Second Order System Impulse Response Generation*

David Waldo

This work is produced by The Connexions Project and licensed under the
Creative Commons Attribution License †

Abstract

This module describes the recursive generation of the impulse response of a second order system.

1 Introduction

This module examines the recursive generation of the impulse response of a second order system. This analysis can be used to determine how to generate a cosine or sine function recursively.

2 Second Order Systems

The transfer function of a general second order system is given as:

$$H(z) = \frac{b_0 z^2 + b_1 z + b_2}{z^2 + a_1 z + a_2} \tag{1}$$

The Direct Form II structure for the implementation of the second order system is shown in the following figure.

*Version 1.3: Jan 6, 2011 4:19 pm -0600
†http://creativecommons.org/licenses/by/3.0/

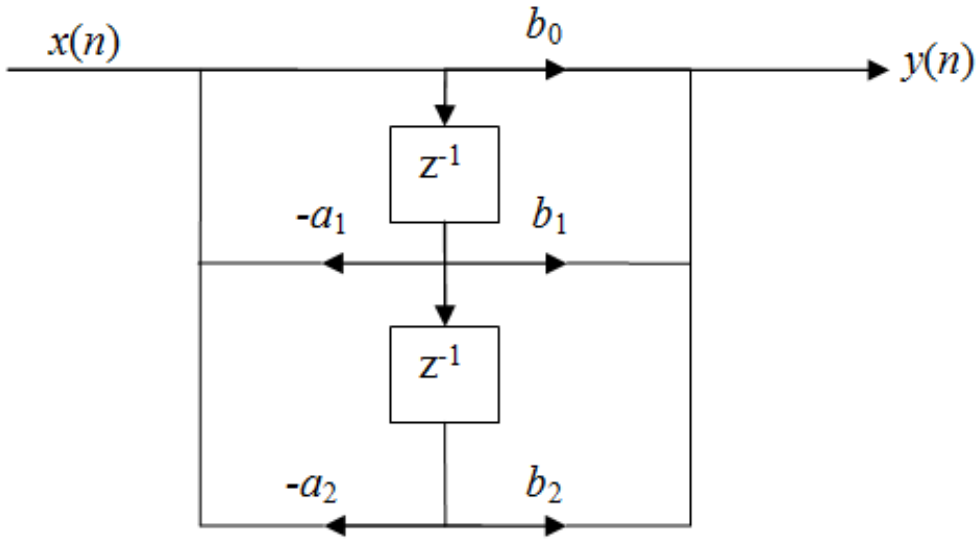

Figure 1

The difference equation for the second order system is

$$y(n) + a_1 y(n-1) + a_2 y(n-2) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) \tag{2}$$

$$y(n) = -a_1 y(n-1) - a_2 y(n-2) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) \tag{3}$$

To recursively determine the impulse response of the system start with the following:

$$h(-1) = h(-2) = 0, x(n) = \delta(n) \tag{4}$$

Then using Equation 3 the impulse response can be found recursively by:

$$h(0) = -a_1 h(-1) - a_2 h(-2) + b_0 \delta(0) + b_1 \delta(-1) + b_2 \delta(-2) = b_0 \tag{5}$$

$$h(1) = -a_1 h(0) - a_2 h(-1) + b_0 \delta(1) + b_1 \delta(0) + b_2 \delta(-1) \tag{6}$$

$$h(1) = -a_1 b_0 + b_1 \tag{7}$$

$$h(2) = -a_1 h(1) - a_2 h(0) + b_0 \delta(2) + b_1 \delta(1) + b_2 \delta(0) \tag{8}$$

$$h(2) = -a_1 (-a_1 + b_1) - a_2 b_0 + b_2 \tag{9}$$

For the values of n>2 the recursive equation reduces to

$$h(n) = -a_1 h(n-1) - a_2 h(n-2) \tag{10}$$

because the values of $x(n), x(n-1)$ and $x(n-2)$ will all be zero for $n>2$.

So, if you know the values of $h(0)$ and $h(1)$, then the Equation 10 can be used to find future values of $h(n)$.