Addition and Subtraction of Fractions, Comparing Fractions, and Complex Fractions: Addition and Subtractions of Fractions with Like Denominators*

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Abstract

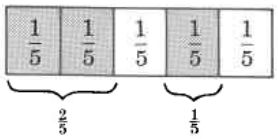
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to add and subtract fractions with like denominators. By the end of the module students should be able to add and subtract fractions with like denominators.

1 Section Overview

- Addition of Fraction With Like Denominators
- Subtraction of Fractions With Like Denominators

2 Addition of Fraction With Like Denominators

Let's examine the following diagram.



2 one-fifths and 1 one fifth is shaded.

It is shown in the shaded regions of the diagram that
(2 one-fifths) + (1 one-fifth) = (3 one-fifths)

*Version 1.2: Aug 18, 2010 8:01 pm -0500
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That is,
$\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$
From this observation, we can suggest the following rule.

Method of Adding Fractions Having Like Denominators

To add two or more fractions that have the same denominators, add the numerators and place the resulting sum over the common denominator. Reduce, if necessary.

2.1 Sample Set A

Find the following sums.

Example 1
$\frac{3}{7} + \frac{2}{7}$. The denominators are the same. Add the numerators and place that sum over 7.
$\frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{5}{7}$

Example 2
$\frac{1}{8} + \frac{3}{8}$. The denominators are the same. Add the numerators and place the sum over 8. Reduce.
$\frac{1}{8} + \frac{3}{8} = \frac{1+3}{8} = \frac{4}{8} = \frac{1}{2}$

Example 3
$\frac{4}{9} + \frac{5}{9}$. The denominators are the same. Add the numerators and place the sum over 9.
$\frac{4}{9} + \frac{5}{9} = \frac{4+5}{9} = \frac{9}{9} = 1$

Example 4
$\frac{7}{8} + \frac{5}{8}$. The denominators are the same. Add the numerators and place the sum over 8.
$\frac{7}{8} + \frac{5}{8} = \frac{7+5}{8} = \frac{12}{8} = \frac{3}{2}$

Example 5
To see what happens if we *mistakenly add the denominators* as well as the numerators, let's add
$\frac{1}{2} + \frac{1}{2}$
Adding the numerators and *mistakenly* adding the denominators produces
$\frac{1}{2} + \frac{1}{2} = \frac{1+1}{2+2} = \frac{2}{4} = \frac{1}{2}$
This means that two $\frac{1}{2}$'s is the same as one $\frac{1}{2}$. Preposterous! **We do not add denominators**.

2.2 Practice Set A

Find the following sums.

Exercise 1 ***(Solution on p. 7.)***
$\frac{1}{10} + \frac{3}{10}$

Exercise 2 ***(Solution on p. 7.)***
$\frac{1}{4} + \frac{1}{4}$

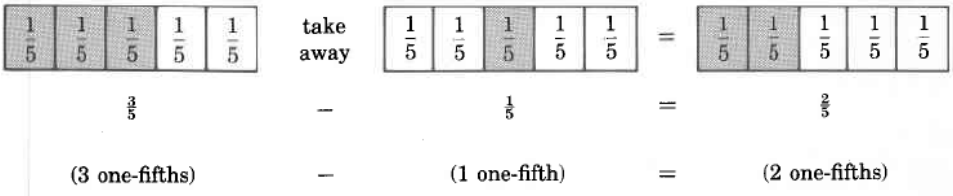
Exercise 3 ***(Solution on p. 7.)***
$\frac{7}{11} + \frac{4}{11}$

Exercise 4 ***(Solution on p. 7.)***
$\frac{3}{5} + \frac{1}{5}$

Exercise 5 ***(Solution on p. 7.)***
Show why adding both the numerators and denominators is preposterous by adding $\frac{3}{4}$ and $\frac{3}{4}$ and examining the result.

3 Subtraction of Fractions With Like Denominators

We can picture the concept of subtraction of fractions in much the same way we pictured addition.



From this observation, we can suggest the following rule for subtracting fractions having like denominators:

Subtraction of Fractions with Like Denominators

To subtract two fractions that have like denominators, subtract the numerators and place the resulting difference over the common denominator. Reduce, if possible.

3.1 Sample Set B

Find the following differences.

Example 6

$\frac{3}{5} - \frac{1}{5}$. The denominators are the same. Subtract the numerators. Place the difference over 5.

$\frac{3}{5} - \frac{1}{5} = \frac{3-1}{5} = \frac{2}{5}$

Example 7

$\frac{8}{6} - \frac{2}{6}$. The denominators are the same. Subtract the numerators. Place the difference over 6.

$\frac{8}{6} - \frac{2}{6} = \frac{8-2}{6} = \frac{6}{6} = 1$

Example 8

$\frac{16}{9} - \frac{2}{9}$. The denominators are the same. Subtract numerators and place the difference over 9.

$\frac{16}{9} - \frac{2}{9} = \frac{16-2}{9} = \frac{14}{9}$

Example 9

To see what happens if we *mistakenly* subtract the denominators, let's consider

$\frac{7}{15} - \frac{4}{15} = \frac{7-4}{15-15} = \frac{3}{0}$

We get division by zero, which is undefined. **We do not subtract denominators.**

3.2 Practice Set B

Find the following differences.

Exercise 6 *(Solution on p. 7.)*

$\frac{10}{13} - \frac{8}{13}$

Exercise 7 *(Solution on p. 7.)*

$\frac{5}{12} - \frac{1}{12}$

Exercise 8 *(Solution on p. 7.)*

$\frac{1}{2} - \frac{1}{2}$

Exercise 9 *(Solution on p. 7.)*

$\frac{26}{10} - \frac{14}{10}$

Exercise 10 *(Solution on p. 7.)*

Show why subtracting both the numerators and the denominators is in error by performing the subtraction $\frac{5}{9} - \frac{2}{9}$.

4 Exercises

For the following problems, find the sums and differences. Be sure to reduce.

Exercise 11 *(Solution on p. 7.)*

$\frac{3}{8} + \frac{2}{8}$

Exercise 12

$\frac{1}{6} + \frac{2}{6}$

Exercise 13 *(Solution on p. 7.)*

$\frac{9}{10} + \frac{1}{10}$

Exercise 14

$\frac{3}{11} + \frac{4}{11}$

Exercise 15 *(Solution on p. 7.)*

$\frac{9}{15} + \frac{4}{15}$

Exercise 16

$\frac{3}{10} + \frac{2}{10}$

Exercise 17 *(Solution on p. 7.)*

$\frac{5}{12} + \frac{7}{12}$

Exercise 18

$\frac{11}{16} - \frac{2}{16}$

Exercise 19 *(Solution on p. 7.)*

$\frac{3}{16} - \frac{3}{16}$

Exercise 20

$\frac{15}{23} - \frac{2}{23}$

Exercise 21 *(Solution on p. 7.)*

$\frac{1}{6} - \frac{1}{6}$

Exercise 22

$\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

Exercise 23 *(Solution on p. 7.)*

$\frac{3}{11} + \frac{1}{11} + \frac{5}{11}$

Exercise 24

$\frac{16}{20} + \frac{1}{20} + \frac{2}{20}$

Exercise 25 *(Solution on p. 7.)*

$\frac{12}{8} + \frac{2}{8} + \frac{1}{8}$

Exercise 26

$\frac{1}{15} + \frac{8}{15} + \frac{6}{15}$

Exercise 27 *(Solution on p. 7.)*

$\frac{3}{8} + \frac{2}{8} - \frac{1}{8}$

Exercise 28

$\frac{11}{16} + \frac{9}{16} - \frac{5}{16}$

Exercise 29 *(Solution on p. 7.)*

$\frac{4}{20} - \frac{1}{20} + \frac{9}{20}$

Exercise 30

$\frac{7}{10} - \frac{3}{10} + \frac{11}{10}$

Exercise 31 *(Solution on p. 7.)*

$\frac{16}{5} - \frac{1}{5} - \frac{2}{5}$

Exercise 32
$\frac{21}{35} - \frac{17}{35} + \frac{31}{35}$

Exercise 33 *(Solution on p. 7.)*
$\frac{5}{2} + \frac{16}{2} - \frac{1}{2}$

Exercise 34
$\frac{1}{18} + \frac{3}{18} + \frac{1}{18} + \frac{4}{18} - \frac{5}{18}$

Exercise 35 *(Solution on p. 7.)*
$\frac{6}{22} - \frac{2}{22} + \frac{4}{22} - \frac{1}{22} + \frac{11}{22}$

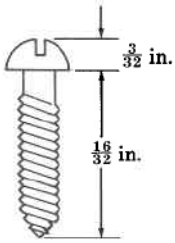
The following rule for addition and subtraction of two fractions is preposterous. Show why by performing the operations using the rule for the following two problems.

Preposterous Rule
To add or subtract two fractions, simply add or subtract the numerators and place this result over the sum or difference of the denominators.

Exercise 36
$\frac{3}{10} - \frac{3}{10}$

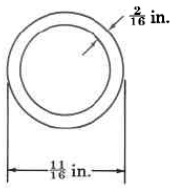
Exercise 37 *(Solution on p. 7.)*
$\frac{8}{15} + \frac{8}{15}$

Exercise 38
Find the total length of the screw.



Exercise 39 *(Solution on p. 7.)*
Two months ago, a woman paid off $\frac{3}{24}$ of a loan. One month ago, she paid off $\frac{5}{24}$ of the total loan. This month she will again pay off $\frac{5}{24}$ of the total loan. At the end of the month, how much of her total loan will she have paid off?

Exercise 40
Find the inside diameter of the pipe.



4.1 Exercises for Review

Exercise 41 *(Solution on p. 8.)*
(here[1]) Round 2,650 to the nearest hundred.

[1] "Addition and Subtraction of Whole Numbers: Rounding Whole Numbers" <http://cnx.org/content/m34780/latest/>

Exercise 42

(here[2]) Use the numbers 2, 4, and 8 to illustrate the associative property of addition.

Exercise 43 *(Solution on p. 8.)*

(here[3]) Find the prime factors of 495.

Exercise 44

(here[4]) Find the value of $\frac{3}{4} \cdot \frac{16}{25} \cdot \frac{5}{9}$.

Exercise 45 *(Solution on p. 8.)*

(here[5]) $\frac{8}{3}$ of what number is $1\frac{7}{9}$?

[2]"Multiplication and Division of Whole Numbers: Properties of Multiplication" <http://cnx.org/content/m34867/latest/>

[3]"Exponents, Roots, Factorization of Whole Numbers: Prime Factorization of Natural Numbers" <http://cnx.org/content/m34873/latest/>

[4]"Introduction to Fractions and Multiplication and Division of Fractions: Multiplication of Fractions" <http://cnx.org/content/m34928/latest/>

[5]"Introduction to Fractions and Multiplication and Division of Fractions: Applications Involving Fractions" <http://cnx.org/content/m34930/latest/>

Solutions to Exercises in this Module

Solution to Exercise (p. 2)

$\frac{2}{5}$

Solution to Exercise (p. 2)

$\frac{1}{2}$

Solution to Exercise (p. 2)

1

Solution to Exercise (p. 2)

$\frac{4}{5}$

Solution to Exercise (p. 2)

$\frac{3}{4} + \frac{3}{4} = \frac{3+3}{4+4} = \frac{6}{8} = \frac{3}{4}$, so two $\frac{3}{4}$'s= one $\frac{3}{4}$ which is preposterous.

Solution to Exercise (p. 3)

$\frac{2}{13}$

Solution to Exercise (p. 3)

$\frac{1}{3}$

Solution to Exercise (p. 3)

0

Solution to Exercise (p. 3)

$\frac{6}{5}$

Solution to Exercise (p. 3)

$\frac{5}{9} - \frac{2}{9} = \frac{5-2}{9-9} = \frac{3}{0}$, which is undefined

Solution to Exercise (p. 4)

$\frac{5}{8}$

Solution to Exercise (p. 4)

1

Solution to Exercise (p. 4)

$\frac{13}{15}$

Solution to Exercise (p. 4)

1

Solution to Exercise (p. 4)

0

Solution to Exercise (p. 4)

0

Solution to Exercise (p. 4)

$\frac{9}{11}$

Solution to Exercise (p. 4)

$\frac{15}{8}$

Solution to Exercise (p. 4)

$\frac{1}{2}$

Solution to Exercise (p. 4)

$\frac{3}{5}$

Solution to Exercise (p. 4)

$\frac{13}{5}$

Solution to Exercise (p. 5)

10

Solution to Exercise (p. 5)

$\frac{9}{11}$

Solution to Exercise (p. 5)

$\frac{16}{30} = \frac{8}{5}$(using the preposterous rule)

Solution to Exercise (p. 5)

$\frac{13}{24}$

Solution to Exercise (p. 5)

2700

Solution to Exercise (p. 6)

$3^2 \cdot 5 \cdot 11$

Solution to Exercise (p. 6)

$\frac{2}{3}$