If we let Q represent a potential quotient, we get
$\frac{0}{0} = Q$
Converting to the multiplication form,
$0 = Q \times 0$
This results in
$0 = 0$
This is a statement that is true regardless of the number used in place of Q. For example,
$\frac{0}{0} = 5$, since $0 = 5 \times 0$.
$\frac{0}{0} = 31$, since $0 = 31 \times 0$.
$\frac{0}{0} = 286$, since $0 = 286 \times 0$.
A *unique* quotient cannot be determined.
Indeterminant
Since the result of the division is inconclusive, we say that $\frac{0}{0}$ is **indeterminant**.
$\frac{0}{0}$ **is Indeterminant**
The division $\frac{0}{0}$ is indeterminant.

5.1 Sample Set B

Perform, if possible, each division.

Example 5
$\frac{19}{0}$. Since division by 0 does not name a whole number, no quotient exists, and we state $\frac{19}{0}$ is undefined

Example 6
$0\overline{)14}$. Since division by 0 does not name a defined number, no quotient exists, and we state

$0\overline{)14}$ is undefined

Example 7

$9\overline{)0}$. Since division into 0 by any nonzero whole number results in 0, we have $9\overline{)0}^{\,0}$

Example 8
$\frac{0}{7}$. Since division into 0 by any nonzero whole number results in 0, we have $\frac{0}{7} = 0$

5.2 Practice Set B

Perform, if possible, the following divisions.

Exercise 7   *(Solution on p. 9.)*
$\frac{5}{0}$

Exercise 8   *(Solution on p. 9.)*
$\frac{0}{4}$

Exercise 9   *(Solution on p. 9.)*
$0\overline{)0}$

Exercise 10   *(Solution on p. 9.)*

$$0\overline{)8}$$

Exercise 11   *(Solution on p. 9.)*

$$\frac{9}{0}$$

Exercise 12   *(Solution on p. 9.)*

$$\frac{0}{1}$$

6 Calculators

Divisions can also be performed using a calculator.

6.1 Sample Set C

Example 9
Divide 24 by 3.

Display Reads		
Type	24	24
Press	÷	24
Type	3	3
Press	=	8

Table 1

The display now reads 8, and we conclude that $24 \div 3 = 8$.

Example 10
Divide 0 by 7.

Display Reads		
Type	0	0
Press	÷	0
Type	7	7
Press	=	0

Table 2

The display now reads 0, and we conclude that $0 \div 7 = 0$.

Example 11
Divide 7 by 0.

Since division by zero is undefined, the calculator should register some kind of error message.

Display Reads		
Type	7	7
Press	÷	7
Type	0	0
Press	=	Error

Table 3

The error message indicates an undefined operation was attempted, in this case, division by zero.

6.2 Practice Set C

Use a calculator to perform each division.

Exercise 13 *(Solution on p. 9.)*
$35 \div 7$

Exercise 14 *(Solution on p. 9.)*
$56 \div 8$

Exercise 15 *(Solution on p. 9.)*
$0 \div 6$

Exercise 16 *(Solution on p. 9.)*
$3 \div 0$

Exercise 17 *(Solution on p. 9.)*
$0 \div 0$

7 Exercises

For the following problems, determine the quotients (if possible). You may use a calculator to check the result.

Exercise 18 *(Solution on p. 9.)*
$4\overline{)32}$

Exercise 19
$7\overline{)42}$

Exercise 20 *(Solution on p. 9.)*
$6\overline{)18}$

Exercise 21
$2\overline{)14}$

Exercise 22 *(Solution on p. 9.)*
$3\overline{)27}$

Exercise 23
$1\overline{)6}$

Exercise 24   *(Solution on p. 9.)*

$4\overline{)28}$

Exercise 25

$\frac{30}{5}$

Exercise 26   *(Solution on p. 9.)*

$\frac{16}{4}$

Exercise 27

$24 \div 8$

Exercise 28   *(Solution on p. 9.)*

$10 \div 2$

Exercise 29

$21 \div 7$

Exercise 30   *(Solution on p. 9.)*

$21 \div 3$

Exercise 31

$0 \div 6$

Exercise 32   *(Solution on p. 9.)*

$8 \div 0$

Exercise 33

$12 \div 4$

Exercise 34   *(Solution on p. 10.)*

$3\overline{)9}$

Exercise 35

$0\overline{)0}$

Exercise 36   *(Solution on p. 10.)*

$7\overline{)0}$

Exercise 37

$6\overline{)48}$

Exercise 38   *(Solution on p. 10.)*

$\frac{15}{3}$

Exercise 39

$\frac{35}{0}$

Exercise 40   *(Solution on p. 10.)*

$56 \div 7$

Exercise 41

$\frac{0}{9}$

Exercise 42   *(Solution on p. 10.)*

$72 \div 8$

Exercise 43

Write $\frac{16}{2} = 8$ using three different notations.

Exercise 44   *(Solution on p. 10.)*

Write $\frac{27}{9} = 3$ using three different notations.

Exercise 45

In the statement $\dfrac{4}{6\overline{)24}}$

6 is called the _____ .

24 is called the _____ .

4 is called the _____ .

Exercise 46   *(Solution on p. 10.)*

In the statement $56 \div 8 = 7$,

7 is called the _____ .

8 is called the _____ .

56 is called the _____ .

7.1 Exercises for Review

Exercise 47

(here[1]) What is the largest digit?

Exercise 48   *(Solution on p. 10.)*

(here[2]) Find the sum. $\begin{array}{r} 8,006 \\ +4,118 \\ \hline \end{array}$

Exercise 49

(here[3]) Find the difference. $\begin{array}{r} 631 \\ -589 \\ \hline \end{array}$

Exercise 50   *(Solution on p. 10.)*

(here[4]) Use the numbers 2, 3, and 7 to illustrate the associative property of addition.

Exercise 51

(here[5]) Find the product. $\begin{array}{r} 86 \\ \times 12 \\ \hline \end{array}$

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[1]"Addition and Subtraction of Whole Numbers: Whole Numbers" <http://cnx.org/content/m34795/latest/>

[2]"Addition and Subtraction of Whole Numbers: Addition of Whole Numbers" <http://cnx.org/content/m34786/latest/>

[3]"Addition and Subtraction of Whole Numbers: Subtraction of Whole Numbers"
<http://cnx.org/content/m34784/latest/>

[4]"Addition and Subtraction of Whole Numbers: Properties of Addition" <http://cnx.org/content/m34802/latest/>

[5]"Multiplication and Division of Whole Numbers: Multiplication of Whole Numbers"
<http://cnx.org/content/m34863/latest/>

Solutions to Exercises in this Module

Solution to Exercise (p. 3)
 4
Solution to Exercise (p. 3)
 2
Solution to Exercise (p. 3)
 5
Solution to Exercise (p. 3)
 6
Solution to Exercise (p. 3)
 4
Solution to Exercise (p. 3)
 9
Solution to Exercise (p. 4)
 undefined
Solution to Exercise (p. 4)
 0
Solution to Exercise (p. 4)
 indeterminant
Solution to Exercise (p. 4)
 undefined
Solution to Exercise (p. 5)
 undefined
Solution to Exercise (p. 5)
 0
Solution to Exercise (p. 6)
 5
Solution to Exercise (p. 6)
 7
Solution to Exercise (p. 6)
 0
Solution to Exercise (p. 6)
 An error message tells us that this operation is undefined. The particular message depends on the calculator.
Solution to Exercise (p. 6)
 An error message tells us that this operation cannot be performed. Some calculators actually set $0 \div 0$ equal to 1. We know better! $0 \div 0$ is indeterminant.
Solution to Exercise (p. 6)
 8
Solution to Exercise (p. 6)
 3
Solution to Exercise (p. 6)
 9
Solution to Exercise (p. 6)
 7
Solution to Exercise (p. 7)
 4
Solution to Exercise (p. 7)
 5
Solution to Exercise (p. 7)
 7

Solution to Exercise (p. 7)
 not defined
Solution to Exercise (p. 7)
 3
Solution to Exercise (p. 7)
 0
Solution to Exercise (p. 7)
 5
Solution to Exercise (p. 7)
 8
Solution to Exercise (p. 7)
 9
Solution to Exercise (p. 7)
 $27 \div 9 = 3; \quad 9\overline{)27} = 3 ; \frac{27}{9} = 3$
Solution to Exercise (p. 8)
 7 is quotient; 8 is divisor; 56 is dividend
Solution to Exercise (p. 8)
 12,124
Solution to Exercise (p. 8)
 $(2+3) + 7 = 2 + (3+7) = 12$

 $5 \quad + \quad 7 \quad = \quad 2 \quad + \quad 10 \quad = \quad 12$