Signed Numbers: Multiplication and Division of Signed Numbers*

Wade Ellis
Denny Burzynski

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Abstract

This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to multiply and divide signed numbers. By the end of the module students should be able to multiply and divide signed numbers and be able to multiply and divide signed numbers using a calculator.

1 Section Overview

- Multiplication of Signed Numbers
- Division of Signed Numbers
- Calculators

2 Multiplication of Signed Numbers

Let us consider first, the product of two positive numbers. Multiply: $3 \cdot 5$.

$3 \cdot 5$ means $5 + 5 + 5 = 15$

This suggests[1] that

(positive number) $\cdot$ (positive number) = (positive number)

More briefly,

$(+)(+) = (+)$

Now consider the product of a positive number and a negative number. Multiply: $(3)(-5)$.

$(3)(-5)$ means $(-5) + (-5) + (-5) = -15$

This suggests that

(positive number) $\cdot$ (negative number) = (negative number)

More briefly,

$(+)(-) = (-)$

By the commutative property of multiplication, we get

(negative number) $\cdot$ (positive number) = (negative number)

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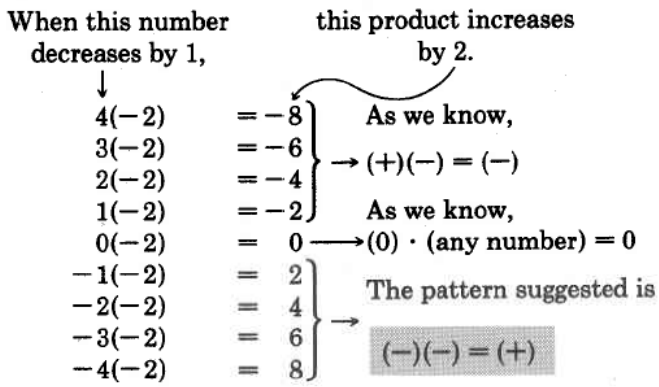
[1] In later mathematics courses, the word "suggests" turns into the word "proof." One example does not prove a claim. Mathematical proofs are constructed to validate a claim for all possible cases.

More briefly,
$(-)(+) = (-)$
The sign of the product of two negative numbers can be suggested after observing the following illustration.
Multiply -2 by, respectively, 4, 3, 2, 1, 0, -1, -2, -3, -4.



We have the following rules for multiplying signed numbers.
Rules for Multiplying Signed Numbers
Multiplying signed numbers:

1. To multiply two real numbers that have the *same sign*, multiply their absolute values. The product is positive.
   $(+)(+) = (+)$
   $(-)(-) = (+)$
2. To multiply two real numbers that have *opposite signs*, multiply their absolute values. The product is negative.
   $(+)(-) = (-)$
   $(-)(+) = (-)$

2.1 Sample Set A

Find the following products.

Example 1
$8 \cdot 6$

$$\left.\begin{array}{rcl} |8| & = & 8 \\ |6| & = & 6 \end{array}\right\} \text{ Multiply these absolute values.}$$

$8 \cdot 6 = 48$
Since the numbers have the same sign, the product is positive.
Thus, $8 \cdot 6 = +48$, or $8 \cdot 6 = 48$.

Example 2
$(-8)(-6)$

$$\left.\begin{array}{rcl} |-8| & = & 8 \\ |-6| & = & 6 \end{array}\right\} \text{ Multiply these absolute values.}$$

$8 \cdot 6 = 48$
Since the numbers have the same sign, the product is positive.
Thus, $(-8)(-6) = +48$, or $(-8)(-6) = 48$.

Example 3

$(-4)(7)$

$$\left.\begin{array}{rcl} |-4| & = & 4 \\ |7| & = & 7 \end{array}\right\} \text{ Multiply these absolute values.}$$

$4 \cdot 7 = 28$

Since the numbers have opposite signs, the product is negative.

Thus, $(-4)(7) = -28$.

Example 4

$6(-3)$

$$\left.\begin{array}{rcl} |6| & = & 6 \\ |-3| & = & 3 \end{array}\right\} \text{ Multiply these absolute values.}$$

$6 \cdot 3 = 18$

Since the numbers have opposite signs, the product is negative.

Thus, $6(-3) = -18$.

2.2 Practice Set A

Find the following products.

Exercise 1 *(Solution on p. 10.)*

$3(-8)$

Exercise 2 *(Solution on p. 10.)*

$4(16)$

Exercise 3 *(Solution on p. 10.)*

$(-6)(-5)$

Exercise 4 *(Solution on p. 10.)*

$(-7)(-2)$

Exercise 5 *(Solution on p. 10.)*

$(-1)(4)$

Exercise 6 *(Solution on p. 10.)*

$(-7)7$

3 Division of Signed Numbers

To determine the signs in a division problem, recall that

$\frac{12}{3} = 4$ since $12 = 3 \cdot 4$

This suggests that

$\frac{(+)}{(+)} = (+)$

$\frac{(+)}{(+)} = (+)$ since $(+) = (+)(+)$

What is $\frac{12}{-3}$?

$-12 = (-3)(-4)$ suggests that $\frac{12}{-3} = -4$. That is,

$\frac{(+)}{(-)} = (-)$

$(+) = (-)(-)$ suggests that $\frac{(+)}{(-)} = (-)$

What is $\frac{-12}{3}$?

$-12 = (3)(-4)$ suggests that $\frac{-12}{3} = -4$. That is,

$\frac{(-)}{(+)} = (-)$

$(-) = (+)(-)$ suggests that $\frac{(-)}{(+)} = (-)$

What is $\frac{-12}{-3}$?

$-12 = (-3)(4)$ suggests that $\frac{-12}{-3} = 4$. That is,

$\frac{(-)}{(-)} = (+)$

$(-) = (-)(+)$ suggests that $\frac{(-)}{(-)} = (+)$

We have the following rules for dividing signed numbers.

Rules for Dividing Signed Numbers

Dividing signed numbers:

1. To divide two real numbers that have the *same sign*, divide their absolute values. The quotient is positive.
   $\frac{(+)}{(+)} = (+)\frac{(-)}{(-)} = (+)$
2. To divide two real numbers that have *opposite signs*, divide their absolute values. The quotient is negative.
   $\frac{(-)}{(+)} = (-)\frac{(+)}{(-)} = (-)$

3.1 Sample Set B

Find the following quotients.

Example 5

$\frac{-10}{2}$

$$\left.\begin{array}{rcl} |-10| & = & 10 \\ |2| & = & 2 \end{array}\right\} \text{Divide these absolute values.}$$

$\frac{10}{2} = 5$

Since the numbers have opposite signs, the quotient is negative.

Thus $\frac{-10}{2} = -5$.

Example 6

$\frac{-35}{-7}$

$$\left.\begin{array}{rcl} |-35| & = & 35 \\ |-7| & = & 7 \end{array}\right\} \text{Divide these absolute values.}$$

$\frac{35}{7} = 5$

Since the numbers have the same signs, the quotient is positive.

Thus, $\frac{-35}{-7} = 5$.

Example 7

$\frac{18}{-9}$

$$\left.\begin{array}{rcl} |18| & = & 18 \\ |-9| & = & 9 \end{array}\right\} \text{Divide these absolute values.}$$

$\frac{18}{9} = 2$

Since the numbers have opposite signs, the quotient is negative.

Thus, $\frac{18}{-9} = 2$.

3.2 Practice Set B

Find the following quotients.

Exercise 7 *(Solution on p. 10.)*

$\frac{-24}{-6}$

Exercise 8 *(Solution on p. 10.)*

$\frac{30}{-5}$

Exercise 9 *(Solution on p. 10.)*

$\frac{-54}{27}$

Exercise 10 *(Solution on p. 10.)*

$\frac{51}{17}$

3.3 Sample Set C

Example 8

Find the value of $\frac{-6(4-7)-2(8-9)}{-(4+1)+1}$.

Using the order of operations and what we know about signed numbers, we get,

$$\begin{aligned}\frac{-6(4-7)-2(8-9)}{-(4+1)+1} &= \frac{-6(-3)-2(-1)}{-(5)+1}\\ &= \frac{18+2}{-5+1}\\ &= \frac{20}{-}\\ &= -5\end{aligned}$$

3.4 Practice Set C

Exercise 11 *(Solution on p. 10.)*

Find the value of $\frac{-5(2-6)-4(-8-1)}{2(3-10)-9(-2)}$.

4 Calculators

Calculators with the $\boxed{+/-}$ key can be used for multiplying and dividing signed numbers.

4.1 Sample Set D

Use a calculator to find each quotient or product.

Example 9

$(-186)\cdot(-43)$

Since this product involves a (negative) $\cdot$ (negative), we know the result should be a positive number. We'll illustrate this on the calculator.

		Display Reads
Type	186	186
Press	+/−	-186
Press	×	-186
Type	43	43
Press	+/−	-43
Press	=	7998

Table 1

Thus, $(-186) \cdot (-43) = 7,998$.

Example 10
$\frac{158.64}{-54.3}$. Round to one decimal place.

		Display Reads
Type	158.64	158.64
Press	÷	158.64
Type	54.3	54.3
Press	+/−	-54.3
Press	=	-2.921546961

Table 2

Rounding to one decimal place we get -2.9.

4.2 Practice Set D

Use a calculator to find each value.

Exercise 12 *(Solution on p. 10.)*
$(-51.3) \cdot (-21.6)$

Exercise 13 *(Solution on p. 10.)*
$-2.5746 \div -2.1$

Exercise 14 *(Solution on p. 10.)*
$(0.006) \cdot (-0.241)$. Round to three decimal places.

5 Exercises

Find the value of each of the following. Use a calculator to check each result.

Exercise 15 *(Solution on p. 10.)*
$(-2)\,(-8)$

Exercise 16
$(-3)(-9)$

Exercise 17 *(Solution on p. 10.)*
$(-4)(-8)$

Exercise 18
$(-5)(-2)$

Exercise 19 *(Solution on p. 10.)*
$(3)(-12)$

Exercise 20
$(4)(-18)$

Exercise 21 *(Solution on p. 10.)*
$(10)(-6)$

Exercise 22
$(-6)(4)$

Exercise 23 *(Solution on p. 10.)*
$(-2)(6)$

Exercise 24
$(-8)(7)$

Exercise 25 *(Solution on p. 10.)*
$\frac{21}{7}$

Exercise 26
$\frac{42}{6}$

Exercise 27 *(Solution on p. 10.)*
$\frac{-39}{3}$

Exercise 28
$\frac{-20}{10}$

Exercise 29 *(Solution on p. 10.)*
$\frac{-45}{-5}$

Exercise 30
$\frac{-16}{-8}$

Exercise 31 *(Solution on p. 10.)*
$\frac{25}{-5}$

Exercise 32
$\frac{36}{-4}$

Exercise 33 *(Solution on p. 10.)*
$8-(-3)$

Exercise 34
$14-(-20)$

Exercise 35 *(Solution on p. 10.)*
$20-(-8)$

Exercise 36
$-4-(-1)$

Exercise 37 *(Solution on p. 11.)*
$0-4$

Exercise 38
$0-(-1)$

Exercise 39 *(Solution on p. 11.)*
$-6+1-7$

Exercise 40
$15-12-20$

Exercise 41 *(Solution on p. 11.)*
$1-6-7+8$

Exercise 42
$2+7-10+2$

Exercise 43 *(Solution on p. 11.)*
$3(4-6)$

Exercise 44
$8(5-12)$

Exercise 45 *(Solution on p. 11.)*
$-3(1-6)$

Exercise 46
$-8(4-12)+2$

Exercise 47 *(Solution on p. 11.)*
$-4(1-8)+3(10-3)$

Exercise 48
$-9(0-2)+4(8-9)+0(-3)$

Exercise 49 *(Solution on p. 11.)*
$6(-2-9)-6(2+9)+4(-1-1)$

Exercise 50
$\frac{3(4+1)-2(5)}{-2}$

Exercise 51 *(Solution on p. 11.)*
$\frac{4(8+1)-3(-2)}{-4-2}$

Exercise 52
$\frac{-1(3+2)+5}{-1}$

Exercise 53 *(Solution on p. 11.)*
$\frac{-3(4-2)+(-3)(-6)}{-4}$

Exercise 54
$-1(4+2)$

Exercise 55 *(Solution on p. 11.)*
$-1(6-1)$

Exercise 56
$-(8+21)$

Exercise 57 *(Solution on p. 11.)*
$-(8-21)$

5.1 Exercises for Review

Exercise 58
(here[2]) Use the order of operations to simplify $\left(5^2 + 3^2 + 2\right) \div 2^2$.

Exercise 59 ***(Solution on p. 11.)***
(here[3]) Find $\frac{3}{8}$ of $\frac{32}{9}$.

Exercise 60
(here[4]) Write this number in decimal form using digits: "fifty-two three-thousandths"

Exercise 61 ***(Solution on p. 11.)***
(here[5]) The ratio of chlorine to water in a solution is 2 to 7. How many mL of water are in a solution that contains 15 mL of chlorine?

Exercise 62
(here[6]) Perform the subtraction $-8 - (-20)$

[2] "Exponents, Roots, Factorization of Whole Numbers: Grouping Symbols and the Order of Operations" <http://cnx.org/content/m34872/latest/>
[3] "Introduction to Fractions and Multiplication and Division of Fractions: Applications Involving Fractions" <http://cnx.org/content/m34930/latest/>
[4] "Decimals: Reading and Writing Decimals" <http://cnx.org/content/m34957/latest/>
[5] "Ratios and Rates: Applications of Proportions" <http://cnx.org/content/m34982/latest/>
[6] "Signed Numbers: Subtraction of Signed Numbers" <http://cnx.org/content/m35032/latest/>

Solutions to Exercises in this Module

Solution to Exercise (p. 3)
-24
Solution to Exercise (p. 3)
64
Solution to Exercise (p. 3)
30
Solution to Exercise (p. 3)
14
Solution to Exercise (p. 3)
-4
Solution to Exercise (p. 3)
-49
Solution to Exercise (p. 5)
4
Solution to Exercise (p. 5)
-6
Solution to Exercise (p. 5)
-2
Solution to Exercise (p. 5)
3
Solution to Exercise (p. 5)
14
Solution to Exercise (p. 6)
1,108.08
Solution to Exercise (p. 6)
1.226
Solution to Exercise (p. 6)
-0.001
Solution to Exercise (p. 6)
16
Solution to Exercise (p. 7)
32
Solution to Exercise (p. 7)
-36
Solution to Exercise (p. 7)
-60
Solution to Exercise (p. 7)
-12
Solution to Exercise (p. 7)
3
Solution to Exercise (p. 7)
-13
Solution to Exercise (p. 7)
9
Solution to Exercise (p. 7)
-5
Solution to Exercise (p. 7)
11

Solution to Exercise (p. 7)
28
Solution to Exercise (p. 7)
-4
Solution to Exercise (p. 8)
-12
Solution to Exercise (p. 8)
-4
Solution to Exercise (p. 8)
-6
Solution to Exercise (p. 8)
15
Solution to Exercise (p. 8)
49
Solution to Exercise (p. 8)
-140
Solution to Exercise (p. 8)
-7
Solution to Exercise (p. 8)
-3
Solution to Exercise (p. 8)
-5
Solution to Exercise (p. 8)
13
Solution to Exercise (p. 9)
$\frac{4}{3} = 1\frac{1}{3}$
Solution to Exercise (p. 9)
$52\frac{1}{2}$