

ADDITION AND SUBTRACTION OF FRACTIONS, COMPARING FRACTIONS, AND COMPLEX FRACTIONS: COMBINATIONS OF OPERATIONS WITH FRACTIONS*

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Abstract

This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses combinations of operations with fractions. By the end of the module students should gain a further understanding of the order of operations.

1 Section Overview

- The Order of Operations

2 The Order of Operations

To determine the value of a quantity such as

$$\frac{1}{2} + \frac{5}{8} \cdot \frac{2}{15}$$

where we have a combination of operations (more than one operation occurs), we must use the accepted order of operations.

The Order of Operations:

1. In the order (2), (3), (4) described below, perform all operations inside grouping symbols: (), [], (), —. Work from the innermost set to the outermost set.
2. Perform exponential and root operations.
3. Perform all multiplications and divisions moving left to right.
4. Perform all additions and subtractions moving left to right.

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2.1 Sample Set A

Determine the value of each of the following quantities.

Example 1

$$\frac{1}{4} + \frac{5}{8} \cdot \frac{2}{15}$$

- (a) Multiply first.

$$\frac{1}{4} + \frac{\frac{5}{8} \cdot \frac{2}{15}}{1} = \frac{1}{4} + \frac{1 \cdot 1}{4 \cdot 3} = \frac{1}{4} + \frac{1}{12}$$

- (b) Now perform this addition. Find the LCD.

$$\left. \begin{array}{l} 4=2^2 \\ 12=2^2 \cdot 3 \end{array} \right\} \text{The LCD} = 2^2 \cdot 3 = 12.$$

$$\begin{aligned} \frac{1}{4} + \frac{1}{12} &= \frac{1 \cdot 3}{12} + \frac{1}{12} = \frac{3}{12} + \frac{1}{12} \\ &= \frac{3+1}{12} = \frac{4}{12} = \frac{1}{3} \end{aligned}$$

$$\text{Thus, } \frac{1}{4} + \frac{5}{8} \cdot \frac{2}{15} = \frac{1}{3}$$

Example 2

$$\frac{3}{5} + \frac{9}{44} \left(\frac{5}{9} - \frac{1}{4} \right)$$

- (a) Operate within the parentheses first, $\left(\frac{5}{9} - \frac{1}{4}\right)$.

$$\left. \begin{array}{l} 9=3^2 \\ 4=2^2 \end{array} \right\} \text{The LCD} = 2^2 \cdot 3^2 = 4 \cdot 9 = 36.$$

$$\frac{5 \cdot 4}{36} - \frac{1 \cdot 9}{36} = \frac{20}{36} - \frac{9}{36} = \frac{20-9}{36} = \frac{11}{36}$$

Now we have

$$\frac{3}{5} + \frac{9}{44} \left(\frac{11}{36} \right)$$

- (b) Perform the multiplication.

$$\frac{3}{5} + \frac{\frac{9}{44} \cdot \frac{11}{36}}{1} = \frac{3}{5} + \frac{1 \cdot 1}{4 \cdot 4} = \frac{3}{5} + \frac{1}{16}$$

- (c) Now perform the addition. The LCD=80.

$$\frac{3}{5} + \frac{1}{16} = \frac{3 \cdot 16}{80} + \frac{1 \cdot 5}{80} = \frac{48}{80} + \frac{5}{80} = \frac{48+5}{80} = \frac{53}{80}$$

$$\text{Thus, } \frac{3}{5} + \frac{9}{44} \left(\frac{5}{9} - \frac{1}{4} \right) = \frac{53}{80}$$

Example 3

$$8 - \frac{15}{426} \left(2 - 1 \frac{4}{15} \right) \left(3 \frac{1}{5} + 2 \frac{1}{8} \right)$$

- (a) Work within each set of parentheses individually.

$$\begin{aligned} 2 - 1 \frac{4}{15} &= 2 \frac{1 \cdot 15 + 4}{15} = 2 - \frac{19}{15} \\ &= \frac{30}{15} - \frac{19}{15} = \frac{30-19}{15} = \frac{11}{15} \end{aligned}$$

$$\begin{aligned} 3 \frac{1}{5} + 2 \frac{1}{8} &= \frac{3 \cdot 5 + 1}{5} + \frac{2 \cdot 8 + 1}{8} \\ &= \frac{16}{5} + \frac{17}{8} \quad \text{LCD} = 40 \\ &= \frac{16 \cdot 8}{40} + \frac{17 \cdot 5}{40} \\ &= \frac{128}{40} + \frac{85}{40} \\ &= \frac{128+85}{40} \\ &= \frac{213}{40} \end{aligned}$$

Now we have

$$8 - \frac{15}{426} \left(\frac{11}{15} \right) \left(\frac{213}{40} \right)$$

(b) Now multiply.

$$8 - \frac{\frac{1}{15}}{\frac{1}{426}} \cdot \frac{11}{15} \cdot \frac{1}{40} = 8 - \frac{1 \cdot 11 \cdot 1}{2 \cdot 1 \cdot 40} = 8 - \frac{11}{80}$$

(c) Now subtract.

$$8 - \frac{11}{80} = \frac{80 \cdot 8}{80} - \frac{11}{80} = \frac{640}{80} - \frac{11}{80} = \frac{640-11}{80} = \frac{629}{80} \text{ or } 7 \frac{69}{80}$$

Thus, $8 - \frac{15}{426} \left(2 - 1 \frac{4}{15} \right) \left(3 \frac{1}{5} + 2 \frac{1}{8} \right) = 7 \frac{69}{80}$

Example 4

$$\left(\frac{3}{4} \right)^2 \cdot \frac{8}{9} - \frac{5}{12}$$

(a) Square $\frac{3}{4}$.

$$\left(\frac{3}{4} \right)^2 = \frac{3}{4} \cdot \frac{3}{4} = \frac{3 \cdot 3}{4 \cdot 4} = \frac{9}{16}$$

Now we have

$$\frac{9}{16} \cdot \frac{8}{9} - \frac{5}{12}$$

(b) Perform the multiplication.

$$\frac{\frac{1}{9}}{\frac{1}{16}} \cdot \frac{1}{9} - \frac{5}{12} = \frac{1 \cdot 1}{2 \cdot 1} - \frac{5}{12} = \frac{1}{2} - \frac{5}{12}$$

(c) Now perform the subtraction.

$$\frac{1}{2} - \frac{5}{12} = \frac{6}{12} - \frac{5}{12} = \frac{6-5}{12} = \frac{1}{12}$$

$$\text{Thus, } \left(\frac{3}{4} \right)^2 \cdot \frac{8}{9} - \frac{5}{12} = \frac{1}{12}$$

Example 5

$$2 \frac{7}{8} + \sqrt{\frac{25}{36}} \div \left(2 \frac{1}{2} - 1 \frac{1}{3} \right)$$

(a) Begin by operating inside the parentheses.

$$\begin{aligned} 2 \frac{1}{2} - 1 \frac{1}{3} &= \frac{2 \cdot 2 + 1}{2} - \frac{1 \cdot 3 + 1}{3} = \frac{5}{2} - \frac{4}{3} \\ &= \frac{15}{6} - \frac{8}{6} = \frac{15-8}{6} = \frac{7}{6} \end{aligned}$$

(b) Now simplify the square root.

$$\sqrt{\frac{25}{36}} = \frac{5}{6} \left(\text{since } \left(\frac{5}{6} \right)^2 = \frac{25}{36} \right)$$

Now we have

$$2 \frac{7}{8} + \frac{5}{6} \div \frac{7}{6}$$

(c) Perform the division.

$$2 \frac{7}{8} + \frac{5}{6} \cdot \frac{1}{7} = 2 \frac{7}{8} + \frac{5 \cdot 1}{1 \cdot 7} = 2 \frac{7}{8} + \frac{5}{7}$$

(d) Now perform the addition.

$$\begin{aligned} 2 \frac{7}{8} + \frac{5}{7} &= \frac{2 \cdot 8 + 7}{8} + \frac{5}{7} = \frac{23}{8} + \frac{5}{7} \quad \text{LCD} = 56. \\ &= \frac{23 \cdot 7}{56} + \frac{5 \cdot 8}{56} = \frac{161}{56} + \frac{40}{56} \\ &= \frac{161+40}{56} = \frac{201}{56} \text{ or } 3 \frac{33}{56} \end{aligned}$$

$$\text{Thus, } 2 \frac{7}{8} + \sqrt{\frac{25}{36}} \div \left(2 \frac{1}{2} - 1 \frac{1}{3} \right) = 3 \frac{33}{56}$$

2.2 Practice Set A

Find the value of each of the following quantities.

Exercise 1

$$\frac{5}{16} \cdot \frac{1}{10} - \frac{1}{32}$$

(Solution on p. 7.)

Exercise 2

$$\frac{6}{7} \cdot \frac{21}{40} \div \frac{9}{10} + 5\frac{1}{3}$$

(Solution on p. 7.)

Exercise 3

$$8\frac{7}{10} - 2\left(4\frac{1}{2} - 3\frac{2}{3}\right)$$

(Solution on p. 7.)

Exercise 4

$$\frac{17}{18} - \frac{58}{30} \left(\frac{1}{4} - \frac{3}{32}\right) \left(1 - \frac{13}{29}\right)$$

(Solution on p. 7.)

Exercise 5

$$\left(\frac{1}{10} + 1\frac{1}{2}\right) \div \left(1\frac{4}{5} - 1\frac{6}{25}\right)$$

(Solution on p. 7.)

Exercise 6

$$\frac{\frac{2}{3} - \frac{3}{8} \cdot \frac{4}{9}}{\frac{7}{16} \cdot 1\frac{1}{3} + 1\frac{1}{4}}$$

(Solution on p. 7.)

Exercise 7

$$\left(\frac{3}{8}\right)^2 + \frac{3}{4} \cdot \frac{1}{8}$$

(Solution on p. 7.)

Exercise 8

$$\frac{2}{3} \cdot 2\frac{1}{4} - \sqrt{\frac{4}{25}}$$

(Solution on p. 7.)

3 Exercises

Find each value.

Exercise 9

$$\frac{4}{3} - \frac{1}{6} \cdot \frac{1}{2}$$

(Solution on p. 7.)

Exercise 10

$$\frac{7}{9} - \frac{4}{5} \cdot \frac{5}{36}$$

Exercise 11

$$2\frac{2}{7} + \frac{5}{8} \div \frac{5}{16}$$

(Solution on p. 7.)

Exercise 12

$$\frac{3}{16} \div \frac{9}{14} \cdot \frac{12}{21} + \frac{5}{6}$$

(Solution on p. 7.)

Exercise 13

$$\frac{4}{25} \div \frac{8}{15} - \frac{7}{20} \div 2\frac{1}{10}$$

Exercise 14

$$\frac{2}{5} \cdot \left(\frac{1}{19} + \frac{3}{38}\right)$$

Exercise 15

$$\frac{3}{7} \cdot \left(\frac{3}{10} - \frac{1}{15}\right)$$

(Solution on p. 7.)

Exercise 16

$$\frac{10}{11} \cdot \left(\frac{8}{9} - \frac{2}{5}\right) + \frac{3}{25} \cdot \left(\frac{5}{3} + \frac{1}{4}\right)$$

Exercise 17

$$\frac{2}{7} \cdot \left(\frac{6}{7} - \frac{3}{28}\right) + 5\frac{1}{3} \cdot \left(1\frac{1}{4} - \frac{1}{8}\right)$$

(Solution on p. 7.)

Exercise 18

$$\frac{\left(\frac{6}{11} - \frac{1}{3}\right) \cdot \left(\frac{1}{21} + 2\frac{13}{42}\right)}{1\frac{1}{5} + \frac{7}{40}}$$

Exercise 19

$$\left(\frac{1}{2}\right)^2 + \frac{1}{8}$$

*(Solution on p. 7.)***Exercise 20**

$$\left(\frac{3}{5}\right)^2 - \frac{3}{10}$$

*(Solution on p. 7.)***Exercise 21**

$$\sqrt{\frac{36}{81}} + \frac{1}{3} \cdot \frac{2}{9}$$

Exercise 22

$$\sqrt{\frac{49}{64}} - \sqrt{\frac{9}{4}}$$

*(Solution on p. 7.)***Exercise 23**

$$\frac{2}{3} \cdot \sqrt{\frac{9}{4}} - \frac{15}{4} \cdot \sqrt{\frac{16}{225}}$$

Exercise 24

$$\left(\frac{3}{4}\right)^2 + \sqrt{\frac{25}{16}}$$

*(Solution on p. 7.)***Exercise 25**

$$\left(\frac{1}{3}\right)^2 \cdot \sqrt{\frac{81}{25}} + \frac{1}{40} \div \frac{1}{8}$$

Exercise 26

$$\left(\sqrt{\frac{4}{49}}\right)^2 + \frac{3}{7} \div 1\frac{3}{4}$$

*(Solution on p. 7.)***Exercise 27**

$$\left(\sqrt{\frac{100}{121}}\right)^2 + \frac{21}{(11)^2}$$

Exercise 28

$$\sqrt{\frac{3}{8} + \frac{1}{64}} - \frac{1}{2} \div 1\frac{1}{3}$$

Exercise 29

$$\sqrt{\frac{1}{4}} \cdot \left(\frac{5}{6}\right)^2 + \frac{9}{14} \cdot 2\frac{1}{3} - \sqrt{\frac{1}{81}}$$

*(Solution on p. 7.)***Exercise 30**

$$\sqrt{\frac{1}{9}} \cdot \sqrt{\frac{6\frac{3}{8} + 2\frac{5}{8}}{16}} + 7\frac{7}{10}$$

Exercise 31

$$\frac{3\frac{3}{4} + \frac{4}{5} \cdot \left(\frac{1}{2}\right)^3}{\frac{67}{240} + \left(\frac{1}{3}\right)^4 \cdot \left(\frac{9}{10}\right)}$$

*(Solution on p. 7.)***Exercise 32**

$$\sqrt{\sqrt{\frac{16}{81}} + \frac{1}{4}} \cdot 6$$

Exercise 33

$$\sqrt{\sqrt{\frac{81}{256}} - \frac{3}{32}} \cdot 1\frac{1}{8}$$

*(Solution on p. 7.)***3.1 Exercises for Review****Exercise 34**

(here¹) True or false: Our number system, the Hindu-Arabic number system, is a positional number system with base ten.

¹"Addition and Subtraction of Whole Numbers: Whole Numbers" <<http://cnx.org/content/m34795/latest/>>

Exercise 35*(Solution on p. 7.)*

(here²) The fact that 1 times any whole number = that particular whole number illustrates which property of multiplication?

Exercise 36

(here³) Convert $8\frac{6}{7}$ to an improper fraction.

Exercise 37*(Solution on p. 7.)*

(here⁴) Find the sum. $\frac{3}{8} + \frac{4}{5} + \frac{5}{6}$.

Exercise 38

(here⁵) Simplify $\frac{6+\frac{1}{8}}{6-\frac{1}{8}}$.

²"Multiplication and Division of Whole Numbers: Properties of Multiplication" <<http://cnx.org/content/m34867/latest/>>

³"Introduction to Fractions and Multiplication and Division of Fractions: Proper Fractions, Improper Fractions, and Mixed Numbers" <<http://cnx.org/content/m34912/latest/>>

⁴"Addition and Subtraction of Fractions, Comparing Fractions, and Complex Fractions: Addition and Subtraction of Fractions with Unlike Denominators" <<http://cnx.org/content/m34935/latest/>>

⁵"Addition and Subtraction of Fractions, Comparing Fractions, and Complex Fractions: Complex Fractions" <<http://cnx.org/content/m34941/latest/>>

Solutions to Exercises in this Module

Solution to Exercise (p. 4)

0

Solution to Exercise (p. 4)

$\frac{35}{6}$ or $5\frac{5}{6}$

Solution to Exercise (p. 4)

$\frac{211}{30}$ or $7\frac{1}{30}$

Solution to Exercise (p. 4)

$\frac{7}{9}$

Solution to Exercise (p. 4)

$2\frac{6}{7}$

Solution to Exercise (p. 4)

$\frac{3}{11}$

Solution to Exercise (p. 4)

$\frac{15}{64}$

Solution to Exercise (p. 4)

$\frac{11}{10}$

Solution to Exercise (p. 4)

$\frac{5}{4}$

Solution to Exercise (p. 4)

$4\frac{2}{7}$

Solution to Exercise (p. 4)

$\frac{2}{15}$

Solution to Exercise (p. 4)

$\frac{1}{10}$

Solution to Exercise (p. 4)

$6\frac{3}{14}$

Solution to Exercise (p. 5)

$\frac{3}{8}$

Solution to Exercise (p. 5)

$\frac{20}{27}$

Solution to Exercise (p. 5)

0

Solution to Exercise (p. 5)

$\frac{2}{5}$

Solution to Exercise (p. 5)

1

Solution to Exercise (p. 5)

$\frac{125}{72}$

Solution to Exercise (p. 5)

$\frac{252}{19}$

Solution to Exercise (p. 5)

$\frac{165}{256}$

Solution to Exercise (p. 6)

multiplicative identity

Solution to Exercise (p. 6)

$\frac{241}{120}$ or $2\frac{1}{120}$