Review of Past Work - Grade 10 [CAPS]*

Free High School Science Texts Project

Based on *Review of Past Work*† by

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1 Introduction

This chapter describes some basic concepts which you have seen in earlier grades and lays the foundation for the remainder of this book. You should feel confident with the content in this chapter, before moving on with the rest of the book.

You can try out your skills on exercises in this chapter and ask your teacher for more questions just like them. You can also try to make up your own questions, solve them and try them out on your classmates to see if you get the same answers.

Practice is the only way to get good at maths!

2 What is a number?

A number is a way to represent quantity. Numbers are not something that you can touch or hold, because they are not physical. But you can touch three apples, three pencils, three books. You can never just touch three, you can only touch three of something. However, you do not need to see three apples in front of you to know that if you take one apple away, there will be two apples left. You can just think about it. That is your brain representing the apples in numbers and then performing arithmetic on them.

A number represents quantity because we can look at the world around us and quantify it using numbers. How many minutes? How many kilometers? How many apples? How much money? How much medicine? These are all questions which can only be answered using numbers to tell us "how much" of something we want to measure.

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A number can be written in many different ways and it is always best to choose the most appropriate way of writing the number. For example, "a half" may be spoken aloud or written in words, but that makes mathematics very difficult and also means that only people who speak the same language as you can understand what you mean. A better way of writing "a half" is as a fraction $\frac{1}{2}$ or as a decimal number $0,5$. It is still the same number, no matter which way you write it.

In high school, all the numbers which you will see are called *real numbers* and mathematicians use the symbol $\mathbb{R}$ to represent the *set of all real numbers*, which simply means all of the real numbers. Some of these real numbers can be written in ways that others cannot. Different types of numbers are described in detail in Section 1.12.

3 Sets

A *set* is a group of objects with a well-defined criterion for membership. For example, the criterion for belonging to a set of apples, is that the object must be an apple. The set of apples can then be divided into red apples and green apples, but they are all still apples. All the red apples form another set which is a *sub-set* of the set of apples. A sub-set is part of a set. All the green apples form another sub-set.

Now we come to the idea of a **union**, which is used to combine things. The symbol for **union** is $\cup$. Here, we use it to combine two or more intervals. For example, if x is a real number such that $1 < x \leq 3$ or $6 \leq x < 10$, then the set of all the possible x values is:

$$(1,3] \cup [6,10) \tag{1}$$

where the $\cup$ sign means the union (or combination) of the two intervals. We use the set and interval notation and the symbols described because it is easier than having to write everything out in words.

4 Letters and Arithmetic

The simplest thing that can be done with numbers is adding, subtracting, multiplying or dividing them. When two numbers are added, subtracted, multiplied or divided, you are performing *arithmetic*[1]. These four basic operations can be performed on any two real numbers.

Mathematics as a language uses special notation to write things down. So instead of:
one plus one is equal to two
mathematicians write

$$1 + 1 = 2 \tag{2}$$

In earlier grades, place holders were used to indicate missing numbers in an equation.

$$\begin{aligned} 1 + \square &= 2 \\ 4 - \square &= 2 \\ \square + 3 - 2\square &= 2 \end{aligned} \tag{3}$$

However, place holders only work well for simple equations. For more advanced mathematical workings, letters are usually used to represent numbers.

$$\begin{aligned} 1 + x &= 2 \\ 4 - y &= 2 \\ z + 3 - 2z &= 2 \end{aligned} \tag{4}$$

[1] Arithmetic is derived from the Greek word *arithmos* meaning *number*.

These letters are referred to as **variables**, since they can take on any value depending on what is required. For example, $x = 1$ in (4), but $x = 26$ in $2 + x = 28$.

A **constant** has a fixed value. The number 1 is a constant. The *speed of light* in a vacuum is also a constant which has been defined to be exactly 299792458 $m \cdot s^{-1}$ (read metres per second). The speed of light is a big number and it takes up space to always write down the entire number. Therefore, letters are also used to represent some constants. In the case of the speed of light, it is accepted that the letter c represents the speed of light. Such constants represented by letters occur most often in physics and chemistry.

Additionally, letters can be used to describe a situation mathematically. For example, the following equation

$$x + y = z \tag{5}$$

can be used to describe the situation of finding how much change can be expected for buying an item. In this equation, y represents the price of the item you are buying, x represents the amount of change you should get back and z is the amount of money given to the cashier. So, if the price is R10 and you gave the cashier R15, then write R15 instead of z and R10 instead of y and the change is then x.

$$x + 10 = 15 \tag{6}$$

We will learn how to "solve" this equation towards the end of this chapter.

5 Addition and Subtraction

Addition (+) and subtraction (−) are the most basic operations between numbers but they are very closely related to each other. You can think of subtracting as being the opposite of adding since adding a number and then subtracting the same number will not change what you started with. For example, if we start with a and add b, then subtract b, we will just get back to a again:

$$\begin{aligned} a + b - b &= a \\ 5 + 2 - 2 &= 5 \end{aligned} \tag{7}$$

If we look at a number line, then addition means that we move to the right and subtraction means that we move to the left.

The order in which numbers are added does not matter, but the order in which numbers are subtracted does matter. This means that:

$$\begin{aligned} a + b &= b + a \\ a - b &\neq b - a \quad \text{if } a \neq b \end{aligned} \tag{8}$$

The sign $\neq$ means "is not equal to". For example, $2+3 = 5$ and $3+2 = 5$, but $5-3 = 2$ and $3-5 = -2$. -2 is a negative number, which is explained in detail in "Negative Numbers" (Section 8: Negative Numbers).

5.1 Commutativity for Addition

The fact that $a + b = b + a$, is known as the *commutative* property for addition.

6 Multiplication and Division

Just like addition and subtraction, multiplication ($\times$, $\cdot$) and division ($\div$, /) are opposites of each other. Multiplying by a number and then dividing by the same number gets us back to the start again:

$$\begin{aligned} a \times b \div b &= a \\ 5 \times 4 \div 4 &= 5 \end{aligned} \tag{9}$$

Sometimes you will see a multiplication of letters as a dot or without any symbol. Don't worry, its exactly the same thing. Mathematicians are efficient and like to write things in the shortest, neatest way possible.

$$\begin{aligned} abc &= a \times b \times c \\ a \cdot b \cdot c &= a \times b \times c \end{aligned} \tag{10}$$

It is usually neater to write known numbers to the left, and letters to the right. So although $4x$ and $x4$ are the same thing, it looks better to write $4x$. In this case, the "4" is a constant that is referred to as the *coefficient* of x.

6.1 Commutativity for Multiplication

The fact that $ab = ba$ is known as the *commutative* property of multiplication. Therefore, both addition and multiplication are described as commutative operations.

7 Brackets

Brackets[2] in mathematics are used to show the order in which you must do things. This is important as you can get different answers depending on the order in which you do things. For example:

$$(5 \times 5) + 20 = 45 \tag{11}$$

whereas

$$5 \times (5 + 20) = 125 \tag{12}$$

If there are no brackets, you should always do multiplications and divisions first and then additions and subtractions[3]. You can always put your own brackets into equations using this rule to make things easier for yourself, for example:

$$\begin{aligned} a \times b + c \div d &= (a \times b) + (c \div d) \\ 5 \times 5 + 20 \div 4 &= (5 \times 5) + (20 \div 4) \end{aligned} \tag{13}$$

If you see a multiplication outside a bracket like this

$$\begin{aligned} a\,(b + c) \\ 3\,(4 - 3) \end{aligned} \tag{14}$$

then it means you have to multiply each part inside the bracket by the number outside

$$\begin{aligned} a\,(b + c) &= ab + ac \\ 3\,(4 - 3) &= 3 \times 4 - 3 \times 3 = 12 - 9 = 3 \end{aligned} \tag{15}$$

unless you can simplify everything inside the bracket into a single term. In fact, in the above example, it would have been smarter to have done this

$$3\,(4 - 3) = 3 \times (1) = 3 \tag{16}$$

[2] Sometimes people say "parentheses" instead of "brackets".

[3] Multiplying and dividing can be performed in any order as it doesn't matter. Likewise it doesn't matter which order you do addition and subtraction. Just as long as you do any $\times \div$ before any $+ -$.

It can happen with letters too

$$3\left(4a-3a\right)=3\times\left(a\right)=3a \tag{17}$$

7.1 Distributivity

The fact that $a\left(b+c\right)=ab+ac$ is known as the *distributive* property.

If there are two brackets multiplied by each other, then you can do it one step at a time:

$$\begin{aligned}\left(a+b\right)\left(c+d\right) &= a\left(c+d\right)+b\left(c+d\right)\\ &= ac+ad+bc+bd\\ \left(a+3\right)\left(4+d\right) &= a\left(4+d\right)+3\left(4+d\right)\\ &= 4a+ad+12+3d\end{aligned} \tag{18}$$

8 Negative Numbers

8.1 What is a negative number?

Negative numbers can be very confusing to begin with, but there is nothing to be afraid of. The numbers that are used most often are greater than zero. These numbers are known as *positive numbers*.

A negative number is a number that is less than zero. So, if we were to take a positive number a and subtract it from zero, the answer would be the negative of a.

$$0-a=-a \tag{19}$$

On a number line, a negative number appears to the left of zero and a positive number appears to the right of zero.

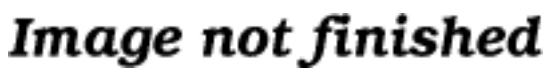

Figure 1: On the number line, numbers increase towards the right and decrease towards the left. Positive numbers appear to the right of zero and negative numbers appear to the left of zero.

8.2 Working with Negative Numbers

When you are adding a negative number, it is the same as subtracting that number if it were positive. Likewise, if you subtract a negative number, it is the same as adding the number if it were positive. Numbers are either positive or negative and we call this their sign. A positive number has a positive sign (+) and a negative number has a negative sign (−).

Subtraction is actually the same as adding a *negative number*.

In this example, a and b are positive numbers, but $-b$ is a negative number

$$\begin{aligned} a - b &= a + (-b) \\ 5 - 3 &= 5 + (-3) \end{aligned} \tag{20}$$

So, this means that subtraction is simply a short-cut for adding a negative number and instead of writing $a + (-b)$, we write $a - b$. This also means that $-b + a$ is the same as $a - b$. Now, which do you find easier to work out?

Most people find that the first way is a bit more difficult to work out than the second way. For example, most people find $12 - 3$ a lot easier to work out than $-3 + 12$, even though they are the same thing. So $a - b$, which looks neater and requires less writing is the accepted way of writing subtractions.

Table 1 shows how to calculate the sign of the answer when you multiply two numbers together. The first column shows the sign of the first number, the second column gives the sign of the second number and the third column shows what sign the answer will be.

a	b	$a \times b$ or $a \div b$
$+$	$+$	$+$
$+$	$-$	$-$
$-$	$+$	$-$
$-$	$-$	$+$

Table 1: Table of signs for multiplying or dividing two numbers.

So multiplying or dividing a negative number by a positive number always gives you a negative number, whereas multiplying or dividing numbers which have the same sign always gives a positive number. For example, $2 \times 3 = 6$ and $-2 \times -3 = 6$, but $-2 \times 3 = -6$ and $2 \times -3 = -6$.

Adding numbers works slightly differently (see Table 2). The first column shows the sign of the first number, the second column gives the sign of the second number and the third column shows what sign the answer will be.

a	b	$a + b$
$+$	$+$	$+$
$+$	$-$	?
$-$	$+$	?
$-$	$-$	$-$

Table 2: Table of signs for adding two numbers.

If you add two positive numbers you will always get a positive number, but if you add two negative numbers you will always get a negative number. If the numbers have a different sign, then the sign of the answer depends on which one is bigger.

8.3 Living Without the Number Line

The number line in Figure 1 is a good way to visualise what negative numbers are, but it can get very inefficient to use it every time you want to add or subtract negative numbers. To keep things simple, we will write down three tips that you can use to make working with negative numbers a little bit easier. These tips will let you work out what the answer is when you add or subtract numbers which may be negative, and will also help you keep your work tidy and easier to understand.

8.3.1 Negative Numbers Tip 1

If you are given an expression like $-a+b$, then it is easier to move the numbers around so that the expression looks easier. In this case, we have seen that adding a negative number to a positive number is the same as subtracting the number from the positive number. So,

$$\begin{aligned} -a+b &= b-a \\ -5+10 &= 10+(-5) \\ &= 10-5 \\ &= 5 \end{aligned} \tag{21}$$

This makes the expression easier to understand. For example, a question like "What is $-7+11$?" looks a lot more complicated than "What is $11-7$?", even though they are exactly the same question.

8.3.2 Negative Numbers Tip 2

When you have two negative numbers like $-3-7$, you can calculate the answer by simply adding together the numbers as if they were positive and then putting a negative sign in front.

$$\begin{aligned} -c-d &= -(c+d) \\ -7-2 &= -(7+2) = -9 \end{aligned} \tag{22}$$

8.3.3 Negative Numbers Tip 3

In Table 2 we saw that the sign of two numbers added together depends on which one is bigger. This tip tells us that all we need to do is take the smaller number away from the larger one and remember to give the answer the sign of the larger number. In this equation, F is bigger than e.

$$\begin{aligned} e-F &= -(F-e) \\ 2-11 &= -(11-2) = -9 \end{aligned} \tag{23}$$

You can even combine these tips: for example, you can use Tip 1 on $-10+3$ to get $3-10$ and then use Tip 3 to get $-(10-3) = -7$.

8.3.3.1 Negative Numbers

1. Calculate:

(a) $(-5)-(-3)$	(b) $(-4)+2$	(c) $(-10)\div(-2)$
(d) $11-(-9)$	(e) $-16-(6)$	(f) $-9\div 3\times 2$
(g) $(-1)\times 24\div 8\times(-3)$	(h) $(-2)+(-7)$	(i) $1-12$
(j) $3-64+1$	(k) $-5-5-5$	(l) $-6+25$
(m) $-9+8-7+6-5+4-3+2-1$		

Table 3

Click here for the solution[4]

[4] http://www.fhsst.org/l3B

2. Say whether the sign of the answer is + or −

(a) $-5+6$	(b) $-5+1$	(c) $-5 \div -5$
(d) $-5 \div 5$	(e) $5 \div -5$	(f) $5 \div 5$
(g) -5×-5	(h) -5×5	(i) 5×-5
(j) 5×5		

Table 4

Click here for the solution[5]

9 Rearranging Equations

Now that we have described the basic rules of negative and positive numbers and what to do when you add, subtract, multiply and divide them, we are ready to tackle some real mathematics problems!

Earlier in this chapter, we wrote a general equation for calculating how much change (x) we can expect if we know how much an item costs (y) and how much we have given the cashier (z). The equation is:

$$x + y = z \tag{24}$$

So, if the price is R10 and you gave the cashier R15, then write R15 instead of z and R10 instead of y.

$$x + 10 = 15 \tag{25}$$

Now that we have written this equation down, how exactly do we go about finding what the change is? In mathematical terms, this is known as solving an equation for an unknown (x in this case). We want to re-arrange the terms in the equation, so that only x is on the left hand side of the = sign and everything else is on the right.

The most important thing to remember is that an equation is like a set of weighing scales. In order to keep the scales balanced, whatever is done to one side must be done to the other.

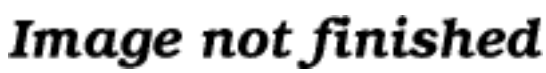

Figure 2: An equation is like a set of weighing scales. In order to keep the scales balanced, you must do the same thing to both sides. So, if you add, subtract, multiply or divide the one side, you must add, subtract, multiply or divide the other side too.

9.1 Method: Rearranging Equations

You can add, subtract, multiply or divide both sides of an equation by any number you want, as long as you always do it to both sides.

[5] http://www.fhsst.org/l3K

So for our example we could subtract y from both sides

$$\begin{aligned} x+y &= z \\ x+y-y &= z-y \\ x &= z-y \\ x &= 15-10 \\ &= 5 \end{aligned} \tag{26}$$

Now we can see that the change is the price subtracted from the amount paid to the cashier. In the example, the change should be R5. In real life we can do this in our heads; the human brain is very smart and can do arithmetic without even knowing it.

When you subtract a number from both sides of an equation, it looks like you just moved a positive number from one side and it became a negative on the other, which is exactly what happened. Likewise, if you move a multiplied number from one side to the other, it looks like it changed to a divide. This is because you really just divided both sides by that number and a number divided by itself is just 1

$$\begin{aligned} a(5+c) &= 3a \\ a(5+c) \div a &= 3a \div a \\ \tfrac{a}{a} \times (5+c) &= 3 \times \tfrac{a}{a} \\ 1 \times (5+c) &= 3 \times 1 \\ 5+c &= 3 \\ c &= 3-5=-2 \end{aligned} \tag{27}$$

However, you must be careful when doing this, as it is easy to make mistakes.

The following is the WRONG thing to do

$$\begin{aligned} 5a+c &= 3a \\ 5+c &= 3 \end{aligned} \tag{28}$$

Can you see why it is wrong? It is wrong because we did not divide the c term by a as well. The correct thing to do is

$$\begin{aligned} 5a+c &= 3a \\ 5+c \div a &= 3 \\ c \div a &= 3-5=-2 \end{aligned} \tag{29}$$

9.1.1 Rearranging Equations

1. If $3(2r-5)=27$, then $2r-5=$ ..... Click here for the solution[6]
2. Find the value for x if $0,5(x-8)=0,2x+11$ Click here for the solution[7]
3. Solve $9-2n=3(n+2)$ Click here for the solution[8]
4. Change the formula $P=A+Akt$ to $A=$ Click here for the solution[9]

[6] http://www.fhsst.org/l3k
[7] http://www.fhsst.org/l30
[8] http://www.fhsst.org/l38
[9] http://www.fhsst.org/l39

5. Solve for x: $\frac{1}{ax} + \frac{1}{bx} = 1$ Click here for the solution[10]

10 Fractions and Decimal Numbers

A fraction is one number divided by another number. There are several ways to write a number divided by another one, such as $a \div b$, a/b and $\frac{a}{b}$. The first way of writing a fraction is very hard to work with, so we will use only the other two. We call the number on the top (left) the *numerator* and the number on the bottom (right) the *denominator*. For example, in the fraction 1/5 or $\frac{1}{5}$, the numerator is 1 and the denominator is 5.

10.1 Definition - Fraction

The word *fraction* means *part of a whole*.

The *reciprocal* of a fraction is the fraction turned upside down, in other words the numerator becomes the denominator and the denominator becomes the numerator. So, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

A fraction multiplied by its reciprocal is always equal to 1 and can be written

$$\frac{a}{b} \times \frac{b}{a} = 1 \tag{30}$$

This is because dividing by a number is the same as multiplying by its reciprocal.

10.2 Definition - Multiplicative Inverse

The reciprocal of a number is also known as the multiplicative inverse.

A decimal number is a number which has an integer part and a fractional part. The integer and the fractional parts are separated by a *decimal point*, which is written as a comma in South African schools. For example the number $3\frac{14}{100}$ can be written much more neatly as $3,14$.

All real numbers can be written as a decimal number. However, some numbers would take a huge amount of paper (and ink) to write out in full! Some decimal numbers will have a number which will repeat itself, such as $0,33333...$ where there are an infinite number of 3's. We can write this decimal value by using a dot above the repeating number, so $0,\dot{3} = 0,33333....$ If there are two repeating numbers such as $0,121212...$ then you can place dots[11] on each of the repeated numbers $0,\dot{1}\dot{2} = 0,121212....$ These kinds of repeating decimals are called *recurring decimals*.

Table 5 lists some common fractions and their decimal forms.

Fraction	Decimal Form
$\frac{1}{20}$	0,05
$\frac{1}{16}$	0,0625
$\frac{1}{10}$	0,1
$\frac{1}{8}$	0,125
$\frac{1}{6}$	$0,16\dot{6}$
$\frac{1}{5}$	0,2
$\frac{1}{2}$	0,5
$\frac{3}{4}$	0,75

Table 5: Some common fractions and their equivalent decimal forms.

[10] http://www.fhsst.org/l3X

[11] or a bar, like $0,\overline{12}$

11 Scientific Notation

In science one often needs to work with very large or very small numbers. These can be written more easily in scientific notation, which has the general form

$$a \times 10^m \tag{31}$$

where a is a decimal number between 0 and 10 that is rounded off to a few decimal places. The m is an integer and if it is positive it represents how many zeros should appear to the right of a. If m is negative, then it represents how many times the decimal place in a should be moved to the left. For example $3,2 \times 10^3$ represents 32 000 and $3,2 \times 10^{-3}$ represents $0,0032$.

If a number must be converted into scientific notation, we need to work out how many times the number must be multiplied or divided by 10 to make it into a number between 1 and 10 (i.e. we need to work out the value of the exponent m) and what this number is (the value of a). We do this by counting the number of decimal places the decimal point must move.

For example, write the speed of light which is 299792458 $m \cdot s^{-1}$ in scientific notation, to two decimal places. First, determine where the decimal point must go for two decimal places (to find a) and then count how many places there are after the decimal point to determine m.

In this example, the decimal point must go after the first 2, but since the number after the 9 is a 7, $a = 3,00$.

So the number is $3,00 \times 10^m$, where $m = 8$, because there are 8 digits left after the decimal point. So, the speed of light in scientific notation to two decimal places is $3,00 \times 10^8\ m \cdot s^{-1}$

As another example, the size of the HI virus is around $1,2 \times 10^{-7}$ m. This is equal to $1,2 \times 0,0000001\ m$, which is $0,00000012\ m$.

12 Real Numbers

Now that we have learnt about the basics of mathematics, we can look at what real numbers are in a little more detail. The following are examples of real numbers and it is seen that each number is written in a different way.

$$\sqrt{3}, \quad 1,2557878, \quad \frac{56}{34}, \quad 10, \quad 2,1, \quad -5, \quad -6,35, \quad -\frac{1}{90} \tag{32}$$

Depending on how the real number is written, it can be further labelled as either rational, irrational, integer or natural. A set diagram of the different number types is shown in Figure 3.

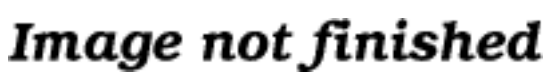

Figure 3: Set diagram of all the real numbers $\mathbb{R}$, the rational numbers $\mathbb{Q}$, the integers $\mathbb{Z}$ and the natural numbers $\mathbb{N}$. The irrational numbers are the numbers not inside the set of rational numbers. All of the integers are also rational numbers, but not all rational numbers are integers.

12.1 Non-Real Numbers

All numbers that are not real numbers have *imaginary* components. We will not see imaginary numbers in this book but they come from $\sqrt{-1}$. Since we won't be looking at numbers which are not real, if you see a number you can be sure it is a real one.

12.2 Natural Numbers

The first type of numbers that are learnt about are the numbers that are used for counting. These numbers are called *natural numbers* and are the simplest numbers in mathematics:

$$0, 1, 2, 3, 4, ... \tag{33}$$

Mathematicians use the symbol $\mathbb{N}_0$ to mean the *set of all natural numbers*. These are also sometimes called *whole numbers*. The natural numbers are a *subset* of the real numbers since every natural number is also a real number.

12.3 Integers

The integers are all of the natural numbers and their negatives:

$$... - 4, -3, -2, -1, 0, 1, 2, 3, 4... \tag{34}$$

Mathematicians use the symbol $\mathbb{Z}$ to mean *the set of all integers*. The integers are a subset of the real numbers, since every integer is a real number.

12.4 Rational Numbers

The natural numbers and the integers are only able to describe quantities that are whole or complete. For example, you can have 4 apples, but what happens when you divide one apple into 4 equal pieces and share it among your friends? Then it is not a whole apple anymore and a different type of number is needed to describe the apples. This type of number is known as a rational number.

A rational number is any number which can be written as:

$$\frac{a}{b} \tag{35}$$

where a and b are integers and $b \neq 0$.

The following are examples of rational numbers:

$$\frac{20}{9}, \quad \frac{-1}{2}, \quad \frac{20}{10}, \quad \frac{3}{15} \tag{36}$$

12.4.1 Notation Tip

Rational numbers are any number that can be expressed in the form $\frac{a}{b}; a, b \in \mathbb{Z}; b \neq 0$ which means "the set of numbers $\frac{a}{b}$ when a and b are integers".

Mathematicians use the symbol $\mathbb{Q}$ to mean *the set of all rational numbers*. The set of rational numbers contains all numbers which can be written as terminating or repeating decimals.

12.4.2 Rational Numbers

All integers are rational numbers with a denominator of 1.

You can add and multiply rational numbers and still get a rational number at the end, which is very useful. If we have 4 integers a, b, c and d, then the rules for adding and multiplying rational numbers are

$$\begin{aligned} \frac{a}{b} + \frac{c}{d} &= \frac{ad+bc}{bd} \\ \frac{a}{b} \times \frac{c}{d} &= \frac{ac}{bd} \end{aligned} \tag{37}$$

12.4.3 Notation Tip

The statement "4 integers a, b, c and d" can be written formally as $\{a, b, c, d\} \in \mathbb{Z}$ because the $\in$ symbol means *in* and we say that a, b, c and d are *in* the set of integers.

Two rational numbers ($\frac{a}{b}$ and $\frac{c}{d}$) represent the same number if $ad = bc$. It is always best to simplify any rational number, so that the denominator is as small as possible. This can be achieved by dividing both the numerator and the denominator by the same integer. For example, the rational number 1000/10000 can be divided by 1000 on the top and the bottom, which gives 1/10. $\frac{2}{3}$ of a pizza is the same as $\frac{8}{12}$ (Figure 4).

Image not finished

Figure 4: $\frac{8}{12}$ of the pizza is the same as $\frac{2}{3}$ of the pizza.

You can also add rational numbers together by finding the *lowest common denominator* and then adding the numerators. Finding a lowest common denominator means finding the lowest number that both denominators are a *factor*[12] of. A factor of a number is an integer which evenly divides that number without leaving a remainder. The following numbers all have a factor of 3

$$3, 6, 9, 12, 15, 18, 21, 24, ... \tag{38}$$

and the following all have factors of 4

$$4, 8, 12, 16, 20, 24, 28, ... \tag{39}$$

The common denominators between 3 and 4 are all the numbers that appear in both of these lists, like 12 and 24. The lowest common denominator of 3 and 4 is the smallest number that has both 3 and 4 as factors, which is 12.

For example, if we wish to add $\frac{3}{4} + \frac{2}{3}$, we first need to write both fractions so that their denominators are the same by finding the lowest common denominator, which we know is 12. We can do this by multiplying $\frac{3}{4}$ by $\frac{3}{3}$ and $\frac{2}{3}$ by $\frac{4}{4}$. $\frac{3}{3}$ and $\frac{4}{4}$ are really just complicated ways of writing 1. Multiplying a number by 1 doesn't change the number.

$$\begin{aligned} \frac{3}{4} + \frac{2}{3} &= \frac{3}{4} \times \frac{3}{3} + \frac{2}{3} \times \frac{4}{4} \\ &= \frac{3 \times 3}{4 \times 3} + \frac{2 \times 4}{3 \times 4} \\ &= \frac{9}{12} + \frac{8}{12} \\ &= \frac{9+8}{12} \\ &= \frac{17}{12} \end{aligned} \tag{40}$$

Dividing by a rational number is the same as multiplying by its reciprocal, as long as neither the numerator nor the denominator is zero:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc} \tag{41}$$

A rational number may be a *proper* or *improper* fraction.

[12] Some people say *divisor* instead of factor.

Proper fractions have a numerator that is smaller than the denominator. For example,

$$\frac{-1}{2}, \frac{3}{15}, \frac{-5}{-20} \tag{42}$$

are proper fractions.

Improper fractions have a numerator that is larger than the denominator. For example,

$$\frac{-10}{2}, \frac{15}{13}, \frac{-53}{-20} \tag{43}$$

are improper fractions. Improper fractions can always be written as the sum of an integer and a proper fraction.

12.4.4 Converting Rationals into Decimal Numbers

Converting rationals into decimal numbers is very easy.

If you use a calculator, you can simply divide the numerator by the denominator.

If you do not have a calculator, then you have to use long division.

Since long division was first taught in primary school, it will not be discussed here. If you have trouble with long division, then please ask your friends or your teacher to explain it to you.

12.5 Irrational Numbers

An *irrational number* is any real number that is not a rational number. When expressed as decimals, these numbers can never be fully written out as they have an infinite number of decimal places which never fall into a repeating pattern. For example, $\sqrt{2} = 1,41421356...$, $\pi = 3,14159265....$ π is a Greek letter and is pronounced "pie".

12.5.1 Real Numbers

1. Identify the number type (rational, irrational, real, integer) of each of the following numbers:
   a. $\frac{c}{d}$ if c is an integer and if d is irrational.
   b. $\frac{3}{2}$
   c. -25
   d. 1,525
   e. $\sqrt{10}$

   Click here for the solution[13]
2. Is the following pair of numbers real and rational or real and irrational? Explain. $\sqrt{4}; \frac{1}{8}$ Click here for the solution[14]

13 Mathematical Symbols

The following is a table of the meanings of some mathematical signs and symbols that you should have come across in earlier grades.

[13] http://www.fhsst.org/l3b
[14] http://www.fhsst.org/l3j

Sign or Symbol	Meaning
$>$	greater than
$<$	less than
$\geq$	greater than or equal to
$\leq$	less than or equal to

Table 6

So if we write $x > 5$, we say that x is greater than 5 and if we write $x \geq y$, we mean that x can be greater than or equal to y. Similarly, $<$ means 'is less than' and $\leq$ means 'is less than or equal to'. Instead of saying that x is between 6 and 10, we often write $6 < x < 10$. This directly means 'six is less than x which in turn is less than ten'.

13.1 Mathematical Symbols

1. Write the following in symbols:
   a. x is greater than 1
   b. y is less than or equal to z
   c. a is greater than or equal to 21
   d. p is greater than or equal to 21 and p is less than or equal to 25

   Click here for the solution[15]

14 Infinity

Infinity (symbol ∞) is usually thought of as something like "the largest possible number" or "the furthest possible distance". In mathematics, infinity is often treated as if it were a number, but it is clearly a very different type of "number" than integers or reals.

When talking about recurring decimals and irrational numbers, the term *infinite* was used to describe *never-ending* digits.

15 End of Chapter Exercises

1. Calculate
   a. $18 - 6 \times 2$
   b. $10 + 3\,(2 + 6)$
   c. $50 - 10\,(4 - 2) + 6$
   d. $2 \times 9 - 3\,(6 - 1) + 1$
   e. $8 + 24 \div 4 \times 2$
   f. $30 - 3 \times 4 + 2$
   g. $36 \div 4\,(5 - 2) + 6$
   h. $20 - 4 \times 2 + 3$
   i. $4 + 6\,(8 + 2) - 3$
   j. $100 - 10\,(2 + 3) + 4$

   Click here for the solution[16]

[15] http://cnx.org/content/m38346/latest/ http://www.fhsst.org/l3I
[16] http://www.fhsst.org/l3D

2. If $p = q + 4r$, then $r =$ ..... Click here for the solution[17]
3. Solve $\frac{x-2}{3} = x - 3$ Click here for the solution[18]

[17] http://www.fhsst.org/l3W
[18] http://www.fhsst.org/l3Z