Force – Moments, Torque, Couple, and Equilibrium*

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Abstract

This module explains moments, torque, and couples in a format that is accessible to blind students.

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2 Preface

2.1 General

This module is part of a collection of modules designed to make physics concepts accessible to blind students.

See http://cnx.org/content/col11294/latest/ [1] for the main page of the collection and http://cnx.org/content/col11294/lat [2] for the table of contents for the collection.

The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

This module explains moments, torque, and couples in a format that is accessible to blind students.

2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.
- The ability to create tactile graphics as described at http://cnx.org/content/m38546/latest/ [7] .

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch? [8]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [9]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/∼djoyce/trig/ [10]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [11] and http://www.w3schools.com/js/default.asp [12]).
- An understanding of all of the material covered in the earlier modules in this collection.
- An understanding of the creation and use of tactile graphics as described at http://cnx.org/content/m38546/latest/ [13] .

[1] http://cnx.org/content/col11294/latest/
[2] http://cnx.org/content/col11294/latest/#cnx_sidebar_column
[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://cnx.org/content/m38546/latest/
[8] http://www.youtube.com/watch?v=c8plj9UsJbg
[9] http://www.youtube.com/watch?v=v-F06HgiUpw
[10] http://www.clarku.edu/∼djoyce/trig/
[11] http://www.dickbaldwin.com/tocjscript1.htm
[12] http://www.w3schools.com/js/default.asp
[13] http://cnx.org/content/m38546/latest/

2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following link to easily find and view the figure while you are reading about it.

2.3.1 Figures

- Figure 1 (p. 7) . Mirror image from the file named Phy1110a1.svg.
- Figure 2 (p. 8) . Non-mirror-image version of the image from the file named Phy1110a1.svg.
- Figure 3 (p. 9) . Key-value pairs for the image in the file named Phy1110a1.svg.
- Figure 4 (p. 13) . Relationship between pound-force and kilogram-force.
- Figure 5 (p. 15) . Mirror image from the file named Phy1110b1.svg.
- Figure 6 (p. 17) . Non-mirror-image version of the image from the file named Phy1110b1.svg.
- Figure 7 (p. 18) . Key-value pairs for the image in the file named Phy1110b1.svg.
- Figure 8 (p. 22) . Mirror image from the file named Phy1110c1.svg.
- Figure 9 (p. 23) . Non-mirror-image version of the image from the file named Phy1110c1.svg.
- Figure 10 (p. 24) . Key-value pairs for the image in the file named Phy1110c1.svg.
- Figure 11 (p. 26) . Mirror image from the file named Phy1110d1.svg.
- Figure 12 (p. 28) . Non-mirror-image version of the image from the file named Phy1110d1.svg.
- Figure 13 (p. 29) . Key-value pairs for the image in the file named Phy1110d1.svg.

2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [14] .

3 General background information

Creation of tactile graphics

The module titled Manual Creation of Tactile Graphics at http://cnx.org/content/m38546/latest/ [15] explained how to create tactile graphics from svg files that I will provide.

If you are going to have an assistant create tactile graphics for this module, you will need to download the file named Phy1110.zip [16] , which contains the svg files for this module. Extract the svg files from the zip file and provide them to your assistant.

Also, if you are going to use tactile graphics, it probably won't be necessary for you to perform the graph board exercises. However, you should still walk through the graph board exercises in your mind because I will often embed important physics concepts in the instructions for doing the graph board exercises.

In each case where I am providing an svg file for the creation of tactile graphics, I will identify the name of the appropriate svg file and display an image of the contents of the file for the benefit of your assistant. As explained at http://cnx.org/content/m38546/latest/ [17] , those images will be mirror images of the actual images so that your assistant can emboss the image from the back of the paper and you can explore it from the front.

I will also display a non-mirror-image version of the image so that your assistant can easily read the text in the image.

Also in those cases, I will provide a table of key-value pairs that explain how the Braille keys in the image relate to text or objects in the image.

Conditions for equilibrium

[14] http://www.dickbaldwin.com/toc.htm
[15] http://cnx.org/content/m38546/latest/
[16] http://cnx.org/content/m38167/latest/Phy1110.zip
[17] http://cnx.org/content/m38546/latest/

You learned in earlier modules that the vector sum of all forces acting on a body must be zero in order for that body to be in equilibrium. There is a second, equally important condition for equilibrium:

The sum of all torques acting upon a body in equilibrium measured about any axis must be zero.

A see saw toy

When I was a child, virtually every playground meant for children had one or more tows commonly called see saws. I never see them any more. They have probably been banned as a safety hazard along with swings and other toys.

Hopefully you have had the privilege of riding on a see saw, because riding on a see saw can provide several different physics lessons.

A simple device

A see saw is a simple device. It can be as simple as a long board balanced on a fulcrum near the center. A child sits on each end. As one child goes down, the other child goes up. Each child uses their feet to propel their end of the board upward. When one end goes up, the other end goes down.

It's the negative acceleration that hurts

Even under the worst of conditions a see saw can prove to be a good physics lesson for a child. The lesson begins when the child on one end jumps off while the child on the other end is high in the air. The remaining child suddenly begins a free fall toward the center of the earth with an acceleration of approximately 32.2 feet/sec^2.

After experiencing that once or twice, the child comes to learn that it is not the high downward velocity that hurts. Instead, it is the very high negative acceleration when the child makes contact with the ground that hurts.

Position relative to the fulcrum

Children also quickly learn that when two children of different body masses play on a see saw, the one with the greatest body mass must sit closer to the fulcrum. Otherwise, the see saw will only be in equilibrium when one end is touching the ground and the other end is high in the air.

Children learn that if each child sits just the right distance from the fulcrum with the more massive child closer to the fulcrum, they can cause the board to balance and be in equilibrium, at least for a short period of time. They also learn that is the seating position that provides the smoothest up and down motion.

Each child exerts a downward force

Although the children probably don't realize it, each child exerts a downward force on the board equal to the body mass of the child multiplied by the acceleration of gravity. Those forces are commonly referred to as the weights of the children.

Balance and equilibrium

When the see saw is in equilibrium (with both ends off the ground), the fulcrum exerts an upward force on the board that is equal to the combined weights of the children. Otherwise, the see saw and the children would either fly off into space, or sink into the earth.

In addition, when the see saw is in equilibrium with both ends off the ground, the product of each child's weight and the distance of the child from the fulcrum is equal to the product of the other child's weight and the distance of the other child from the fulcrum. Children usually figure out how that works but I doubt that they understand why it works.

The crank

I have an emergency light that has a crank on the side. When I turn the crank, an electrical generator inside the light turns, which generates electrical power that illuminates a light bulb in the end of the light.

There are many other examples of cranks in common use. For example, I have an automobile jack with a crank that is used to raise the automobile in order to replace a flat tire.

Sailboats have hand-operated windlasses with cranks that are used to shorten the length of ropes to raise the sails. Awnings have cranks that are used to raise or lower the awning to create more or less shade from the sun.

A toggle switch on the wall is a small crank with about a 30-degree angle of travel that is used to turn the lights on and off. Pushing the handle in one direction or the other causes an axel to turn inside the switch causing an electrical contact to be closed or open.

The turning effect of a crank

When analyzing a crank, there are at least two factors that must be considered. One factor is the force exerted on the handle. The second is the length of the crank arm. The turning effect depends on the product of the force and the length of the crank arm.

Moments or torques

Cranks are often used, (along with other mechanical devices such as screws) to provide mechanical advantage. The same turning effect (torque) can be achieved with a long crank arm and a small force, or a short crank arm and a large force. (Remember, the torque is the product of the two.) For example, much more torque is required to raise my automobile off the ground than is required to open or close the electrical contacts inside a toggle switch. Therefore, the crank arm on my automobile jack is much longer than the crank arm on a toggle switch.

By definition, (see *College Physics by Mendenhall, Keys, Eve, and Sutton)*

The torque or moment of a force about an axis through the point O is defined as the product of the force and the perpendicular distance from O to the line of action of the force.

Do torque and moment of a force mean the same thing?

This definition would have you believe that the torque of a force and the moment of a force mean the same thing. If you Google the difference between torque and moment, you will find many who agree that they mean the same thing and many who disagree. However, for practical purposes in this module, we will assume that they mean the same thing.

The turning action has a direction

Like force, the turning action of torque has a direction. Unlike force, however, for which the direction is a line through a point, the direction of the turning action of a torque is rotational about a point. The direction can be either clockwise or counter-clockwise. In this module, we will assume that a counter-clockwise direction is positive and a clockwise direction is negative.

(Although the turning action of a torque is rotational about a point, the true direction of a torque is said to be perpendicular to the plane containing the forces. That topic is beyond the scope of this module.)

A machine that converts force to torque

A crank is a machine that converts force along a line into torque about a point. When the crank arm on my automobile jack is horizontal with the crank handle to the right, and I push down on the crank handle, a clockwise torque is developed about the axle at the other end of the crank arm.

That axle is actually a large screw, and the torque causes the screw to turn. The other end of that screw is threaded through a mechanism that is often referred to as a "scissors jack." The rotational motion of the screw causes a small platform to raise underneath my car to lift it off the ground.

Oops, need to change direction

When I push down on the crank handle and the screw begins to turn, the crank arm rotates along with the screw. Therefore, the crank arm is no longer horizontal but is oriented at an angle slightly south of east. In order to keep the screw turning, I must adjust the direction of my force to being a little west of south instead of being straight down. As the process continues, I must continually adjust the direction of the force to be perpendicular to the length of the crank arm.

4 Discussion and sample code

Let's begin the discussion by expanding the conditions for equilibrium beyond the conditions that we have considered in earlier modules.

4.1 Conditions for equilibrium

In order for a body to be in equilibrium,

The vector sum of all forces acting on the body must be zero and the sum of all torques acting on the body measured about any axis must be zero.

4.2 Equilibrium for a body with an arbitrary shape

Achieving equilibrium may be easier said than done for a body with an arbitrary shape. Let's consider what first appears to be a simple example. Using your graph board, mark out a square that is four units on each side. You should probably make it rather large using four or five divisions on the graph board to represent one unit. Let this square represent a square board that is free to move horizontally or vertically and is also free to rotate.

Mark four points with pins

Assume that the lower left corner of the square is the origin with coordinates of x=0 and y=0. Place pins at the following coordinate positions:

- A. x=1, y=1
- B.x=1, y=2
- C. x=3, y=2
- D. x=2, y=1

Draw vectors

Now use rubber bands or pipe cleaners to draw the following force vectors originating at the locations of the pins. (Positive angles are measured counter-clockwise relative to the positive x axis.)

- A. 5 units magnitude at 180 degrees
- B. 2.8 units magnitude at 45 degrees
- C. 3 units magnitude at 0 degrees
- D. 2 units magnitude at -90 or +270 degrees

Tactile graphics

The svg file that is required to create tactile graphics for this exercise is named Phy1110a1.svg. You should have downloaded (p. 3) that file earlier. This file contains a vector diagram that represents the instructions given above (p. 6) .

Figure 1 (p. 7) shows the mirror image that is contained in that file for the benefit of your assistant who will create the tactile graphic for this exercise.

Mirror image from the file named Phy1110a1.svg.

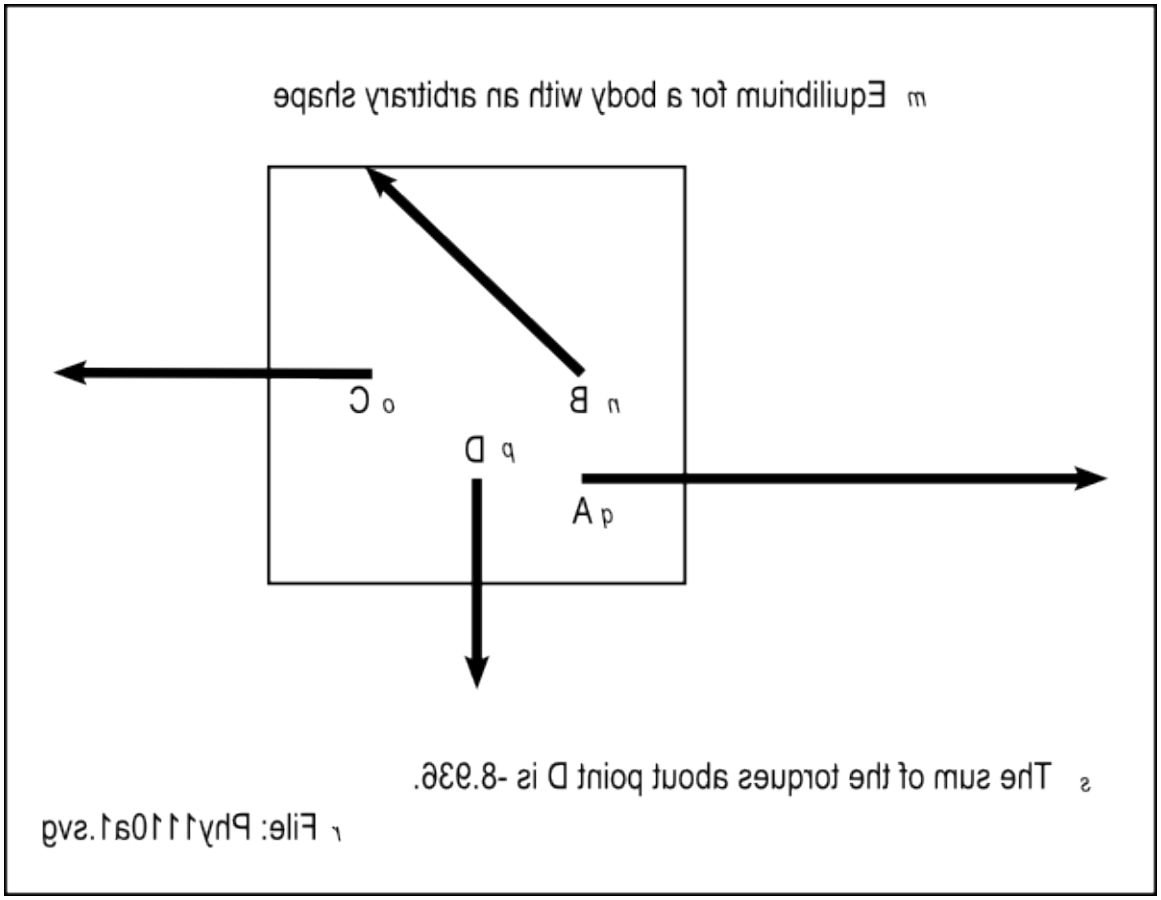

Figure 1: Mirror image from the file named Phy1110a1.svg.

Figure 2 (p. 8) shows a non-mirror-image version of the same image.

Non-mirror-image version of the image from the file named Phy1110a1.svg.

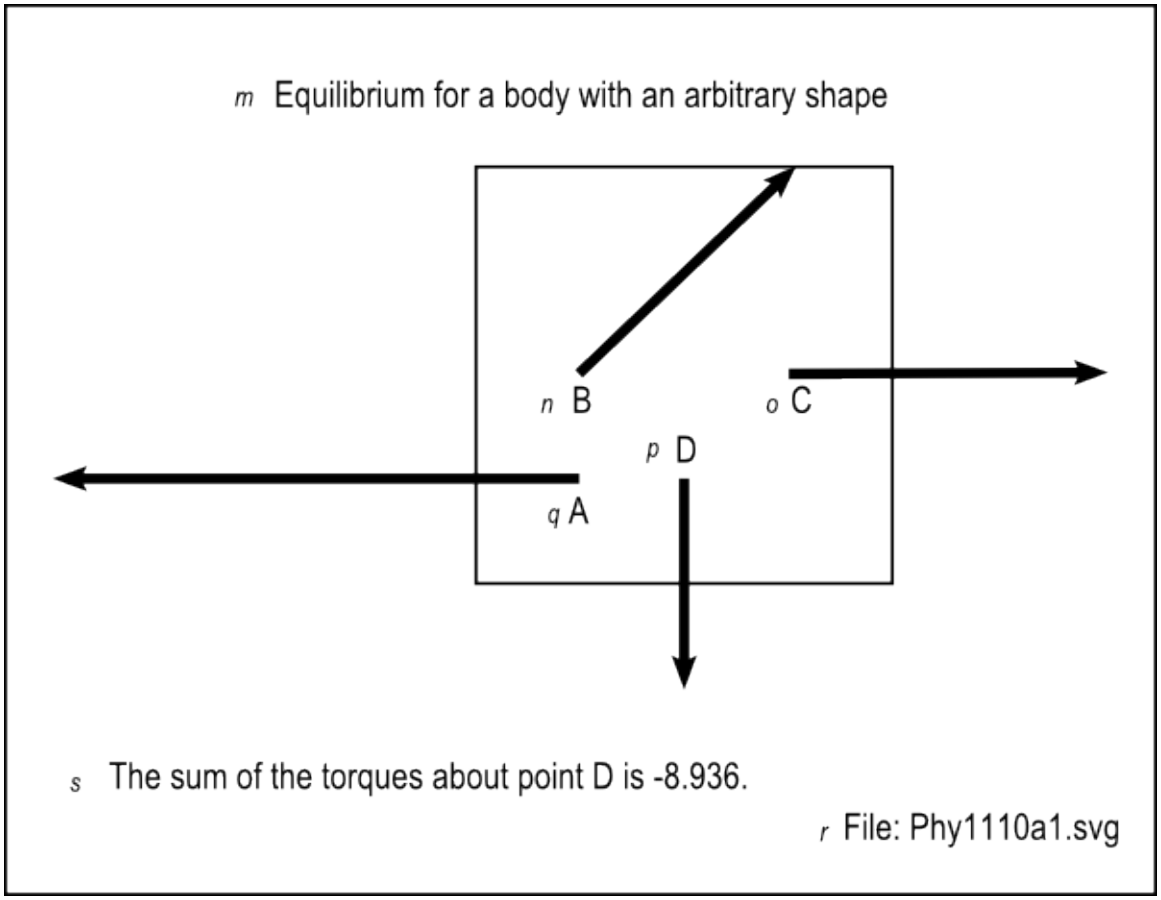

Figure 2: Non-mirror-image version of the image from the file named Phy1110a1.svg.

Figure 3 (p. 9) shows the key-value pairs that go with the image in the file named Phy1110a1.svg.

Key-value pairs for the image in the file named Phy1110a1.svg.

```
    m: Equilibrium for a body with an arbitrary shape
n: B
o: C
p: D
q: A
r: File: Phy1110a1.svg
s: The sum of the torques about point D is - 8.936.
```

Figure 3: Key-value pairs for the image in the file named Phy1110a1.svg.

Compute the vector sum of the forces

Begin by computing the horizontal and vertical components of the force at B. As you know by now, the horizontal component is equal to the magnitude of the vector (2.8) multiplied by the cosine of 45 degrees and the vertical component is equal to the magnitude of the vector multiplied by the sine of 45 degrees. Both the sine and the cosine of 45 degrees is 0.707, so the horizontal and vertical components are both equal to 2. (Obviously, I planned it that way.)

The horizontal and vertical components

The remaining vectors are either horizontal or vertical so no trigonometry is required to compute their components.

The horizontal components of all the vectors consist of:

- A. 5 units at 180 degrees = -5
- Bx. 2 units at 0 degrees = +2
- C. 3 units at 0 degrees = +3

The sum of the horizontal components is 0.

The vertical components of all the vectors consist of:

- By. 2 units at 90 degrees = +2
- D. 2 units at -90 or +270 degrees = -2

The sum of the vertical components is also 0. Therefore, the system exhibits the first condition of equilibrium.

Compute the moment or torque

The second condition for equilibrium states that the sum of all torques acting on the body measured about any axis must be zero.

You can often reduce the complexity of the computation for computing torque by judicious selection of the point about which the torque will be computed.

Compute the torque about point D

In this case, we can simplify the arithmetic by choosing an axis that goes through the point at D.

If you examine your graph, you will see that the line of action for forces A and D both go through the point at D. Therefore, neither of those forces creates a moment about point D so we can exclude them from our computation.

Forces B and C create a torque about D

Further examination of your graph will reveal that forces B and C both create a torque about D, and they both create a clockwise or negative torque. Since there are no more forces being exerted on the board, the sum of the torques cannot be zero.

Thus, we can tell by inspection that this body is not in equilibrium.

For completeness...

Just for completeness, let's compute the torque about the point at D. If you extend the line of action for force B down and to the left, you will see that the distance from point D to that line, along a path that is perpendicular to the line, is 2.12 units. (You can compute that distance using either trigonometry or the Pythagorean theory.)

The components of the torque about the point at D are:

- B. 2.12 * (-2.8) = -5.936
- C. 1 * (-3) = -3

The sum of the torques about point D is not zero. Instead, it is -8.936. Therefore, the system does not satisfy the second condition for equilibrium.

Can equilibrium be achieved through the addition or modification of forces?

Obviously, if you add four more forces that are equal in magnitude and opposite in direction to the four existing forces, the system will be in equilibrium.

Also, if you allow the board to turn clockwise by several degrees, there is a point at which it will stop turning and will be in equilibrium.

Beyond those possibilities, I am unable, simply by inspection, to identify any forces that could be modified or added to bring the system into equilibrium. If it is required to add or modify forces to achieve equilibrium, I would need to construct and solve some simultaneous equations involving trigonometry. I have decided to opt out of that and will leave that as an exercise for the student.

The examples that follow will be less complicated than that.

4.3 A spring balance or spring scale

A fisherman's scale

What is a spring balance or spring scale? A spring scale is often called a fisherman's scale because a lot of fishermen carry one in their tackle box. They use it weigh the fish that they catch.

An old-fashioned spring scale consists of a stiff coil spring with a hook connected to each end. The spring is enclosed in a case that has a vertical slot with numbers along the side of the slot. There is a pointer attached to the spring that sticks out through the slot.

When you hold the top hook with one hand and hang something on the bottom hook, the spring extends causing the pointer to move downward. The distance that the pointer moves is an indication of the weight of the mass connected to the bottom hook.

A luggage scale

Because of the high fees that the airlines charge for overweight luggage, I carry a modern version of a spring scale when I travel so that I can weigh my luggage before check-in and make adjustments to the contents of each piece of luggage as appropriate. My spring scale is very similar to a fisherman's scale except that it is designed to weigh much heavier items than fish and is more modern in appearance.

My spring scale is designed to make it easy to measure the weight of a piece of luggage. (It also has a built-in tape measure, calibrated in both inches and centimeters that can be used to measure the dimensions of a piece of luggage, but that is not germane to this discussion.)

Construction of my spring scale

My spring scale has:

- A handle on the top
- A hook on the bottom

- A very stiff spring on the inside, and
- A circular dial with a red hand and a black hand on the front. The dial is calibrated both in lb (pounds-force) and in kg (kilograms-force).

The red and black hands

The red hand is normally in rotational equilibrium pointing to zero on the dial. The black hand can be rotated 360 degrees using a knob on the front of the scale.

Weighing a piece of luggage

You begin the process of weighing a piece of luggage by turning the black hand to point to zero. Then you hook the scale onto the handle on the luggage and pull up on the scale, causing the luggage to be lifted off the floor.

The red and black hands move

When you do that, the spring extends very slightly allowing the hook to move relative to the body of the scale, which in turn causes the entire scale to become slightly longer from top to bottom. The change in the state of the spring is proportional to the downward force (weight) exerted by the luggage on the hook.

The internal mechanism causes the red hand to rotate around the center of the dial. The position of the red hand indicates the weight of the luggage when compared to the printed calibrations on the face of the dial.

Equilibrium

When you are supporting the weight of the luggage with the spring scale, the luggage is being pulled down by the force of gravity and is being pulled up by the hook and your muscles. This causes the system to be in momentary equilibrium.

The black hand also moves

For convenience, the black hand moves along with the red hand, but unlike the red hand, the black hand doesn't return to zero when the luggage is allowed to settle back onto the floor. The position of the black hand, therefore, indicates the maximum angular excursion of the red hand, which indicates the weight of the luggage in either lb or kg.

Applying a force manually

When I hold the handle in one hand and pull on the hook with the other hand so that the overall length of the scale increases by about 1/4 inch, the red hand moves to a position indicating a force of approximately 20 lb or 9 kg.

When a spring balance is used in the following exercises, we will assume that the change in the overall length of the scale is negligible relative to the overall dimensions of the problem.

The difference between force and mass

The SI unit for mass is the kilogram with a symbol of kg.

An often-used non-SI unit for mass is the pound.

An area of potential confusion

Does the relationship between 20 lb and 9 kg as printed on the dial of my spring scale make sense? It only makes sense if you know what is meant by those abbreviations. (They do not indicate a 20-pound mass or a 9-kg mass as the abbreviations might lead you to believe.)

This is an area of physics that can be confusing – the difference between force and mass.

4.4 Exercise relating pound-force to kilogram-force

Let's work through an exercise using the following abbreviations and see if we can justify the relationship between lb and kg indicated by the dial on my spring scale:

- lbf means pounds-force
- pound means pounds mass
- kgf means kilograms-force
- kg means kilograms mass
- N means newtons

- m means meters
- s means seconds
- ft means feet

We will compute the relationship between pounds-force and kilograms-force using the force unit of newtons an intermediary.

We will assume that the acceleration of gravity is either 32.17 ft/s^2 or 9.81 m/s^2.

The calculations are shown in Figure 4 (p. 13) .

Relationship between pound-force and kilogram-force.

```
Begin with known conversion factors:
1 lbf = 1 pound * 32.17 * ft/s^2
1 pound = 0.45 * kg

Substitute kg for pound.
1 lbf = 0.45 * kg * 32.17 * ft/s^2

Substitute meters for feet.
1 ft = 0.30 m
1 lbf = 0.45 * kg * 32.17 * 0.30 * m/s^2

Do the arithmetic and substitute (1 N)
for (1 kg*m/s^2)
1 lbf = 4.34 * kg*m/s^2 = 4.34 N



Now compute the relationship between kgf and N
Begin with some more known quantities.
1 kgf = 1 kg * 9.81 * m/s^2
1 N = 1 kg*m/s^2

Substitute (1 N) for (1 kg  * m/s^2)
1 kgf = 9.81 N

Now form a ratio between 1 lbf and 1 kgf
1 lbf/1 kgf = (4.34 N)/(9.81 N)

Cancel like terms, do the arithmetic, and
multiply both sides by kgf.
1 lbf = 0.44 kgf
```

Figure 4: Relationship between pound-force and kilogram-force.

Is this the correct answer?

According to the online units converter [18] ,
1 pound-force (lbf) = 0.45359237 kilogram-force (kgf)

[18] http://www.translatorscafe.com/cafe/units-converter/force/calculator/kilogram-force-[kgf]-to-newton-[N]/

Considering that I rounded all computations to two decimal digits, my result is pretty close.

Apply conversion factor to the spring scale calibration

We now have a conversion factor that allows us to convert between lbf and kgf:

1 pound-force (lbf) = 0.45359237 kilogram-force (kgf)

If we multiply 20 lbf by0.45 to convert that value to force in units of kgf, we get 9.0 kgf. This agrees with the calibrations on my spring scale.

What do these terms really mean?

One lbf is the amount of force that is required to cause a one-pound mass to accelerate at 32.2 ft/s^2 due to the force of gravity.

One kgf is the amount of force required to cause a one-kg mass to accelerate at 9.81 m/s^2 due to the force of gravity.

4.5 Exercise involving a trapeze bar

Use your graph board to construct a picture that looks something like a very long trapeze bar. By this, I mean a horizontal bar with a rope tied to each end. The other end of each rope is firmly attached to the ceiling. Make the horizontal bar ten meters in length.

Tactile graphics

The file named Phy1110b1.svg contains an image that represents this scenario. The image shows the trapeze bar, the ropes, and the spring scales. The image also shows vectors that represent the forces acting on the trapeze bar.

Figure 5 (p. 15) shows the mirror image that is contained in that file for the benefit of your assistant who will create the tactile graphic for this exercise.

Mirror image from the file named Phy1110b1.svg.

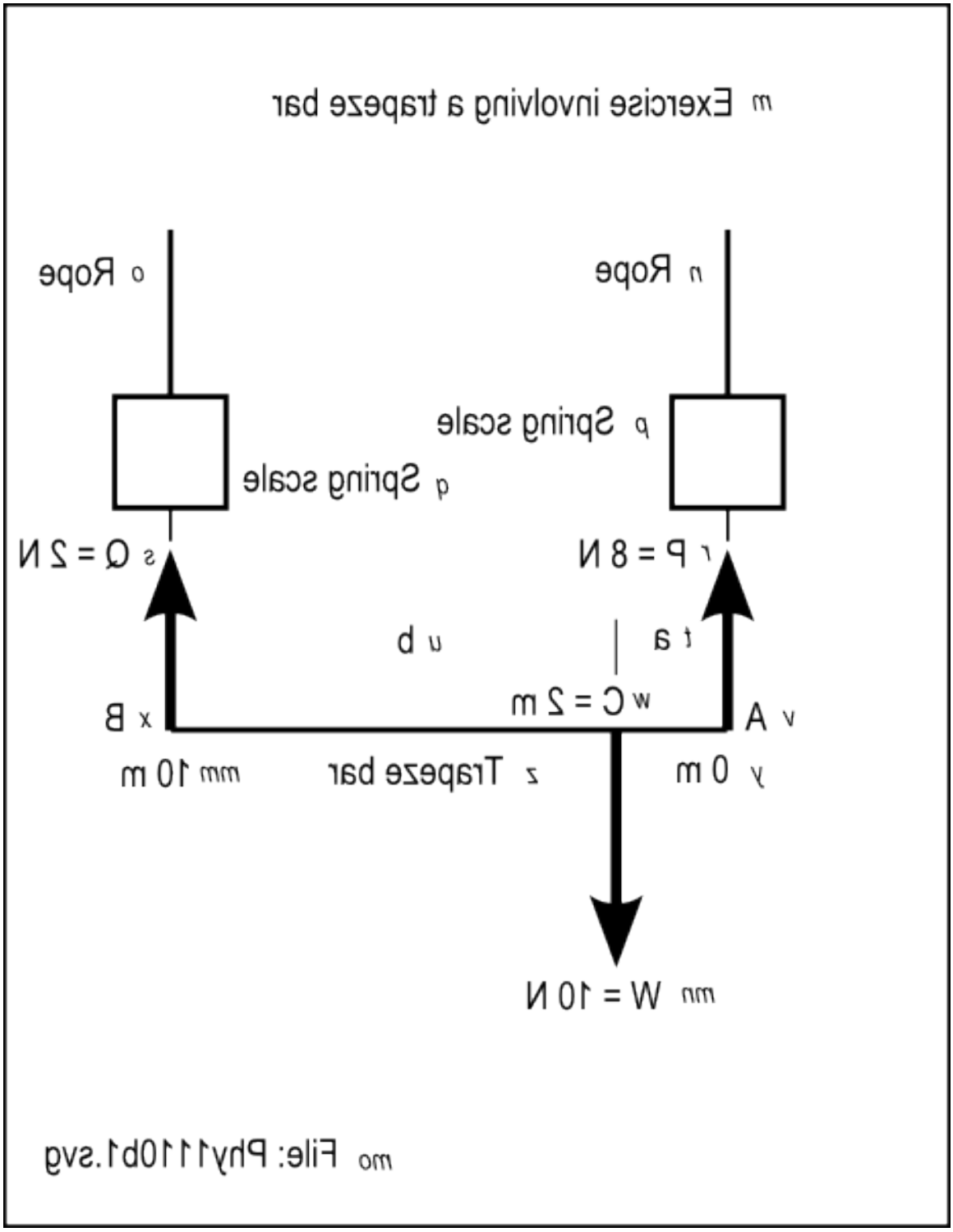

Figure 5: Mirror image from the file named Phy1110b1.svg.

Figure 6 (p. 17) shows a non-mirror-image version of the same image.

Non-mirror-image version of the image from the file named Phy1110b1.svg.

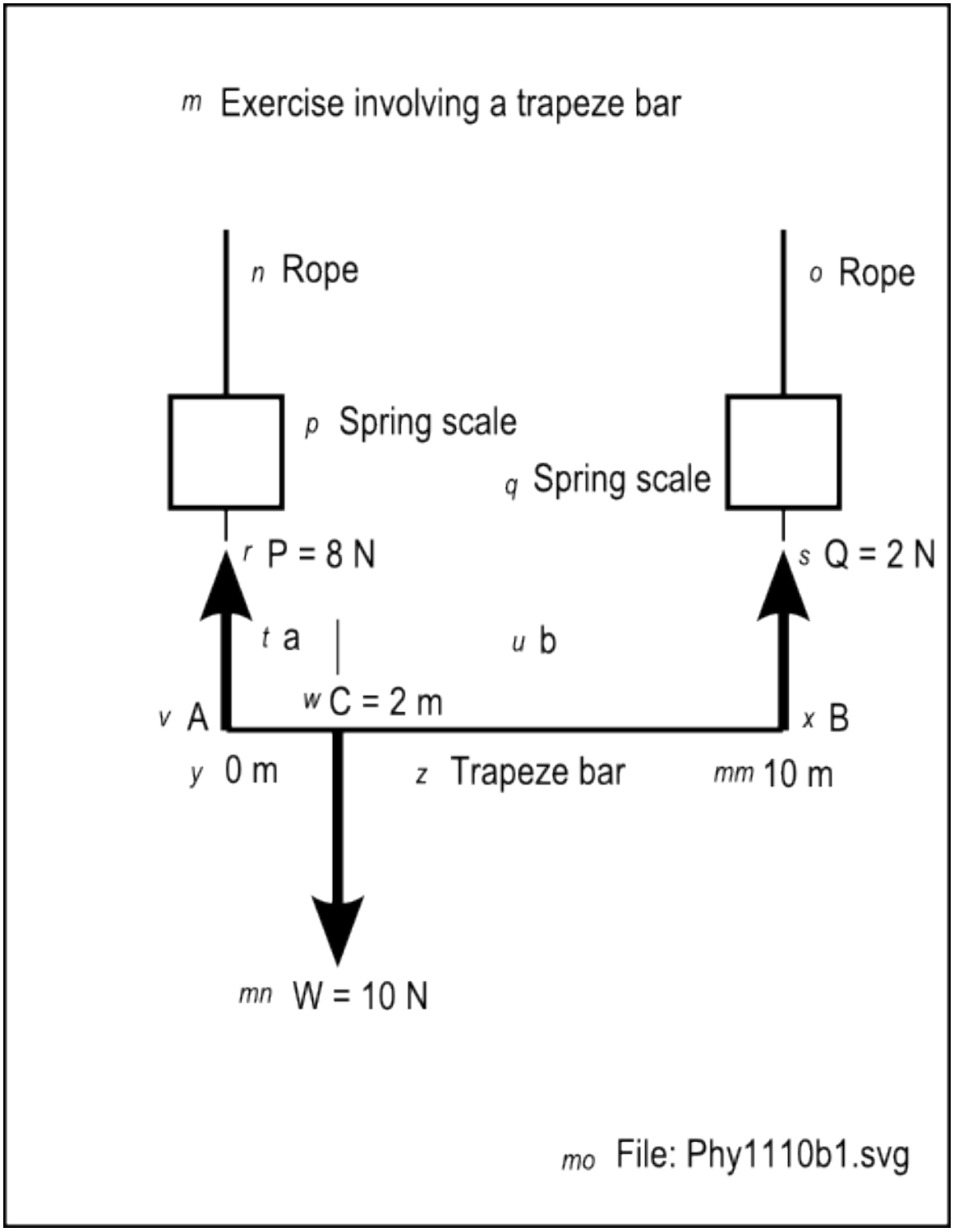

Figure 6: Non-mirror-image version of the image from the file named Phy1110b1.svg.

Figure 7 (p. 18) shows the key-value pairs that go with the image in the file named Phy1110b1.svg.

Key-value pairs for the image in the file named Phy1110b1.svg.

```
    m: Exercise involving a trapeze bar
n: Rope
o: Rope
p: Spring scale
q: Spring scale
r: P = 8 N
s: Q = 2 N
t: a
u: b
v: A
w: C = 2 m
x: B
y: 0 m
z: Trapeze bar
mm: 10 m
mn: W = 10 N
mo: File: Phy1110b1.svg
```

Figure 7: Key-value pairs for the image in the file named Phy1110b1.svg.

Insert a hypothetical spring scale in each rope

Now cut each rope somewhere along its length and insert something that is meant to replicate a spring scale in each rope. Assume that the weights of the ropes, the spring scales, and the bar are negligible. Also assume that the extension of the spring scales caused by loading is negligible. (In other words, the bar will remain horizontal even when downward forces are applied.)

Each spring scale will exert an upward force on the bar and will, at the same time indicate the amount of force with which that end of the bar is pulling down on the rope that supports it.

The bar is in equilibrium

By default, the bar is in equilibrium. By that I mean that the bar doesn't accelerate in any direction, nor does it rotate about any axis. Furthermore, the weight being registered on each spring scale is negligible.

Label points and distances on your drawing

Use your Braille labeler to label the left end of the bar A and the right end of the bar B. Use pins to subdivide the bar into ten units.

Now assume that you will hang a 10 N weight somewhere on the bar between the ropes. Label that point C. Label the distance from C to A as a, and label the distance from C to B as b. Label the weight as W.

Add the weight to the bar

The situation changes when you add the 10 N weight to the bar. Each spring scale now exerts an upward force on its end of the bar that is no longer negligible. Label the upward force on the left end P and label

the upward force on the right end Q.

Is the bar in equilibrium

I believe that most of you will know, based on experience with the trapeze bar on the playground, that the bar will still be in equilibrium. When you hung quietly on a trapeze bar as a child, it didn't accelerate off in some direction, nor did it rotate about any axis. So, we know from experience that the bar is still in equilibrium.

Three forces

There are now three forces being exerted on the bar: P, Q, and W. We know from earlier modules that the vector sum of those three forces must be zero in order for the bar to be in equilibrium. This system is simple enough that we can perform the vector sum in our heads. We don't need to draw a vector diagram. We see that

P + Q - W = 0

Adding W to both sides of the equation gives us:

P + Q = W (**eq. a1**)

The values for P and Q

What are the values for P and Q? We know that wherever we place the weight along the horizontal length of the bar, the bar will continue to be in equilibrium. Therefore, the sum of P and Q must be equal to W regardless of the position of W (so long as the position of W is between the ropes and the ropes don't break).

Compute the torque about C

Assume that positive coordinates are to the right with the origin at the left end of the bar. Computing the torque about the point C gives us:

(C-A)*(P) + (C-B)*(Q) = 0

Let's put some numbers in the problem now.

- Let C = 2
- Let A = 0
- Let B = 10
- Let W = 10

Substituting numbers in the above equation gives us:

(2)*(P) + (-8)*(Q) = 0

Simplify and rearrange terms

Simplifying, rearranging terms, and dividing both sides by 2 gives us:

P = 4*Q (**eq. a2**)

Two equations and two unknowns

Including eq. a1 (p. 19) , (repeated below) we now have two equations and two unknowns.

P + Q = W

Inserting the numeric value for W gives us

P + Q = 10, or

Q = 10 - P (**eq. a3**)

Substituting this value of Q into eq. a2 (p. 19) gives us

P = 4*(10 - P), or

P = 40 - 4*P, or

5*P = 40, or

P = 8

which is one of the answers that we are looking for.

Inserting the value for P into eq. a3 (p. 19) gives us

Q = 10 - 8, or

Q = 2

which is the other answer that we are looking for.

4.6 A more general case of the trapeze bar

Let's pick another point, label it X, and compute the moments about that point. Those moments must also sum to zero for the bar to be in equilibrium. (The moments computed about any point on the bar must sum to zero for the bar to be in equilibrium.)

The moments about X produced by the three forces are:

- P: (X-A)*(P)
- W: (X-C)*(W)
- Q: (X-B)*(Q)

Let X = 5

Substituting for X gives us:
(5)*(P) - (3)*(10) + (-5)*(Q) = 0
Simplifying, rearranging terms, and diving both sides by 5 gives us:
P - Q - 6 = 0, or
P = Q + 6 (**eq. a4**)
Now we can use this equation along with eq.a1 (p. 19) to solve for Q.
P + Q = 10, from eq.a1 (p. 19) , or
Q = 10 - P
Substituting this value for Q in eq. a4 (p. 20) gives us,
P = 10 - P + 6
Simplifying and dividing both sides by 2 gives us,
P = 8 , which is the same answer as before (which it should be)
Substitution of P back into eq. a1 gives us,
Q = 2

4.7 Apply the weight at different locations on the trapeze

If you solve for P and Q for any location of the weight between the ropes, you will find that the values for the upward forces at each end of the bar are inversely proportional to the distance from the weight to that end of the bar.

For example, for equilibrium, using the dimension symbols established for your drawing earlier:
a*P = b*Q
Dividing both sides by a gives:
P = (b/a)*Q
Weight is centered
If b/a = 1 (weight is centered), then:
P = Q meaning that both upward forces are equal.
Weight towards the left end
If b/a = 4 (weight at 2), then:
P = 4*Q meaning that most of the force is being exerted by the rope on the left end.
Weight toward the right
If b/a = 1/4 (weight at 8), then
P = Q/4 meaning that most of the force is being exerted by the rope on the right end.
Weight at the right end
If b/a = 0 (weight at 10, right end of the bar)
P = 0 meaning that all of the force is being exerted by the rope on the right end of the bar.
Because P + Q = W, we conclude that Q = W
The bar is essentially eliminated

The scenario where b/a = 0 essentially eliminates the bar from consideration. The weight is hanging directly on the rope on the right end of the bar and the weightless bar and the other rope are simply floating in the air.

4.8 Hypothetical replacement by a single upward force

If we were to draw an imaginary force labeled R pointing directly up from the point C where R is equal to P + Q, we could imagine the forces P and Q as being replaced by R. In that case, R would be the resultant of P and Q and would be equal in magnitude and opposite in direction to the downward force W.

In the absence of the forces P and Q, the force R would produce the same turning effect on the bar as do the forces P and Q jointly.

If the magnitude of R were not equal to the magnitude of W, the vector sum of the forces would not be zero, there would be a torque about any point on the bar other than at the point C, and the bar would not be in equilibrium.

4.9 Exercise involving a bar supported at only one point

Tactile graphics

The file named Phy1110c1.svg contains a vector diagram that represents this scenario. All of the action occurs near the right end of the bar, so only the right end is shown in the image.

Figure 8 (p. 22) shows the mirror image that is contained in that file for the benefit of your assistant who will create the tactile graphic for this exercise.

Mirror image from the file named Phy1110c1.svg.

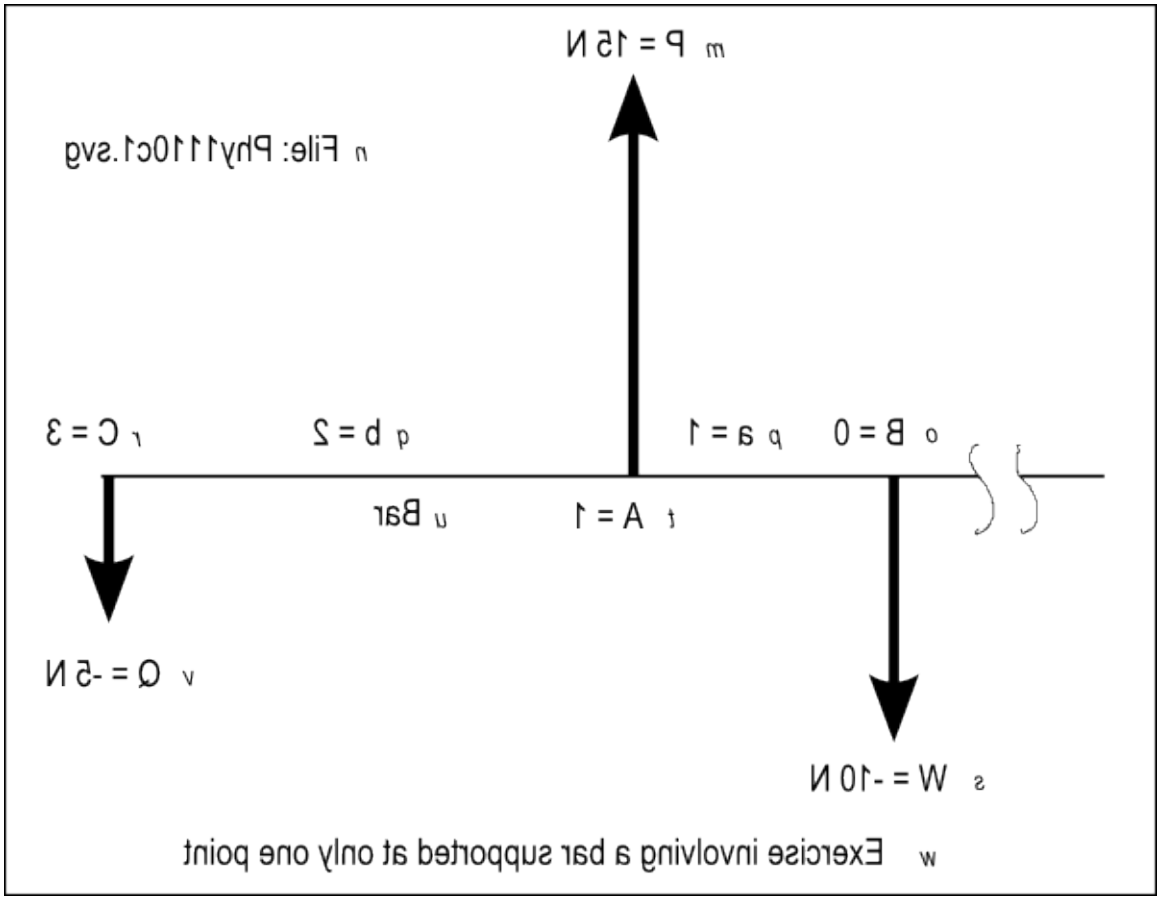

Figure 8: Mirror image from the file named Phy1110c1.svg.

Figure 9 (p. 23) shows a non-mirror-image version of the same image.

Non-mirror-image version of the image from the file named Phy1110c1.svg.

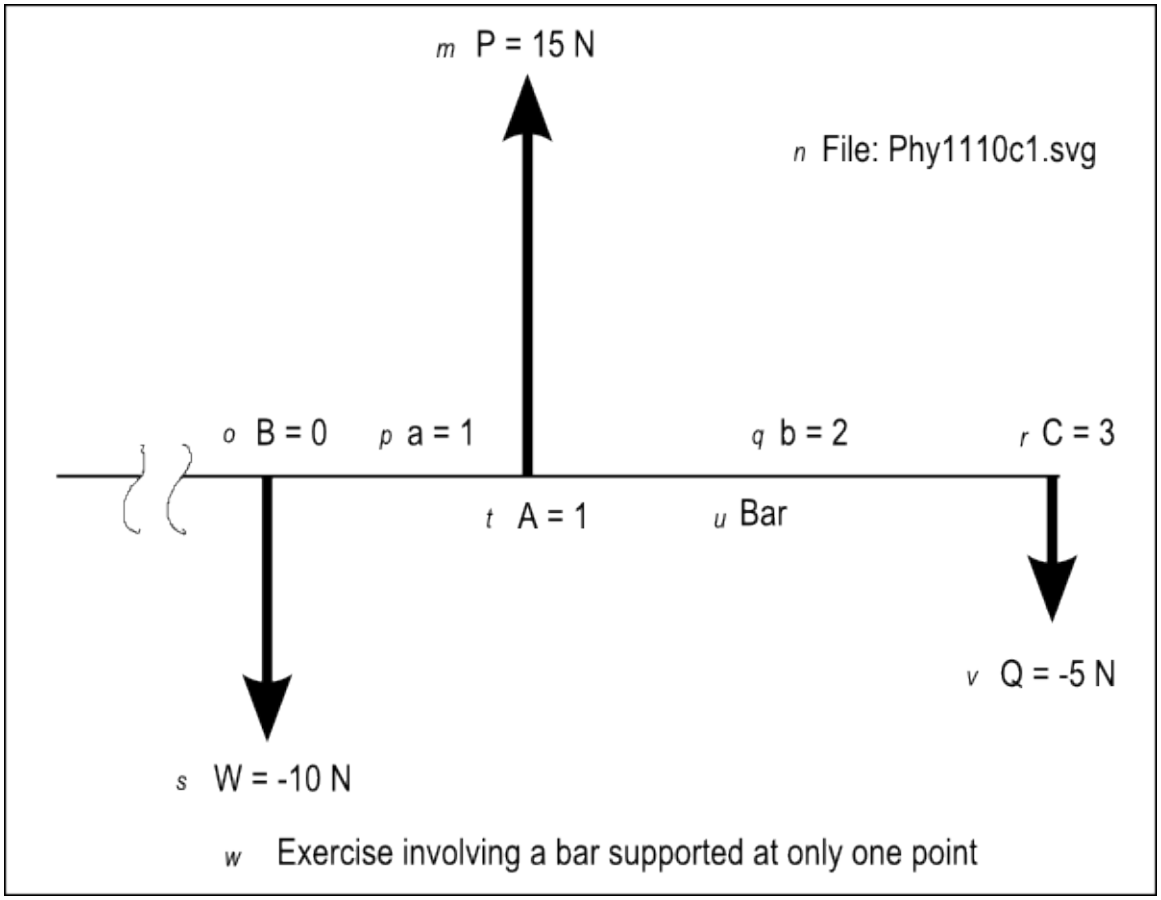

Figure 9: Non-mirror-image version of the image from the file named Phy1110c1.svg.

Figure 10 (p. 24) shows the key-value pairs that go with the image in the file named Phy1110c1.svg.

Key-value pairs for the image in the file named Phy1110c1.svg.

```
    m: P = 15 N
n: File: Phy1110c1.svg
o: B = 0
p: a = 1
q: b = 2
r: C = 3
s: W = -10 N
t: A = 1
u: Bar
v: Q = -5 N
w: Exercise involving a bar supported at only one point
```

Figure 10: Key-value pairs for the image in the file named Phy1110c1.svg.

Unlike or **antiparallel**

It's time to modify your drawing to reflect a different scenario.

Make your bar 20 units in length. Draw a downward force labeled Q at the right end of the bar. Label this as point C on the bar.

Draw an upward force labeled P somewhere on the bar. Label the location of this point on the bar as A

In this case, we have two parallel forces, P and Q pulling in opposite directions but not in the same line of action. These forces are said to be **unlike** or **antiparallel** .

Add another downward force

Draw a downward force labeled W somewhere to the left of A. Label this point as B and consider it to be the horizontal origin. Directions to the right of B are positive directions.

Label the horizontal distance from B to A as a. Label the horizontal distance from A to C as b.

What is required for equilibrium?

We can look at this scenario and tell that the forces labeled W and P, both of which point down, must be on opposite sides of the force labeled P. Otherwise, the bar will rotate in a clockwise direction.

The following two conditions must be true for the bar to be in equilibrium.

P + W + Q = 0, or

P = - (W + Q) (vector sum must be zero)

a*P + (a+b) * Q = 0 (moments around point B)

The plan

We will set values for all the variables in the two conditions other than a and P and then solve for values of a and P that will result in the bar being **in equilibrium** .

- Let W = -10
- Let Q = -5
- Let b = 2

The computations

Substituting these values into the two conditions and rearranging terms gives us:
P = -(W + Q), or
P = 15 , this is one of the answers
a*P +(a+2)*(-5) = 0, or
15*a -5*a -10 = 0, or
10*a = 10, or
a = 1 , this is the other answer

The answers

Therefore, for the values of W, Q, and b given above (p. 24) , P must be equal to 15 and a must be equal to 1 for the bar to be in equilibrium.

Variation of a and P with changes in Q

Now let's see how P and a vary with respect to the value of Q

- Let W = -10
- Let Q = -20
- Let b = 2

P = -(W + Q) = 30
a*P +(a+2)*(-20) = 0, or
30*a -20*a -40 = 0, or
10*a = 40, or
a = 4

As you can see, keeping the location of P constant and increasing the value for Q requires the value of P to increase and the location of W to move further to the left.

4.10 Couples

Now draw a new scenario where antiparallel forces, each having a magnitude of 10 newtons are acting in opposite directions on a bar, with each force in the same plane and both forces perpendicular to the bar. Label one force P and the other force Q.

Tactile graphics

The file named Phy1110d1.svg contains a vector diagram that represents this scenario. It shows two 10 N forces labeled P and Q along with an unknown force labeled X.

Figure 11 (p. 26) shows the mirror image that is contained in that file for the benefit of your assistant who will create the tactile graphic for this exercise.

Mirror image from the file named Phy1110d1.svg.

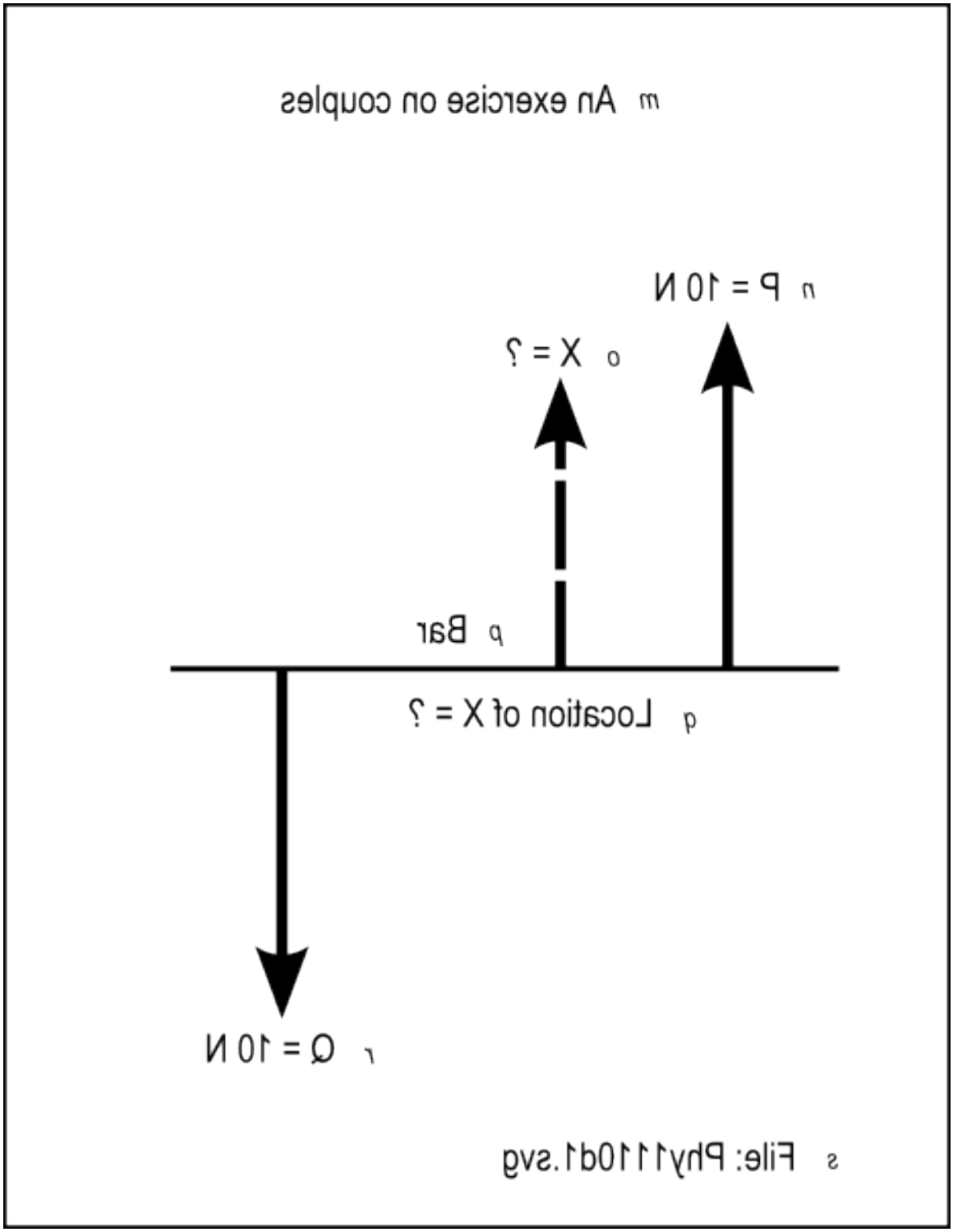

Figure 11: Mirror image from the file named Phy1110d1.svg.

Figure 12 (p. 28) shows a non-mirror-image version of the same image.

Non-mirror-image version of the image from the file named Phy1110d1.svg.

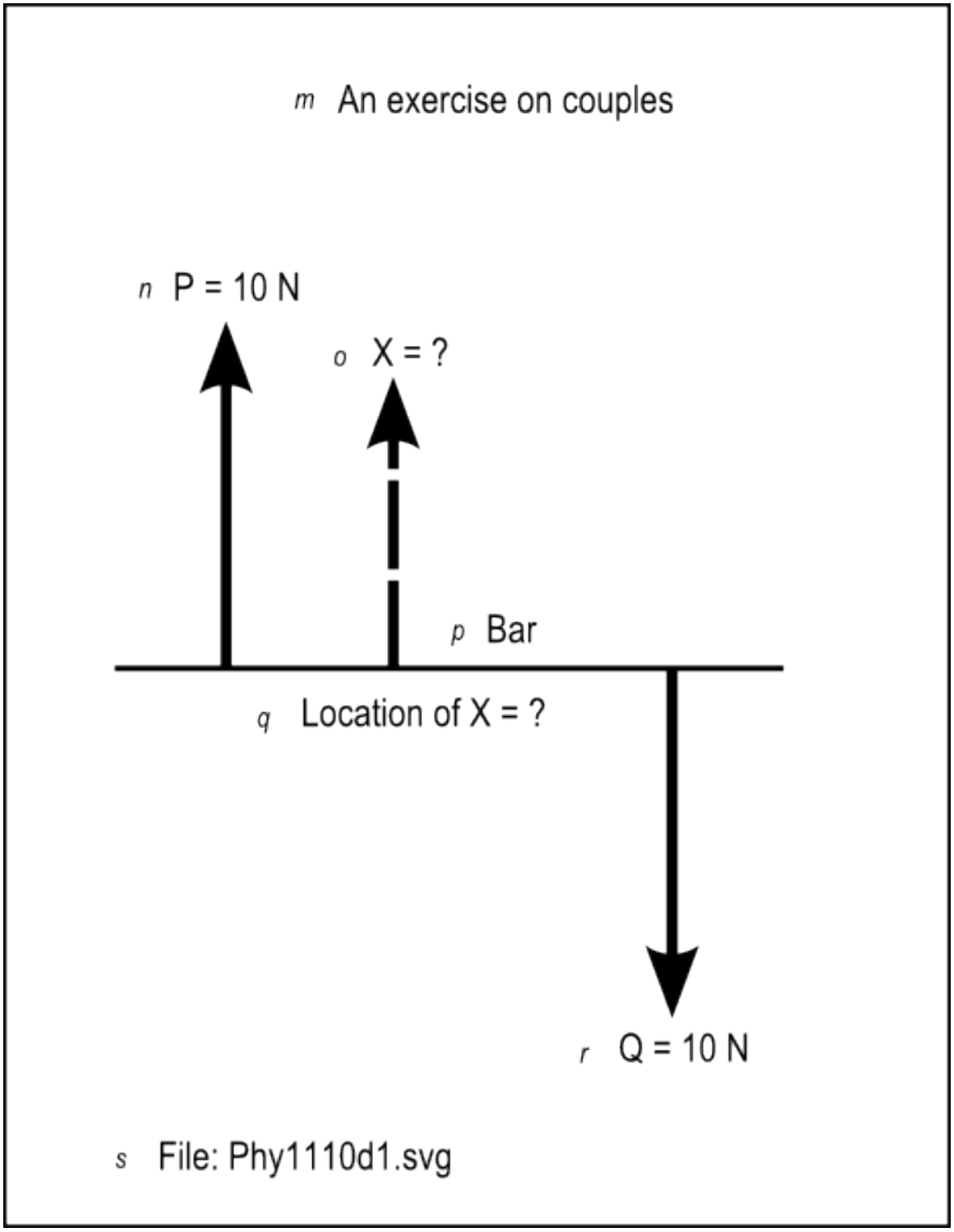

Figure 12: Non-mirror-image version of the image from the file named Phy1110d1.svg.

Figure 13 (p. 29) shows the key-value pairs that go with the image in the file named Phy1110d1.svg.

Key-value pairs for the image in the file named Phy1110d1.svg.

```
    m: An exercise on couples
n: P = 10 N
o: X = ?
p: Bar
q: Location of X = ?
r: Q = 10 N
s: File: Phy1110d1.svg
```

Figure 13: Key-value pairs for the image in the file named Phy1110d1.svg.

The moments about a point

If you compute the moments about any point on this bar, they won't sum to zero. Therefore, the bar isn't in equilibrium. Can you find the location and magnitude of a single force that can be applied to the bar to cause it to be in equilibrium?

Consider the first condition for equilibrium

The vector sum of the forces must be zero.

In this case, the vector sum of the forces is

P - Q + X = 0

10 - 10 + X = 0

where X is an unknown force.

As you can see, there is no value for X that will satisfy this condition other than X=0. If X=0, there is no force, so there is no single force that can be applied to the bar to cause it to be in equilibrium.

A couple

Two forces equal in magnitude but opposite in direction and not in the same line constitute a ***couple*** .

Stated differently,

A couple is a system of forces with a non-zero moment but no resultant force. In other words, the vector sum of the forces is zero but the sum of the moments is not zero.

A simple couple

The simplest kind of couple consists of two equal and opposite forces whose lines of action do not coincide. This is often called a "simple couple."

The units for a couple

The SI unit for the torque is the newton meter, which when expanded into SI base units is kg*m^2/s^2.

The moment of force for a couple is unique

The moment of a force is only defined with respect to a certain point P (it is said to be the "moment about P"). In general when P is changed, the moment changes.

However, the moment (torque) of a couple is independent of the reference point P. Any point will give the same moment.

Let's prove that

The moment of a couple is independent of the point about which it is measured. Can we prove the truth of that statement?

Imagine a horizontal bar that is five units long. Imagine an upward force at the right end of the bar with a magnitude of P. Imagine a downward force with the same magnitude d units to the left of the first force. Imagine that the location of this force is labeled as X.

Imagine that the left end of the bar is labeled A and the distance from point A to point X is labeled as b. Imagine another point somewhere between A and X that is labeled as B. Imagine that the distance from A to B is labeled as c.

Compute the moments about the point A

Ma = (b+d)*P-b*P, or
Ma = b*P+d*P-b*p, or
Ma = d*p
Now compute the moments about the point B.
Mb = (b-c+d)*P-(b-c)*P, or
Mb = b*P-c*P+d*P-b*P+c*P, or
Mb = d*P

The moments are the same regardless of the point about which they are computed. The value d*P is called the moment or torque of a couple.

The torque of a couple

The torque of a couple is the product of either force and the perpendicular distance between the lines of actions of the forces.

5 Complete the exercises

I encourage you to complete the exercises that I have presented in this lesson to confirm that you get the same results. Experiment with the procedures, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

7 Miscellaneous

This section contains a variety of miscellaneous information.

NOTE: **Housekeeping material**

- Module name: Force – Moments, Torque, Couple, and Equilibrium
- File: Phy1110.htm
- Revised: 07/07/2011
- Keywords:
  - physics
  - accessible
  - blind
  - graph board
  - protractor
  - screen reader
  - refreshable Braille display
  - JavaScript

· trigonometry
· moment
· torque
· couple
· equilibrium

NOTE: **Disclaimers: Financial** : Although the Connexions site makes it possible for you to download a PDF file for this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

I also want you to know that I receive no financial compensation from the Connexions website even if you purchase the PDF version of the module.

Affiliation : I am a professor of Computer Information Technology at Austin Community College in Austin, TX.

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