

# UNITS AND DIMENSIONAL ANALYSIS\*

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## Abstract

The purpose of this module is to explain units and dimensional analysis in a format that is accessible to blind students.

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## 2 Preface

### 2.1 General

This module is part of a collection of modules designed to make physics concepts accessible to blind students.

See <http://cnx.org/content/col11294/latest/><sup>1</sup> for the main page of the collection and <http://cnx.org/content/col11294/latest/><sup>2</sup> for the table of contents for the collection.

The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

The purpose of this module is to explain units and dimensional analysis in a format that is accessible to blind students.

### 2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams ( <http://www.youtube.com/watch?v=c8plj9UsJbg><sup>3</sup> ).
- A protractor for measuring angles ( <http://www.youtube.com/watch?v=v-F06HgiUpw><sup>4</sup> ).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at <http://www.nvda-project.org/><sup>5</sup> .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor ( <http://www.userite.com/ecampus/lesson1/tools.php><sup>6</sup> ).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.
- The ability to create tactile graphics as described at <http://cnx.org/content/m38546/latest/><sup>7</sup> .

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams ( <http://www.youtube.com/watch?v=c8plj9UsJbg><sup>8</sup> ).
- An understanding of the use of a protractor for measuring angles ( <http://www.youtube.com/watch?v=v-F06HgiUpw><sup>9</sup> ).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry ( <http://www.clarku.edu/~djoyce/trig/><sup>10</sup> ).
- An introductory understanding of JavaScript programming ( <http://www.dickbaldwin.com/tocjavascript1.htm><sup>11</sup> and <http://www.w3schools.com/js/default.asp><sup>12</sup> ).
- An understanding of all of the material covered in the earlier modules in this collection.
- An understanding of the creation and use of tactile graphics as described at <http://cnx.org/content/m38546/latest/><sup>13</sup> .

<sup>1</sup><http://cnx.org/content/col11294/latest/>

<sup>2</sup>[http://cnx.org/content/col11294/latest/#cnx\\_sidebar\\_column](http://cnx.org/content/col11294/latest/#cnx_sidebar_column)

<sup>3</sup><http://www.youtube.com/watch?v=c8plj9UsJbg>

<sup>4</sup><http://www.youtube.com/watch?v=v-F06HgiUpw>

<sup>5</sup><http://www.nvda-project.org/>

<sup>6</sup><http://www.userite.com/ecampus/lesson1/tools.php>

<sup>7</sup><http://cnx.org/content/m38546/latest/>

<sup>8</sup><http://www.youtube.com/watch?v=c8plj9UsJbg>

<sup>9</sup><http://www.youtube.com/watch?v=v-F06HgiUpw>

<sup>10</sup><http://www.clarku.edu/~djoyce/trig/>

<sup>11</sup><http://www.dickbaldwin.com/tocjavascript1.htm>

<sup>12</sup><http://www.w3schools.com/js/default.asp>

<sup>13</sup><http://cnx.org/content/m38546/latest/>

## 2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures and listings while you are reading about them.

### 2.3.1 Figures

- Figure 1 (p. 4) . Single-line format.
- Figure 2 (p. 5) . SI base units.
- Figure 3 (p. 6) . Examples of SI derived units.
- Figure 4 (p. 7) . A sampling of SI prefixes.
- Figure 5 (p. 12) . Screen output for Listing #1.
- Figure 6 (p. 15) . Screen output for Listing #2.
- Figure 7 (p. 18) . Screen output for Listing #3.
- Figure 8 (p. 19) . Mirror image from the file named Phy1050a1.svg.
- Figure 9 (p. 20) . Key-value table for the image shown in Figure 8.
- Figure 10 (p. 22) . Non-mirror-image version of the image from the file named Phy1050a1.svg.

### 2.3.2 Listings

- Listing 1 (p. 10) . Convert from paces to miles.
- Listing 2 (p. 14) . Free fall exercise.
- Listing 3 (p. 16) . A plotting exercise.

## 2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at [www.DickBaldwin.com](http://www.DickBaldwin.com) <sup>14</sup> .

## 3 General background information

As a young engineering student many years ago, I was told that when evaluating mathematical expressions, I should always embed the units in the expression, manipulate them algebraically, and confirm that the units that survive until the end of the evaluation are correct relative to the problem specifications.

### Three good reasons

There are at least three good reasons for following this procedure:

- It shows what the units of the result are. A common mistake is to get the correct numerical result of a calculation but to write it with the wrong units.
- It shows where unit conversions must be performed. If units that should have canceled do not cancel, we need to go back and perform the necessary conversions. For example, when a speed in miles per hour is being calculated and the result comes out with units of miles per second, we should convert seconds to hours.
- It helps to locate mistakes. If a velocity (meters per second) is being calculated and the units come out as seconds per meter, we know to look for an error.

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<sup>14</sup><http://www.dickbaldwin.com/toc.htm>

### 3.1 Formatting mathematical expressions

#### A significant disadvantage

This is an area where blind students may be at a significant disadvantage relative to sighted students due mainly to the fact that blind students using screen readers and Braille displays can only see one line of text at a time. It is much easier (for me anyway) to correctly manipulate the units when the expression is written on two or more lines.

#### Multiple-line format

What I mean by this is that a sighted student would typically write a fraction as a horizontal line with the numerator above the line and the denominator below the line using superscripts to indicate exponents. The product of two fractions would typically be written in the same format with a multiplication indicator joining the two fractions.

#### Single-line format

A blind student constrained to seeing only one line of text at a time would probably need to write and evaluate the product of the two fractions using a format something like that shown in Figure 1 (p. 4) .

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#### Single-line format.

$$((3*x^2)/(4*x) * (6/(4*x) = (18*x^2)/(16*x^2) = 18/16$$

**Figure 1:** Single-line format.

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With enough time and practice, I might become almost as proficient in evaluating expressions as shown in Figure 1 (p. 4) as in evaluating expressions in a multiple-line format, but I'm afraid that I would need a lot of time and a lot of practice to achieve that proficiency.

#### We must work with what we are given

However, we must work with what we are given so the mathematical expressions in this module (and most of the other modules as well) will all be in a single-line format. The good news is that the single-line format can usually be easily translated into a format that is suitable for evaluation using JavaScript.

### 3.2 What do we mean by units?

Let me try to answer this important question with an example.

If you were to walk from your home to your school and count your steps, you might report that the distance from your home to your school is 2112 steps or paces. If you are a male student, this would probably be approximately one mile, using assumptions that I will explain later. If you are a female, it would probably be somewhat less than a mile because your pace would probably be a little shorter than your male friend's pace.

#### A pace is a unit of measurement

In this case, the pace would be a unit of measurement. However, it would not be a standard unit because the length of a pace for one student is often different from the length of a pace for a different student. A male student, for example, typically has a longer pace length than a female student.

#### Standard units

Standard units are units whose value has been set by some official agency that has the authority to set standards. One such standard unit is the mile, which as you may know is equal to 5280 feet (more correctly pronounced feet).

A foot is also a standard unit, which you may also know is equal to 12 inches. An inch is another standard unit.

### 3.3 SI units

The discrepancy between the pronunciation of the unit **foot** and its plural **feet** is an example of the need for more consistency in the use of units. Many physics books use a system of units called **SI units**. SI is an abbreviation for a French name, which I am unable to pronounce, and which is probably also not compatible with your screen reader and your Braille display.

I won't attempt to explain much about SI units. You can probably find a good explanation in your textbook, and if not, you can Google SI units and find hundreds of web pages that explain the system in varying levels of detail.

#### Tables of SI units

Most of those references will probably also provide tables for the units, but those tables may be partially incompatible with your screen reader and Braille display due to the extensive use of superscripts. Therefore, I will attempt to provide accessible tables in Figure 2 (p. 5) and Figure 3 (p. 6).

#### Don't ignore the details

Note, however, that you should not ignore the online SI-unit tables altogether. They are likely to contain important information that I won't reproduce here, such as the fact that the meter is a unit of length and its value is the length of the path travelled by light in a vacuum during a time interval of  $1/299792458$  of a second.

#### Base units and derived units

When reading about SI units, you will find that they are often divided into base units and derived units. I will put the base units in Figure 2 (p. 5) and some sample derived units in Figure 3 (p. 6).

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#### SI base units.

Base Quantity	Name	Symbol
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

**Figure 2:** SI base units.

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Note that the list of derived units in Figure 3 (p. 6) is only a sampling of different units that can be derived from the base units.

### The exponentiation indicator

As is the case throughout these modules, the character "^" that you see used extensively in Figure 3 (p. 6) indicates that the character following the ^ is an exponent. Note also that when the exponent is negative, it is enclosed along with its minus sign in parentheses for clarity.

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### Examples of SI derived units.

area	square meter	$m^2$
volume	cubic meter	$m^3$
speed, velocity	meter per second	$m/s$
acceleration	meter per second squared	$m/s^2$
wave number	reciprocal meter	$m^{-1}$
mass density	kilogram per cubic meter	$kg/m^3$
specific volume	cubic meter per kilogram	$m^3/kg$
current density	ampere per square meter	$A/m^2$

**Figure 3:** Examples of SI derived units.

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### 3.4 Prefixes for powers of ten

As an alternative to explicitly writing powers of ten, SI uses prefixes for units to indicate power of ten factors. Figure 4 (p. 7) shows some of the powers of ten and the SI prefixes used for them.

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### A sampling of SI prefixes.

Prefix-(abbreviation)	Power of Ten
peta-(P)	$10^{15}$
tera-(T)	$10^{12}$
giga-(G)	$10^9$
mega-(M)	$10^6$
kilo-(k)	$10^3$
deci-(d)	$10^{-1}$
centi-(c)	$10^{-2}$
milli-(m)	$10^{-3}$
micro-(Greek letter mu)	$10^{-6}$
nano-(n)	$10^{-9}$
pico-(p)	$10^{-12}$
femto-(f)	$10^{-15}$

**Figure 4:** A sampling of SI prefixes.

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### 3.5 Dimensional analysis

As typically used in physics, the word *dimensions* means basic types of units such as time, length, and mass (see Figure 2 (p. 5) ).

There are many different units that are used to express length, for example, such as foot, mile, meter, inch, etc.

Because they all have dimensions of length, you can convert from one to another. For example one mile is equal to 5280 feet.

#### **Cannot convert between units of different dimensions**

When evaluating mathematical expressions, we can add, subtract, or equate quantities only if they have the same dimensions (although they may not necessarily be given in the same units). For example, it is possible to add 3 meters to 2 inches (after converting units), but it is not possible to add 3 meters to 2 kilograms.

To analyze dimensions, you should treat them as algebraic quantities using the same procedures that we will use in the upcoming exercise on manually converting units.

## 4 Discussion and sample code

Let's manually work through an exercise to determine the distance from your home to your school in miles given that we know the distance in paces. (I will convert this to a JavaScript script later in the module.)

As we discussed earlier, you have already determined that the distance from your home to your school is 2112 paces. We have also recognized that the length of one of your paces is likely to be different from the length of someone else's paces.

#### 4.1 Exercise on manually converting units

This is a somewhat long and tedious explanation, but unless you already have quite a lot of experience in this area, you should probably bear with me as I walk you through it.

##### Convert your pace length to standard units

First we must convert the length of your pace to standard units, which we can then convert to miles.

##### A calibration exercise

Assume that you go into an empty parking lot, mark your starting position, walk 100 paces, and then mark your ending position on the pavement. Assume that you then use a tactile measuring tape to measure the distance from the starting position to the ending position and you find that distance to be 3000 inches.

From that, we could conclude that for your pace length,

$$100 \text{ paces} = 3000 \text{ inches}$$

Thus, we have calibrated your pace length in terms of the standard unit inch.

##### The plural versus the singular

Note that the use of the plural such as **inches** and the singular such as **inch** has no impact on the results of our computations. we can switch back and forth from the plural to the singular at will as long as it makes sense to do so.

##### A conversion factor

If we divide both sides of the above equation (p. 8) by 100 paces, we get

$$1 = 3000 \text{ inches}/100 \text{ paces} = 30 \text{ inches}/\text{pace}$$

Note that we now have a 1 on the left side and a fraction on the right side.

##### Multiplication by 1

Multiplying a value by 1 doesn't change the value. This means that if we multiply a term in an algebraic expression by

$$30 \text{ inch}/\text{pace}$$

we won't change the intrinsic value of the term, although we might change the units in which that value is expressed.

The term

$$30 \text{ inch}/\text{pace}$$

can be used to convert a quantity in units of paces to the same quantity in units of inches.

##### Move your body forward

So now we know that on the average, each time you execute one pace, you have moved your body forward by 30 inches. The next task is to determine how far you have moved your body when you have executed 2112 paces (the distance from home to school measured in paces).

##### Convert from paces to inches

We can begin by determining the distance from home to school in inches as follows:

$$\text{distance} = (2112 \text{ pace}) * (30 \text{ inch}/\text{pace})$$

##### A review of algebra

At this point, let's review a little algebra:

Given the expression:  $((w/x)*(y/z))$

we can rewrite this as

$$(w*y)/(x*z)$$

Given this expression, we can reverse the process by rearranging and factoring out the terms producing our original expression:

$$((w/x)*(y/z))$$

##### An algebraic manipulation

We determined earlier that

$$\text{distance} = (2112 \text{ pace}) * (30 \text{ inch}/\text{pace})$$

Applying the algebraic process shown above to the problem at hand, we can rearrange terms and factor our expression to produce

$$\text{distance} = (2112 * 30 * \text{pace} * \text{inch})/\text{pace}$$



**The distance in inches**

At this point, we have a fraction that contains the unit pace in both the numerator and the denominator. We can cancel those two terms leaving us with

$$\text{distance} = (2112 * 30 * \text{inch}) = 63360 * \text{inch}$$

So good so far. We now know the distance from home to school as expressed in units of inches (even though the original measurement was made in paces and not in inches). The distance hasn't changed. What has changed is the units in which we are expressing that distance.

**Still not what we are looking for**

While this is interesting, it still isn't what we are looking for. We want to know the distance in miles. Therefore, we need to convert the distance in inches to the distance in miles.

**How many inches are in a mile?**

At this point, we don't know how many inches are in a mile (but we could calculate it if we wanted to). However, we do know how many inches are in a foot and we know how many feet are in a mile.

**A conversion factor for inches to feet**

For starters, let's calculate a factor that we can use to convert from inches to feet. We know that

$$12 * \text{inches} = 1 * \text{foot}$$

If we divide each side by 12\*inches, we get

$$1 = 1 * \text{foot} / 12 * \text{inches}$$

Let's refer to this as a conversion factor

**Multiplication by 1**

Since the expression on the right side is equal to 1, we can multiply any expression in an equation with the conversion factor

$$1 * \text{foot} / 12 * \text{inches}$$

without changing the intrinsic value of that expression. (Once again, although we won't be changing the intrinsic value of the expression, we may cause that intrinsic value to be expressed in different units.)

We can use this conversion factor to convert a distance value expressed in inches into the value for the same distance expressed in feet.

**A conversion factor for feet to inches**

We could also use the reciprocal of the expression to convert a distance value expressed in feet into the value for the same distance expressed in inches. I will have more to say on this later.

**A conversion factor for feet to miles**

Similarly, we could use the same procedure to show that

$$1 = 1 * \text{mile} / 5280 * \text{foot}$$

Again, since the expression on the right side is equal to 1, we can multiply any expression in an equation with the expression on the right without changing the intrinsic value of the original expression. This conversion expression can be used to convert from feet to miles. Its reciprocal could be used to convert from miles to feet.

**Back to the problem at hand**

We determined earlier that the distance expressed in inches from home to school is

$$\text{distance} = 63360 * \text{inch}$$

We can probably agree that multiplying the expression on the right as follows would not change the intrinsic value for the distance.

$$\text{distance} = 63360 * \text{inch} * 1 * 1$$

In fact, we could probably agree that it wouldn't change anything at all.

**Substitute the two conversion factors**

Having agreed on that, we can substitute the two conversion factors derived earlier, each of which has a value of 1, for the last two factors in our distance equation.

$$\text{distance} = 63360 * \text{inch} * (1 * \text{foot} / 12 * \text{inch}) * (1 * \text{mile} / 5280 * \text{foot})$$

**Rearranging the terms**

We can rearrange the terms and rewrite this equation as

$$\text{distance} = (63360 * \text{inch} * 1 * \text{foot} * 1 * \text{mile}) / (12 * \text{inch} * 5280 * \text{foot})$$

**Canceling like terms**

Canceling out like terms in the numerator and denominator leaves us with

$$\text{distance} = (63360 \cdot \text{mile}) / (12 \cdot 5280)$$

Doing the arithmetic, we get

$$\text{distance} = 1 \cdot \text{mile}$$

(Obviously, I started out with a set of numbers that were designed to cause the final answer to be one mile, but that was simply for convenience.)

**There you have it**

You have seen a detailed procedure for converting from a distance expressed in paces (for a specific individual) to the same distance expressed in miles. Obviously, once you understand the overall procedure, you could omit and/or combine some of the steps to simplify the process.

**4.2 Using JavaScript to convert units**

Let's write, execute, and analyze a script that solves the same problem.

The objective of the script is to convert a distance of 2112 paces to a distance in miles where it is given that

- 100 paces = 3000 inches
- 12 inches = 1 foot
- 5280 feet = 1 mile

Please copy the code from Listing 1 (p. 10) into an html file and open the file in your browser.

**Listing 1: Convert from paces to miles.**

```
>!-- File JavaScript01.html ----- >
>html >>>body >
>script language="JavaScript1.3" >

var d = 2112 //distance, units = paces

document.write(
"d = " + d + " paces" + ">/br >")

var d2 = 3000 //distance, units = inches
var d3 = 100 //distance, units = paces

var f1 = d2/d3 //factor, units = inches/pace
document.write(
"f1 = " + f1 + " inches/pace" + ">/br >")

//pace*inch/pace = inch
var d = d *f1 //distance, units = inches
document.write(
"d = " + d + " inches" + ">/br >")

var f2 = 1/12 //factor, units = feet/inch
document.write(
"f2 = " + f2 + " feet/inch" + ">/br >")

//inch*feet/inch = feet
```

```
var d = d *f2 //distance, units = feet
document.write(
"d = " + d + " feet" + ">/br >")

var f3 = 1/5280 //factor, units = miles/foot
document.write(
"f3 = " + f3 + " miles/foot" + ">/br >")

//feet*mile/feet = mile
var d = d *f3 //distance, units = miles
document.write(
"d = " + d + " mile" + ">/br >")

//Now reverse the process using reciprocals
//(pace/inch)*(inch/foot)*(foot/mile) = pace/mile
var f4 =(1/f1)*(1/f2)*(1/f3) //factor, units = paces/mile
document.write(
"f4 = " + f4 + " paces/mile" + ">/br >")

//miles*paces/mile = paces
var d = d *f4 //distance, units = paces
document.write(
"d = " + d + " paces" + ">/br >")

document.write("The End")

>/script >
>/body >>/html >
```

### Screen output

When you open the html file in your browser, the text shown in Figure 5 (p. 12) should appear in your browser window.

---

### Screen output for Listing #1.

```
d = 2112 paces
f1 =30 inches/pace
d = 63360 inches
f2 = 0.08333333333333333 feet/inch
d = 5280 feet
f3 = 0.0001893939393939394 miles/foot
d = 1 mile
f4 = 2112 paces/mile
d = 2112 paces
The End
```

**Figure 5:** Screen output for Listing #1.

---

#### Discuss the output first

Let's begin our discussion with the screen output shown in Figure 5 (p. 12) and then go back and examine the code that produced that output.

Figure 5 (p. 12) begins with a distance (labeled **d**) expressed in units of paces. The value is 2112 paces, which is the distance from your home to your school discussed earlier.

#### Convert paces to inches

Then Figure 5 (p. 12) shows the conversion factor (labeled **f1**) that can be used to convert a value expressed in paces to a value expressed in inches.

That conversion factor is applied to the original distance producing an output consisting of the same distance expressed in inches instead of paces. The distance is now 63360 inches.

#### Convert from inches to feet

Following that, **f2** shows the conversion factor that can be used to convert a value expressed in inches to the same value expressed in feet.

That conversion factor is applied to the current distance value producing an output consisting of the same distance expressed in feet instead of inches. The distance is now 5280 feet.

#### Convert from feet to miles

Finally, Figure 5 (p. 12) shows the conversion factor (labeled **f3**) that can be used to convert a value expressed in feet to the same value expressed in miles.

That conversion factor is applied to the current distance value producing a final output consisting of the same distance expressed in miles instead of feet. That distance is now one mile (obviously I chose the numbers to make it come out that way).

That is the number that we were looking for.

#### Reverse the process

If a conversion factor **X** can be used to convert from A-units to B-units, the reciprocal of **X** can be used to convert from B-units back to A-units.

As you will see later, the conversion factor **f4** shown in Figure 5 (p. 12) was computed as the product of the reciprocals of **f1**, **f2**, and **f3**. The result is that the conversion factor labeled **f4** can be used

to convert a value expressed in miles to the same value expressed in paces (according to the pace-length for one individual).

That conversion factor was applied to the distance expressed in miles to produce the same distance expressed in paces. As you might have guessed, this value of 2112 paces shown near the end of Figure 5 (p. 12) matches the value shown at the beginning of Figure 5 (p. 12) .

### Analysis of the script

Now lets analyze the script code in Listing 1 (p. 10) that produced the text output shown in Figure 5 (p. 12) .

Listing 1 (p. 10) begins by declaring and initializing the variables **d** , **d2** , and **d3** , which are respectively,

- The home to school distance in paces.
- The two values that represent the relationship between paces and inches for one individual.

Various values are displayed as the script executes using calls to the **document.write** method. You should have no difficulty identify the code that displays the values, so I will skip over that code in this discussion.

### Compute, save, and apply the conversion factor f1

Then Listing 1 (p. 10) declares a variable named **f1** and populates it with the value of a conversion factor that can be used to convert paces to inches. This conversion factor is computed from the known relationship between paces and inches mentioned earlier.

The conversion factor named **f1** is applied to the distance in paces converting it to a value that represents the same distance in inches (63360 inches).

### Analysis of the units

The comment that reads  
`//pace*inch/pace = inch`

is an analysis that shows the units that will result from applying the conversion factor to the distance at this point. As you can see, the pace terms will cancel and the result will be in inches.

### Compute, save, and apply the conversion factor f2

Following that, a conversion factor named **f2** is computed that can be used convert a value expressed in inches to the same value expressed in feet. This conversion factor is based on the known fact that there are 12 inches in a foot.

The factor named **f2** is applied to the distance that is now expressed in inches converting it to a new value that expresses the same distance in feet (5280 feet).

### Analysis of the units

The comment that reads  
`//inch*feet/inch = feet`

is an analysis that shows the units that will result from applying the conversion factor to the distance at this point. As you can see, the inch terms will cancel and the result will be in feet.

### Compute, save, and apply the conversion factor f3

Following that, a conversion factor named **f3** is computed that can be used to convert a value expressed in feet to the same value expressed in miles. This conversion factor is based on the known fact that there are 5280 feet in a mile.

The factor named **f3** is applied to the distance that is now expressed in feet converting it to a new value that expresses the same distance in miles (1 mile).

### Analysis of the units

The comment that reads  
`//feet*mile/feet = mile`

is an analysis that shows the units that will result from applying the conversion factor to the distance at this point. As you can see, the feet terms will cancel and the result will be in miles.

### That satisfies the specifications

That satisfies the original program specification. However, I mentioned earlier that if a conversion factor  $X$  can be used to convert from A-units to B-units, the reciprocal of  $X$  can be used to convert from B-units back to A-units.

### Reversing the process

Continuing with Listing 1 (p. 10) , the comment that reads

```
//(pace/inch)*(inch/foot)*(foot/mile) = pace/mile
```

shows the units that survive from a product of the reciprocals of **f1** , **f2** , and **f3** . As you can see, after canceling out inches and feet, the result of multiplying the reciprocals of those three conversion factors is a new conversion factor that can be used to convert a value expressed in miles to a new value that represents the same distance expressed in paces.

That conversion factor is applied to the distance in miles producing an output of 2112, which unsurprisingly, is the distance in paces that we started off with.

## 4.3 More substantive JavaScript examples

Assume that you drop a rock from a balloon at a height of 10000 feet above the ground, Further assume that the positive direction is up and the negative direction is down.

What would be the height of the rock above the ground at the end of ten seconds? What would be the height of the rock at the end of twenty seconds?

### An equation relating distance, acceleration, and time

As you will learn in a future module, the following equation gives the distance that the rock will travel as a function of time assuming that the **initial velocity is zero** . (The assumption is that the rock was simply dropped **and not thrown** .)

$$d = 0.5 * g * t^2$$

where

- $d$  is the distance traveled
- $g$  is the acceleration of gravity (approximately  $-32.2 \text{ ft/s}^2$ )
- $t$  is time in seconds

### 4.3.1 Free fall exercise

Please copy the code from Listing 2 (p. 14) into an html file and open it in your browser.

#### Listing 2: Free fall exercise.

```
>!-- File JavaScript02.html --<
<html<>body<
<script language="JavaScript1.3"<

//Function to compute free-fall distance for a given time
// in seconds.
function distance(time){
  var g = -32.174; //acceleration of gravity in feet/sec^2
  var d = 0.5 * g * time * time;//(feet/sec^2)*sec^2 = feet
  return new Number(d.toFixed(0));
} //end function

//Compute and display height at ten seconds.
var t1 = 10;//time in seconds
var d1 = distance(t1);//distance traveled in feet
```

```
var h1 = 10000 + d1;//height in feet

document.write("At " + t1 + " seconds:" + ">/br<");
document.write("distance traveled = " + d1 + " feet>/br<")
document.write("height = " + h1 + " feet>/br<")

//Compute and display height at twenty seconds.
var t2 = 20;//time in seconds
var d2 = distance(t2);//distance traveled in feet
var h2 = 10000 + d2;//height in feet

document.write(">/br<At " + t2 + " seconds:" + ">/br<");
document.write("distance traveled = " + d2 + " feet>/br<")
document.write("height = " + h2 + " feet>/br<")

document.write(">/br<The End")

>/script<>/body<>/html<
```

### Screen output

When you open the html file in your browser, the text shown in Figure 6 (p. 15) should appear in your browser window.

---

### Screen output for Listing #2.

```
At 10 seconds:
distance traveled = -1609 feet
height = 8391 feet

At 20 seconds:
distance traveled = -6435 feet
height = 3565 feet

The End
```

**Figure 6:** Screen output for Listing #2.

As you can see from Figure 6 (p. 15) , after ten seconds, the rock will have traveled a distance of -1609 feet. (The minus sign indicates downward motion.)

Adding this value to the initial height of 10000 feet produces a value of 8391 for the height of the rock at the end of 10 seconds.

Similarly, Figure 6 (p. 15) gives us 3565 for the height of the rock at 20 seconds.

### Analysis of the code

The code in Listing 2 (p. 14) begins by defining a function that computes and returns the distance traveled for a rock falling toward the earth for a given time interval since release assuming that the initial velocity was zero (the rock was simply dropped).

This function implements the equation (p. 14) shown earlier, and expects to receive the time as an input parameter. It returns the distance traveled during that time interval.

Note that the interaction of units is shown in the comments with the result that the returned value is in feet.

### Call the function twice in succession

Then the code calls that function twice in succession for two different time intervals (10 seconds and 20 seconds) to determine the distance traveled during each time interval.

In both cases, the distance traveled is added to the initial height of 10000 feet to determine the height of the rock at the end of the time interval.

Also, in both cases, the time interval, the distance traveled, and the resulting height of the rock above the ground are displayed as shown in Figure 6 (p. 15) .

Note again that in all cases, the interactions of the various units are shown in the comments.

### 4.3.2 A plotting exercise

Let's do another exercise and this time plot the results.

Please prepare your plot board to plot a curve defined by several points. Interpret the grid lines such that you can plot values ranging from 0 to 10000 feet on the vertical axis and you can plot values ranging from 0 to 30 seconds on the horizontal axis.

#### A tactile graph

If you have an assistant who will download the zip file named Phy1050.zip<sup>15</sup>, extract the file named Phy1050a1.svg, and use that file to create a tactile graph as described in the module named Manual Creation of Tactile Graphics at <http://cnx.org/content/m38546/latest/><sup>16</sup>, you won't need to draw your graph on the graph board. You can explore the tactile graph instead. Regardless, you should write the script and examine the printed output as described in the following paragraphs.

#### JavaScript code

Copy the code from Listing 3 (p. 16) into an html file and open it in your browser.

#### Listing 3: A plotting exercise.

```
<!-- File JavaScript03.html -->
<html<<body<
<script language="JavaScript1.3"<

//Function to compute free-fall distance for a given time
// interval in seconds.
function distance(time){
  var g = -32.174; //acceleration of gravity in feet/sec^2
  var d = 0.5 * g * time * time;//(feet/sec^2)*sec^2 = feet
  return new Number(d.toFixed(0));
} //end function

//Compute the height of the rock every two seconds from
// release until the rock hits the ground.
var t = 0;//seconds
```

<sup>15</sup><http://cnx.org/content/m37480/latest/Phy1050.zip>

<sup>16</sup><http://cnx.org/content/m38546/latest/>



```
while(t <= 30){
  d = distance(t);//distance traveled in feet
  h = 10000 + d;//height in feet

  //Don't allow the height to go negative (underground)
  if(h < 0){
    h = 0;
  }//end if

  //Display time and height at current time.
  document.write("time = " + t +
    " height = " + h + "<br/<");
  t = t + 2;
} //end while

document.write("</br<The End")

</script<</body<</html<
```

### The screen output

When you open the html file in your browser, the text shown in Figure 7 (p. 18) should appear in your browser window.

---

**Screen output for Listing #3.**

```
time = 0 height = 10000
time = 2 height = 9936
time = 4 height = 9743
time = 6 height = 9421
time = 8 height = 8970
time = 10 height = 8391
time = 12 height = 7683
time = 14 height = 6847
time = 16 height = 5882
time = 18 height = 4788
time = 20 height = 3565
time = 22 height = 2214
time = 24 height = 734
time = 26 height = 0
time = 28 height = 0
time = 30 height = 0
```

The End

**Figure 7:** Screen output for Listing #3.

---

**Plot the data**

Using pushpins, rubber bands, pipe cleaners, or whatever works best for you, plot the 16 points shown in Figure 7 (p. 18) on your graph board. Your horizontal axis should be the time axis and your vertical axis should be the height axis.

If we have both done everything correctly, the shape of your curve should be an upside-down parabola with its high point at a value of 10000. (If the word parabola is new to you, don't worry about it. It's not important for the purpose of this module.)

**The tactile graph**

For the benefit of your assistant who will create the tactile graph, Figure 8 (p. 19) shows the image that is contained in the file named Phy1050.svg. As usual, this is a mirror image of the actual image in anticipation of the fact that your assistant will emboss the image from the back side of the paper. A non-mirror-image version of this same image is shown in Figure 10 (p. 22) .

Image from the file named Phy1050a1.svg.

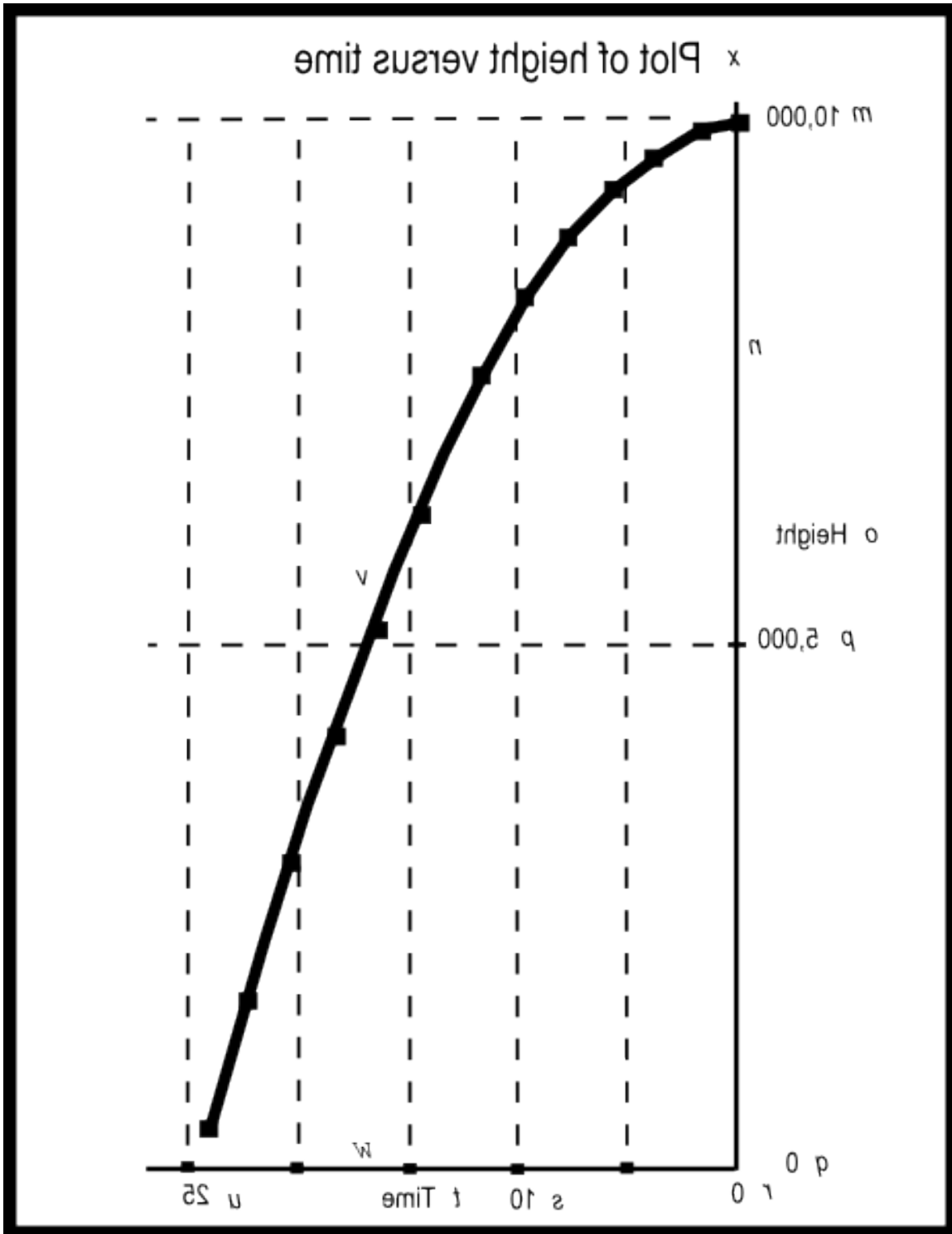


Figure 8: Image from the file named Phy1050a1.svg.

### Key-value pairs

Hopefully, by now you are familiar with the concept of key-value pairs as used in these modules. If not, please go back and study the section on Key-value pairs in conjunction with Figure 2 of the module named Manual Creation of Tactile Graphics at [http://cnx.org/content/m38546/latest/#Figure\\_2](http://cnx.org/content/m38546/latest/#Figure_2)<sup>17</sup>.

Figure 9 (p. 20) below contains the key-value table for the image shown in Figure 8 (p. 19).

---

### Key-value table for the image shown in Figure 8.

```
m: 10,000
n: Vertical axis
o: Height
p: 5,000
q: 0
r: 0
s: 10
t: Time
u: 25
v: Height versus time curve
w: Horizontal axis
x: Plot of height versus time
```

**Figure 9:** Key-value table for the image shown in Figure 8.

---

### Change in height increases with time

Getting back to the height versus time values shown in Figure 7 (p. 18) and plotted on your graph board or tactile graph, the value for height should decrease for each successive time value.

However, the change in height should increase for each successive time interval until the height goes to zero. The height goes to zero when the rock has landed on the ground somewhere between 24 and 26 seconds. It continues at zero after that.

(Note that I put code into the script to prevent the value of height from going negative.)

The increase in the change in the height value during each successive time interval is the result of the time being squared in the equation given earlier (p. 14). It is also the result of the acceleration of gravity that causes the downward speed of the rock to increase as the rock falls.

### Analysis of the code

Listing 3 (p. 16) calls a function copied from the previous exercise inside a **while** loop to compute the distance traveled and the resulting height for a series of time values ranging from 0 to 30 seconds in two-second intervals.

Those values are displayed as shown in Figure 7 (p. 18).

Understanding the code in Listing 2 (p. 14) and Listing 3 (p. 16) is important. However, it is more important at this point that you understand the handling of units as shown in the comments in the code.

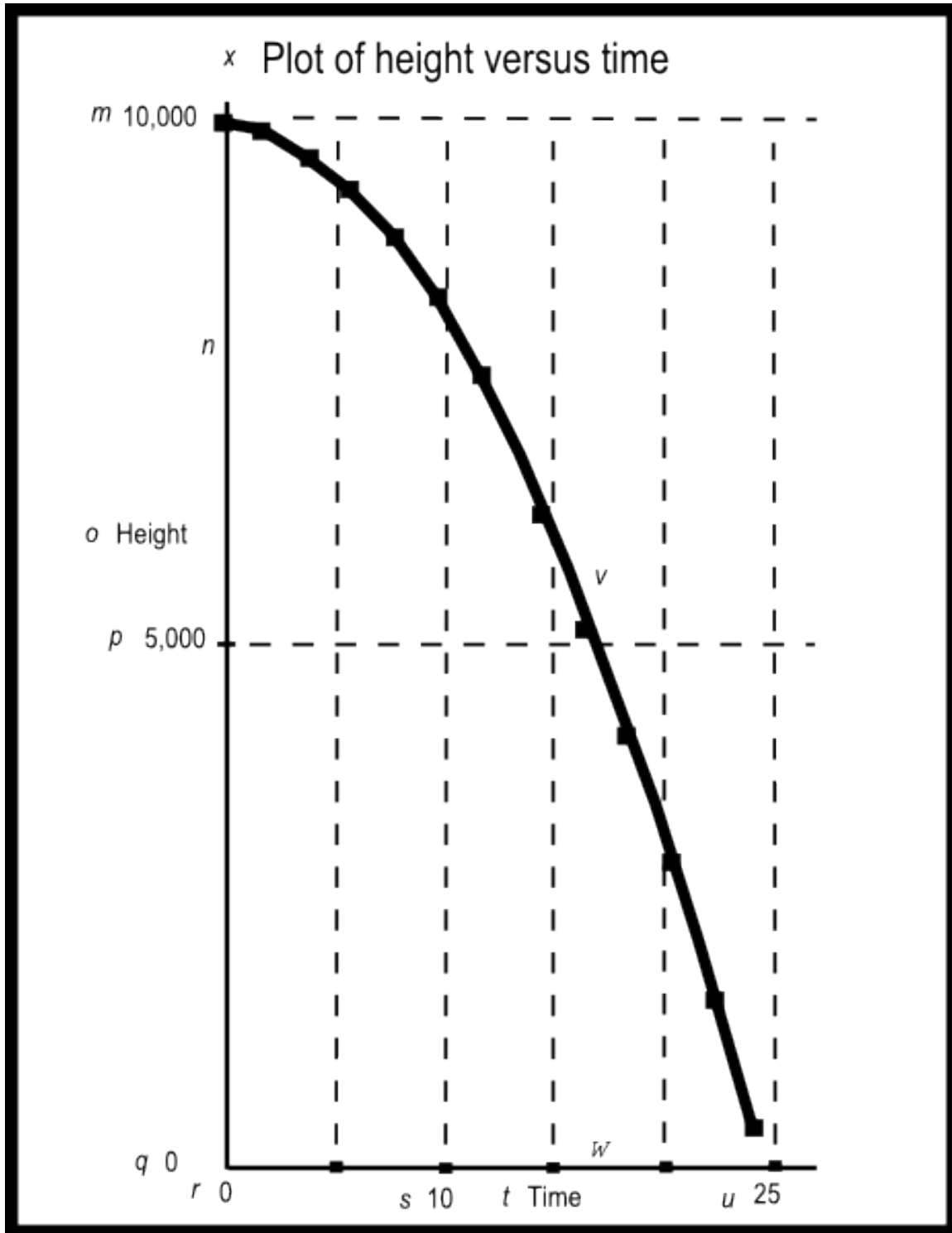
---

<sup>17</sup>[http://cnx.org/content/m38546/latest/#Figure\\_2](http://cnx.org/content/m38546/latest/#Figure_2)

#### 4.4 Non-mirror-image graphics

Figure 8 (p. 19) shows the actual mirror image contained in the file named Phy1050a1.svg. Figure 10 (p. 22) shows a non-mirror-image version of that same image.

Non-mirror-image version of the image from the file named Phy1050a1.svg.



**Figure 10:** Non-mirror-image version of the image from the file named Phy1050a1.svg.

## 5 Run the scripts

I encourage you to run the scripts that I have presented in this lesson to confirm that you get the same results. Copy the code for each script into a text file with an extension of .html. Then open that file in your browser. Experiment with the code, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

## 6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

## 7 Miscellaneous

This section contains a variety of miscellaneous information.

**NOTE: Housekeeping material**

- Module name: Units and Dimensional Analysis
- File: Phy1050.htm
- Revised: 07/02/2011
- Keywords:
  - physics
  - accessible
  - blind
  - graph board
  - screen reader
  - Braille display
  - JavaScript
  - units
  - dimensional analysis
  - SI units
  - powers of ten
  - acceleration of gravity

**NOTE: Disclaimers: Financial** : Although the Connexions site makes it possible for you to download a PDF file for this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

I also want you to know that I receive no financial compensation from the Connexions website even if you purchase the PDF version of the module.

**Affiliation** : I am a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-