Relationships Among Kinematics, Newton's Laws, Vectors, 2D Motion, 2D Forces, Momentum, Work, Energy, and Power*

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Abstract

This module illustrates relationships among kinematics, Newton's laws, vectors, 2D motion, 2D forces, momentum, work, energy, and power in a format that is accessible to blind students.

1 Table of Contents

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2 Preface

2.1 General

This module is part of a collection (see http://cnx.org/content/col11294/latest/ [1]) of modules designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

This module illustrates relationships among kinematics, Newton's laws, vectors, 2D linear motion, 2D forces, momentum, work, energy, and power in a format that is accessible to blind students.

2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [2]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [3]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [4] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [5]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch? [6]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [7]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/∼djoyce/trig/ [8]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [9] and http://www.w3schools.com/js/default.asp [10]).
- An understanding of all of the material covered in the earlier modules in this collection.

2.3 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [11] .

[1] http://cnx.org/content/col11294/latest/
[2] http://www.youtube.com/watch?v=c8plj9UsJbg
[3] http://www.youtube.com/watch?v=v-F06HgiUpw
[4] http://www.nvda-project.org/
[5] http://www.userite.com/ecampus/lesson1/tools.php
[6] http://www.youtube.com/watch?v=c8plj9UsJbg
[7] http://www.youtube.com/watch?v=v-F06HgiUpw
[8] http://www.clarku.edu/∼djoyce/trig/
[9] http://www.dickbaldwin.com/tocjscript1.htm
[10] http://www.w3schools.com/js/default.asp
[11] http://www.dickbaldwin.com/toc.htm

3 Discussion

A wrap-up module

The next module following this one will involve circular motion, which will be a major change in direction (no pun intended). Therefore, in this module, I will work through a major example involving a rocket that will tie together much of what you have learned in earlier modules.

Before we get to that example, however, let's do a quick review on external and internal forces.

A quick review of external forces

You learned in an earlier module that when work is done on an object by *external* forces, the total mechanical energy possessed by the object, consisting of kinetic energy plus potential energy, must change.

The work done on the object by external forces can be positive, in which case the total mechanical energy will increase. The work can be negative, in which case the total mechanical energy will decrease. The change in mechanical energy will be equal to the net work that is done on the object.

In this case, the total mechanical energy is not *conserved* . Therefore, external forces are often referred to as *non-conservative* forces.

A quick review of internal forces

On the other hand, you also learned that if work is done on an object only by *internal* forces, the total mechanical energy possessed by the object cannot change. However, it can be transformed from potential energy to kinetic energy and vice versa.

In this case, the total mechanical energy is *conserved* . Therefore, internal forces are often referred to as *conservative* forces.

A quantitative relationship

The quantitative relationship between work and mechanical energy can be stated as follows:

MEf = MEi + We

where

- MEf and MEi represent the final and initial total mechanical energy possessed by the object respectively.
- We represents the work done on the object by external forces.

This equation states that the final amount of mechanical energy possessed by an object is equal to the initial mechanical energy plus the work done on the object by external forces.

Potential energy plus kinetic energy

The total mechanical energy at any point in time can be the sum of potential energy *(gravitational or elastic potential energy)* and kinetic energy due to motion.

Given that, we can rewrite the earlier equation as:

KEf + PEf = KEi + PEi + We

where

- KEf and PEf represent the final kinetic and potential energy respectively.
- KEi and PEi represent the initial kinetic and potential energy respectively.
- We represents the work done on the object by external forces.

As mentioned, the work done by external forces can be either positive or negative work. Whether the work is positive or negative depends on the cosine of the angle between the direction of the force and the direction of the displacement of the object.

3.1 An ideal rocket example

Consider the following scenario. The owners of an experimental rocket lift the rocket onto a platform above ground level and set it up for firing.

Later, when they fire the rocket, it goes straight up while the rocket engine is burning. When the rocket engine runs out of fuel and stops burning, the rocket coasts to its apex and stops climbing. Then it falls back to the surface of the earth in an unglamorous free fall.

Simplifying assumptions

We will make some simplifying assumptions:

- The mass of the fuel is insignificant relative to the combined mass of the rocket and its payload. Therefore, expenditure of fuel doesn't affect the mass of the rocket in a significant way.
- Air resistance is negligible. The rocket acts as if in a vacuum.

Initial conditions

Here are the initial conditions for the rocket experiment:

- Platform height = 15 meters.
- Mass of rocket and payload = 10kg.
- Thrust of rocket is constant at 150 newtons during burn.
- Burn time for the rocket = 10 seconds.

Legs of the trip

We will analyze the rocket's round trip from the ground, into the air, and back to the ground in several legs as described below:

- Leg A: Manually lifting the rocket from the ground to the platform.
- Leg B: Displacement of the rocket under rocket-engine power straight up.
- Leg C: Displacement of the rocket without power while coasting to the apex.
- Leg D: Displacement of the rocket in free fall from the apex back to the ground.

We will analyze several aspects of the state of the rocket at the end of each leg. We will also compare alternative ways of computing the state of the rocket.

3.1.1 Leg A

During this leg, the rocket is manually lifted from the ground to the platform. The rocket has no potential or kinetic energy while on the ground, so it begins with zero mechanical energy.

An external force must be provided to lift the rocket from the ground to the platform in order to overcome the internal force of gravity. As an external force, this force is capable of changing its mechanical energy, which it does.

When the rocket has been lifted onto the platform, the mechanical energy of the rocket consists of its gravitational potential energy, which is equal to the work done to lift it to the platform. The kinetic energy will be 0 at that point because the rocket isn't moving.

Weight of the rocket = m*g = 10kg*9.8m/s^2 = 98 newtons

Work = f * d = 98N * 15m = 1470 joules

State at the end of Leg A

Thus, the total mechanical energy possessed by the rocket at the end of Leg A is 1470 joules.

3.1.2 Leg B

During this leg, which begins when the rocket engine fires, the rocket flies straight up as a result of a constant upward force exerted by the rocket engine.

The net acceleration

For this leg, we need to determine the net acceleration that is applied to the rocket. The net acceleration consists of the upward or positive acceleration due to the force of the rocket engine and the downward or negative acceleration of gravity.

Aup = 150N/10kg = 15 m/s^2
Ag = -9.8 m/s^2
Anet = 15 m/s^2 - 9.8 m/s^2 = 5.2 m/s^2

How far will the rocket go?

The initial velocity of the rocket is zero. Given that, you learned in an earlier module that the distance that the rocket will travel during the burn is

d = 0.5*Anet*t^2 = 0.5 * (5.2 m/s^2) * (10s)^2, or
d = 260 meters

260 meters straight up

In other words, when the rocket runs out of fuel at the end of the 10-second burn, the rocket has traveled straight up by 260 meters. Given that it started 15 meters above the ground, it is at a height of 275 meters above the ground at that point in time.

Total mechanical energy

At that point in time, the total mechanical energy possessed by the rocket consists of its gravitational potential energy plus its kinetic energy.

The kinetic energy

To compute the kinetic energy, we need to know the velocity. You learned in an earlier module that we can compute the velocity as

v = Anet * t = (5.2 m/s^2) * 10s = 52 m/s

You also learned earlier that the kinetic energy is equal to

KE = 0.5 * m * v^2 = 0.5 * 10 kg * (52 m/s)^2 = 13520 joules

Gravitational potential energy

You learned in an earlier module that the gravitational potential energy of an object due to its height above the surface of the earth is equal to

PEg = m * g * h = 10 kg * (9.8m/s^2) * 275 m = 26950 joules

Note that this value is computed using the height above the ground and not the height above the platform, which is 260 meters.

The total mechanical energy

Thus, the total mechanical energy at this point in time is

ME = 13520 joules + 26950 joules = 40470 joules

Validation

Let's see if we can validate that result in some other way.

We know that the mechanical energy of the rocket at rest on the platform was equal to 1470 joules.

We can compute the work done in moving the rocket up by 260 meters by multiplying that distance by the upward force. Thus,

work = distance * thrust, or
work = 260 m * 150N = 39000 joules

Additional mechanical energy

This is the mechanical energy added to the rocket after it left the platform while the engine was burning. The total mechanical energy when the burn ends is the sum of that value and the mechanical energy that it had while at rest on the platform. Thus, the total mechanical energy at the end of the burn is:

ME = 1470 joules + 39000 joules = 40470 joules

A good match

This value matches the value computed earlier (p. 5) on the basis of the height of the rocket above the surface of the earth and the velocity of the rocket. Thus, the two approaches agree with one another up to this point.

State at the end of Leg B

Therefore, at the end of Leg B,

- **The total mechanical energy** possessed by the rocket is equal to 40470 joules.
- The gravitational potential energy is 26950 joules

- The kinetic energy is 13520 joules
- The rocket is out of fuel and is coasting upward with a velocity of 52 m/s.
- The only force acting on the rocket is an internal downward force due to gravity, which is equal to 10kg * 9.8m/s^2 = 98 newtons.

3.1.3 Leg C

This is the part of the trip where the rocket coasts from its height at the end of Leg B to the apex of its trip. The continued upward motion is due solely to its kinetic energy at the end of Leg B.

From the end of Leg B when the rocket engine stops burning, until the rocket crashes on the surface of the earth, the only forces acting on the rocket will be the internal force of gravity.

Total mechanical energy is conserved

Since internal forces cannot change the mechanical energy possessed by an object, the total mechanical energy for the rocket must remain at **40470 joules** for the remainder of the trip.

How long to reach the apex?

We can compute the time required for the rocket to reach the apex as

t = v/g = (52m/s)/(9.8m/s^2) = 5.31 seconds

How far will the rocket travel?

Knowing the time required to reach the apex, we can compute the distance to the apex (during this leg only) as

d = v0*t - 0.5*g*t^2, or

d = (52m/s) * (5.31s) - (0.5) * (9.8m/s^2)*(5.31s)^2, or

d = 138 meters

An additional 138 meters

In other words, the rocket travels an additional 138 meters straight up after the rocket-engine stops burning. This additional travel is due solely to the kinetic energy possessed by the rocket at the end of the burn.

The total height of the apex

Adding 138 more meters to the height at the end of the burn causes the height at the apex to be

height at apex = 275m + 138m = 413 meters

Mechanical energy equals potential energy alone

At that point, the total mechanical energy is equal to the gravitational potential energy because the rocket isn't moving and the kinetic energy has gone to zero.

Validation

We can compute the total mechanical energy at this point as

PEg = m*g*h = 10kg * (9.8m/s^2) * 413m = 40474 joules

This result is close enough to the total mechanical energy (p. 5) at the end of Leg B to validate the computations. In this case, we determined the height using time, velocity, and acceleration, and validated that height using work/energy concepts.

State at the end of Leg C

At the completion of Leg C:

- The rocket is at the apex at a height of 413 meters.
- The total mechanical energy is 40470 joules.
- The kinetic energy is 0 because for an instant, the rocket isn't moving.
- The mechanical energy consists totally of gravitational potential energy.

3.1.4 Leg D

Leg D of the trip is fairly simple. The rocket falls for a distance of 413 meters under the influence of the internal gravitational force.

No change in mechanical energy

Once again, because the force is an internal force, the total mechanical energy cannot be changed by the work done by the force. However, the mechanical energy can be transformed from potential energy to kinetic energy.

At the instant before the rocket strikes the ground, it must still have a total mechanical energy value of 40470 joules.

Kinetic energy: 40470, potential energy: 0

At the instant before the rocket strikes the ground, all of the mechanical energy has been transformed into kinetic energy. We can use that knowledge to compute the velocity of the rocket right before it strikes the ground.

KE = 0.5*m*v^2, or

v^2 = KE/(0.5*m), or

v = (KE/(0.5*m))^(1/2) = (40470 joules/(0.5*10kg))^(1/2), or

terminal velocity = v = 90 meters/sec

Thus, the terminal velocity of the rocket when it strikes the ground is 90 meters/sec straight down.

Validation

Let's see if we can validate that result using a different approach. Given the height of the apex and the acceleration of gravity, we can computer the transit time as

413m = 0.5*g*t^2, or

t^2 = 413m/(0.5*g), or

t = (413m/(0.5*g))^(1/2) = (413m/(0.5*9.8m/s^2))^(1/2), or

t = 9.18 seconds

Compute the terminal velocity

Knowing the time to make the trip to the ground along with the acceleration, we can compute the terminal velocity as

v = g * t = (9.8m/s^2)*9.18s = 90 m/s

which matches the terminal velocity (p. 7) arrived at on the basis of work and energy.

State at the end of Leg D

Therefore, at the end of Leg D, the rocket crashes into the ground. However, an instant before the crash,

- The total mechanical energy is 40470 joules.
- The gravitational potential energy is 0.
- The kinetic energy is 40470 joules.
- The velocity is 90 m/s straight down toward the center of the earth.

4 Do the calculations

I encourage you to repeat the calculations that I have presented in this lesson to confirm that you get the same results. Experiment with the scenarios, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

5 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

6 Miscellaneous

This section contains a variety of miscellaneous information.

NOTE: **Housekeeping material**

- Module name: Relationships Among Kinematics, Newton's Laws, Vectors, 2D Motion, 2D Forces, Momentum, Work, Energy, and Power for Blind Students
- File: Phy1220.htm
- Keywords:
  - physics
  - accessible
  - accessibility
  - blind
  - graph board
  - protractor
  - screen reader
  - refreshable Braille display
  - JavaScript
  - trigonometry
  - potential energy
  - work
  - gravitational potential energy
  - elastic potential energy
  - kinetic energy
  - mechanical energy
  - total mechanical energy
  - power
  - watt
  - internal force
  - conservative force
  - external force
  - non-conservative force

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Affiliation : I am a professor of Computer Information Technology at Austin Community College in Austin, TX.

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