The Fundamental Theorem of Algebra, and the Fundamental Theorem of Analysis*

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Abstract

An introduction to the fundamental theorem of algebra, with links to important theorems on the topic.

In this chapter we will discover the incredible difference between the analysis of functions of a single complex variable as opposed to functions of a single real variable. Up to this point, in some sense, we have treated them as being quite similar subjects, whereas in fact they are extremely different in character. Indeed, if f is a differentiable function of a complex variable on an open set $U \subseteq C$, then we will see that f is actually expandable in a Taylor series around every point in U. In particular, a function $f$of a complex variable is guaranteed to have infinitely many derivatives on U if it merely has the first one on U. This is in marked contrast with functions of a real variable. See part (3) of here[1].

The main points of this chapter are:

1. **The Cauchy-Riemann Equations** (here[2]),
2. **Cauchy's Theorem** (here[3]),
3. **Cauchy Integral Formula** (here[4]),
4. **A complex-valued function that is differentiable on an open set is expandable in a Taylor series around each point of the set** (here[5]),
5. **The Identity Theorem** (here[6]),
6. **The Fundamental Theorem of Algebra** (here[7]),

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[1] "Differentiation, Local Behavior: Higher Order Derivatives", Theorem 1 <http://cnx.org/content/m36192/latest/#fs-id7582339>

[2] "Fundamental Theorem of Algebra, Analysis: Cauchy's Theorem", Theorem 1: Cauchy-Riemann equations <http://cnx.org/content/m36235/latest/#fs-id1170810388270>

[3] "Fundamental Theorem of Algebra, Analysis: Cauchy's Theorem", Theorem 3: Cauchy's Theorem, Fundamental Theorem of Analysis <http://cnx.org/content/m36235/latest/#fs-id1170827240654>

[4] "Fundamental Theorem of Algebra, Analysis: Cauchy's Theorem", Theorem 4: Cauchy Integral Formula <http://cnx.org/content/m36235/latest/#fs-id1170819963435>

[5] "Fundamental Theorem of Algebra, Analysis: Basic Applications of the Cauchy Integral Formula", Theorem 1 <http://cnx.org/content/m36237/latest/#fs-id1167472693549>

[6] "Fundamental Theorem of Algebra, Analysis: Basic Applications of the Cauchy Integral Formula", Theorem 2: Identity Theorem <http://cnx.org/content/m36237/latest/#fs-id1853397>

[7] "Fundamental Theorem of Algebra, Analysis: The Fundamental Theorem of Algebra", Theorem 1: Fundamental Theorem of Algebra <http://cnx.org/content/m36238/latest/#fs-id1172549276378>

7. **Liouville's Theorem** (here[8]),
8. **The Maximum Modulus Principle** (corollary to here[9]),
9. **The Open Mapping Theorem** (here[10]),
10. **The uniform limit of analytic functions is analytic** (here[11]), and
11. **The Residue Theorem** (here[12]).

[8] "Fundamental Theorem of Algebra, Analysis: The Fundamental Theorem of Algebra", Theorem 2: Liouville <http://cnx.org/content/m36238/latest/#fs-id1172552488682>

[9] "Fundamental Theorem of Algebra, Analysis: The Maximum Modulus Principle", Corollary 1: Maximum Modulus Principle <http://cnx.org/content/m36239/latest/#fs-id1172068491562>

[10] "Fundamental Theorem of Algebra, Analysis: The Open Mapping Theorem and the Inverse Function Theorem", Theorem 1: Open Mapping Theorem <http://cnx.org/content/m36240/latest/#fs-id1167524484222>

[11] "The Fundamental Theorem of Algebra, Analysis: Uniform Convergence of Analytic Functions", Theorem 1 <http://cnx.org/content/m36241/latest/#fs-id1167522586798>

[12] "Fundamental Theorem of Algebra, Analysis: Isolated Singularities, and the Residue Theorem", Theorem 5: Residue Theorem <http://cnx.org/content/m36242/latest/#fs-id1165909735907>