Vector Subtraction*

Richard Baldwin

Abstract

This module explains vector subtraction in a format that is accessible to blind students.

1 Table of Contents

*Version 1.2: Oct 8, 2012 2:02 pm -0500

2 Preface

2.1 General

This module is part of a collection (see http://cnx.org/content/col11294/latest/ [1]) of modules designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

This module explains vector subtraction in a format that is accessible to blind students.

2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [2]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [3]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [4] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [5]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch? [6]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [7]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/∼djoyce/trig/ [8]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [9] and http://www.w3schools.com/js/default.asp [10]).
- An understanding of all of the material covered in the earlier modules in this collection.

2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures and listings while you are reading about them.

2.3.1 Figures

[1]http://cnx.org/content/col11294/latest/
[2]http://www.youtube.com/watch?v=c8plj9UsJbg
[3]http://www.youtube.com/watch?v=v-F06HgiUpw
[4]http://www.nvda-project.org/
[5]http://www.userite.com/ecampus/lesson1/tools.php
[6]http://www.youtube.com/watch?v=c8plj9UsJbg
[7]http://www.youtube.com/watch?v=v-F06HgiUpw
[8]http://www.clarku.edu/∼djoyce/trig/
[9]http://www.dickbaldwin.com/tocjscript1.htm
[10]http://www.w3schools.com/js/default.asp

2.3.2 Listings

- Listing 1 (p. 5) . Add and subtract vectors using trigonometry.

2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [11] .

3 Discussion

Earlier lessons have dealt quite a lot with vector addition, but I have had very little to say about vector subtraction. That is because we haven't had much need for vector subtraction until now. The next module will deal with circular motion and the need for understanding vector subtraction will be paramount in understanding the material in that lesson.

My objective in this lesson is to explain vector subtraction in sufficient depth that you can visualize what it means when the text says that vector A is subtracted from vector B.

A simple rule

If A and B are scalars and you are asked to subtract A from B, you should already know the rule that says change the sign of A and then add.

The same rule also applies to vector subtraction. If we need to subtract vector A from vector B, we need to change the sign on the vector named A and then add it to the vector named B.

Changing the sign of a vector

So the question is, how do you change the sign of a vector? I will answer the question in three ways that really mean the same thing:

1. Change the sign for the horizontal and vertical components of the vector.
2. Draw the vector so that it points in exactly the opposite direction.
3. Add 180 degrees to the angle that defines the vector.

If all else fails, just remember this simple rule and apply it using one of the three ways described above.

Three ways to add vectors

Let's review **three ways to add two vectors** :

1. Draw them sequentially tail-to-head. The sum will be a vector that extends from the tail of the first vector to the head of the last vector.
2. Draw them tail-to-tail. Then draw a parallelogram where the two vectors form two sides of the parallelogram. The sum will be a vector that extends from the point where the tails join to the opposite corner of the parallelogram.
3. Using trigonometry, decompose both vectors into horizontal and vertical components. Add the horizontal components and add the vertical components. The result will be the horizontal and vertical components of the sum vector. If needed, use trigonometry to convert the components of the sum vector into a magnitude and angle.

Creating vectors with a graph board is useful

I have stated in earlier modules that the third approach using trigonometry is probably the most practical for blind students.

However, I also believe that it is important to get a picture in the mind's eye as to what happens when we add or subtract vectors. Therefore, I believe it is also useful for blind students to

- Use a graph board, a protractor, pipe cleaners, and pushpins to "draw" vectors.

[11] http://www.dickbaldwin.com/toc.htm

- Add them using one of the first two approaches listed above (p. 3) .
- Develop a picture of the result in the mind's eye.

(Being able to see the addition and subtraction of vectors in the mind's eye is very important. As a sighted person, I often close my eyes and draw vectors on the palm of my hand in order to get a better feel for what happens when vectors are added or subtracted.)

4 Examp les of vector subtraction

With that as an introduction, I am going to discuss some examples that are intended to help you to get a good feel for what it means to add, and more importantly to subtract two vectors. I hope that you will not only follow along and work through the examples, but that you will also use your graph board and "draw" the examples as I describe them.

4.1 The parallelogram method

Before getting into the details of vector subtraction, let me make a few comments about the parallelogram method of vector addition.

Blind students should be able to do a pretty good job of adding two vectors with this method using a graph board, some pushpins, and five pipe cleaners. Try to keep the pipe cleaners as straight as practical. Consider making a loop at each end of four of the pipe cleaners that you can use to pin them down to the graph board.

Given two vectors...

Given two vectors A and B, which are specified in terms of the magnitude of each vector and the angle that each vector makes relative to the horizontal axis, follow these steps:

1. Cut or bend two pipe cleaners to the correct length for one of the vectors. Set one aside temporarily.
2. Cut or bent two pipe cleaners to the correct length for the other vector. Set one aside temporarily.
3. Position two of the pipe cleaners so as to represent the pair of vectors connected at their tails with the correct angle between them. (Make each vector form the correct angle relative to the horizontal axis.) Pin each end of each vector down to hold it at the correct location with the correct angle. Think of the point where their tails join as the origin of a Cartesian coordinate system.
4. Use the other two pipe cleaners to form a parallelogram. Pin the ends of those pipe cleaners down using the pushpins at the ends of the vectors plus one additional pushpin. If you pin the ends of those two pipe cleaners to the ends of the vectors first, you should be able to find the point where those two pipe cleaners meet and pin them down at that point. That will be the "opposite corner" of the parallelogram. If you end up with a four-sided geometric shape that is not a parallelogram, you have used the final two pipe cleaners in the wrong order. Switch them and draw the parallelogram again.
5. Place a fifth pipe cleaner from the origin to the opposite corner of the parallelogram. The length of that vector will be the magnitude of the sum of the other two vectors. The angle that vector forms with the horizontal axis will be the angle of the sum of the other two vectors.

4.2 Angles, approaches, and vector naming convention

Make one vector horizontal

In order to make the computations and the manipulations of the pipe cleaners in the following examples easier, I will cause one of the two vectors to lie on the horizontal axis with an angle of 0 degrees. That won't make the results any less general, but it will make it easier for you to get accurate results.

The parallelogram method

I may be wrong, but I believe that the parallelogram method for adding two vector is more practical for blind students than the tail-to-tip method. Therefore, I will walk you through a parallelogram solution and a trigonometry solution for several of the example scenarios.

Vector naming convention

I will define vectors in the following way:

- Bm = 10 units
- Ba = 45 degrees
- Cs = B + A
- Cd = B - A

where

- Bm is the magnitude of a vector named B.
- Ba is the angle that the vector named B makes with the horizontal axis.
- Cs and Cd are the sum and difference of two vectors respectively.
- Csm and Cdm are the magnitudes of the sum and difference of two vectors.
- Csa and Cda are the angles (relative to the horizontal axis) of the sum and difference of two vectors.

4.3 Trigonometric solutions

I'm going to write a JavaScript program such that we can simply plug numbers into the values of variables in order to solve for the sum and difference of two vectors. This will be easier and less error prone than doing lots of calculations using the Google calculator.

The code for the JavaScript program is shown in Listing 1 (p. 5) . There is nothing in Listing 1 (p. 5) that you haven't seen in earlier modules, so it shouldn't require an explanation.

Listing 1: Add and subtract vectors using trigonometry.

```
    <!------------ File JavaScript01.html ------------------>
<html><body>
<script language="JavaScript1.3">

document.write("Start Script </br>");

//The purpose of this function is to receive the adjacent
// and opposite side values for a right triangle and to
// return the angle in degrees in the correct quadrant.
function getAngle(x,y){
  if((x == 0) && (y == 0)){
    //Angle is indeterminate. Just return zero.
    return 0;
  }else if((x == 0) && (y > 0)){
    //Avoid divide by zero denominator.
    return 90;
  }else if((x == 0) && (y < 0)){
    //Avoid divide by zero denominator.
    return -90;
  }else if((x < 0) && (y >= 0)){
    //Correct to second quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else if((x < 0) && (y <= 0)){
    //Correct to third quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
```

```
  }else{
    //First and fourth quadrants. No correction required.
    return Math.atan(y/x)*180/Math.PI;
  }//end else
}//end function getAngle


//Modify these values and run for different cases.
var Bm = 10;
var Ba = 90;
var Am = 10;
var Aa = 0;

//Do not modify any of the following code.
//Convert angles to radians
var bAng = Ba*Math.PI/180;
var aAng = Aa*Math.PI/180;

//Compute horizontal and vertical components
// for each vector.
var Bx = Bm*Math.cos(bAng);
var By = Bm*Math.sin(bAng);

var Ax = Am*Math.cos(aAng);
var Ay = Am*Math.sin(aAng);

//Compute sum of vectors
var Cx = Bx + Ax;
var Cy = By + Ay;
var Ca = getAngle(Cx,Cy);
var Cm = Math.sqrt(Cx*Cx + Cy*Cy);

//Compute difference between vectors
var Dx = Bx - Ax;
var Dy = By - Ay;
var Da = getAngle(Dx,Dy);
var Dm = Math.sqrt(Dx*Dx + Dy*Dy);

document.write("Bm = " + Bm.toFixed(2) + "</br>");
document.write("Ba = " + Ba.toFixed(2) + " deg</br>");
document.write("Am = " + Am.toFixed(2) + "</br>");
document.write("Aa = " + Aa.toFixed(2) + " deg</br>");

document.write("Bx = " + Bx.toFixed(2) + "</br>");
document.write("By = " + By.toFixed(2) + "</br>");
document.write("Ax = " + Ax.toFixed(2) + "</br>");
document.write("Ay = " + Ay.toFixed(2) + "</br>");
document.write("Cx = " + Cx.toFixed(2) + "</br>");
document.write("Cy = " + Cy.toFixed(2) + "</br>");
document.write("Ca = " + Ca.toFixed(2) + " deg</br>");
document.write("Cm = " + Cm.toFixed(2) + "</br>");
```

```
document.write("Da = " + Da.toFixed(2) + " deg</br>");
document.write("Dm = " + Dm.toFixed(2) + "</br>");

document.write("End Script");

</script>
</body></html>
```

The one unique thing

The only thing that is unique about Listing 1 (p. 5) is that we can plug values into the variables named Bm, Ba, Am, and Aa to specify the magnitudes and angles for the vectors named B and A. (The same naming scheme is used for the variables as was described earlier.) Then when we open the script in our browser, the sum and difference of the vectors B and A (plus some additional information) will be computed and displayed.

4.4 Orthogonal vectors

Orthogonal vectors are vectors that form a right angle when placed tail-to-tail. In other words, the angle between them is 90 degrees. Since this is a fairly easy case to work with, lets begin with a couple of examples of this sort.

Specification for two vectors

- Bm = 10 units
- Ba = 90 degrees
- Am = 10 units
- Aa = 0 degrees

Use the graphical method and the trigonometric method to find Cs=B+A and Dd=B-A.

4.4.1 Graphical solution for Cs=B+A

Draw the two vectors on your graph board and construct the parallelogram as described earlier.

In this case, the parallelogram simply becomes a square. You should find that the angle for the vector C is 45 degrees. Using the Pythagorean theorem, you should find that the magnitude of the vector C is 14.14. Thus,

- Cm = 14.14 units
- Ca = 45 degrees

4.4.2 Graphical solution for Dd=B-A

To subtract the vector A from the vector B, flip vector A over and draw it pointing in exactly the opposite direction. Stated differently, add 180 degrees to the angle for A and draw it. Then add the modified vector A to the original vector B.

Once again, the parallelogram is a square. Now you should find that the magnitude for vector D is 14.14 units, and the angle for vector D is 135 degrees. Thus

- Dm = 14.14 units
- Dz = 135 degrees

Sum and difference magnitudes are the same

Note that the magnitude of the difference vector is the same as the magnitude of the sum vector in this case. As you will see later, that is not the case in general.

Difference vector is perpendicular to the sum vector

Also, the angle of the difference vector is 90 degrees greater than the angle of the sum vector. In other words, the two vectors are perpendicular. As you will see later, that is the case in general.

4.4.3 Trigonometric solution for Cs=B+A and Dd=B-A

Figure 1 (p. 8) shows the program output for the sum and difference of a pair of orthogonal vectors. These are the same vectors for which you estimated the magnitudes and angles of the sum and difference vectors earlier.

Program output for orthogonal vectors.

```
Start Script
Bm = 10.00
Ba = 90.00 deg
Am = 10.00
Aa = 0.00 deg
Bx = 0.00
By = 10.00
Ax = 10.00
Ay = 0.00
Cx = 10.00
Cy = 10.00
Ca = 45.00 deg
Cm = 14.14
Da = 135.00 deg
Dm = 14.14
End Script
```

Figure 1: Program output for orthogonal vectors.

The last five lines of output text in Figure 1 (p. 8) show the same results that you got using graphical methods to add and subtract the vectors.

4.5 Same magnitude, 45-degree angle

Now let's modify the problem and reduce the angle between the two vectors to 45 degrees. Assume that

- Bm = 10 units
- Ba = 45 degrees

- Am = 10 units
- Aa = 0 degrees

Compute B+A and B-A as before.

4.5.1 Graphical solutions for sum and difference vectors

When you draw your parallelograms, you should find that:

- The angle for the sum vector is now 22.5 degrees (the sum vector falls half way between the two vectors being added).
- The angle for the difference vector is still equal to the angle for the sum vector plus 90 degrees, or 112.5 degrees. (The difference vector is perpendicular to the sum vector.)
- The magnitude of the sum vector is now longer than before.
- The magnitude of the difference vector is now shorter than before.

We could continue with graphical solutions

We could continue this process for smaller and smaller angles between two vectors with the same or different magnitudes, and we would find that

- The direction of the sum vector continues to be between the two vectors being added.
- The magnitude of the sum vector approaches the sum of the magnitudes of the two vectors as the angle between them approaches zero.
- The magnitude of the difference vector approaches the difference between the magnitudes of the two vectors as the angle approaches zero.

When the two vectors have the same magnitude

For the special case where the magnitudes of the two vectors being added and subtracted are the same,

- The magnitude of the sum vector approaches twice the magnitude of either vector as the angle between the two vectors approaches zero.
- The magnitude of the difference vector approaches zero as the angle between the two vectors approaches zero.
- The angle for the difference vector continues to be equal to the angle for the sum vector plus 90 degrees as the angle between the two vectors approaches zero. (The difference vector is perpendicular to the sum vector.)

This case will be very important in the modules on circular motion,

What happens to the angles?

Getting back to your graph board drawing of the most recent scenario, there is another very important characteristic that we might be able to recognize. When you add the negative of vector A to vector B in order to subtract vector A from vector B, the direction of the resulting vector points at an angle that bisects the angle made by vectors B and -A.

An almost straight line

As the angle between the vectors B and A approaches zero, the angle between the vectors B and - A approaches 180 degrees. Therefore, those two vectors tend to describe a straight line when joined at their tails as the angle between B and A approaches 0.

Perpendicularity

The magnitude of the vector that is the sum of B and -A approaches 0, while the direction of that vector approaches perpendicularity with the (almost) straight line. That means that the difference vector approaches perpendicularity with each of the original vectors, B and A, as the angle between them approaches 0.

Please remember this when we discuss centripetal force in a future module.

4.6 Sum and difference for smaller and smaller angles

It is difficult and time consuming for blind students to do vector addition and subtraction with the graph board. Therefore, I will make some minor modifications to the code in Listing 1 (p. 5) to cause the output to be more compact and then run the script for a series of decreasing angles between the vectors A and B while keeping the magnitudes of the two vectors the same. The results are shown in Figure 2 (p. 11) .

Program output for smaller and smaller angles.

```
    Start Script
Bm = 10.00 Ba = 90.00 deg
Am = 10.00 Aa = 0.00 deg
Cm = 14.14 Ca = 45.00 deg
Dm = 14.14 Da = 135.00 deg

Start Script
Bm = 10.00 Ba = 45.00 deg
Am = 10.00 Aa = 0.00 deg
Cm = 18.48 Ca = 22.50 deg
Dm = 7.65 Da = 112.50 deg

Start Script
Bm = 10.00 Ba = 22.50 deg
Am = 10.00 Aa = 0.00 deg
Cm = 19.62 Ca = 11.25 deg
Dm = 3.90 Da = 101.25 deg

Start Script
Bm = 10.00 Ba = 11.25 deg
Am = 10.00 Aa = 0.00 deg
Cm = 19.90 Ca = 5.62 deg
Dm = 1.96 Da = 95.63 deg

Start Script
Bm = 10.00 Ba = 5.63 deg
Am = 10.00 Aa = 0.00 deg
Cm = 19.98 Ca = 2.81 deg
Dm = 0.98 Da = 92.81 deg

Start Script
Bm = 10.00 Ba = 2.81 deg
Am = 10.00 Aa = 0.00 deg
Cm = 19.99 Ca = 1.41 deg
Dm = 0.49 Da = 91.41 deg

Start Script
Bm = 10.00 Ba = 1.41 deg
Am = 10.00 Aa = 0.00 deg
Cm = 20.00 Ca = 0.70 deg
Dm = 0.25 Da = 90.70 deg

Start Script
Bm = 10.00 Ba = 0.07 deg
Am = 10.00 Aa = 0.00 deg
Cm = 20.00 Ca = 0.04 deg
Dm = 0.01 Da = 90.04 deg
```


Figure 2: Program output for smaller and smaller angles.

The results

Note in particular, the values of the magnitude of the difference vector (Dm) and the angle of the difference vector (Da) in Figure 2 (p. 11) as the angle between the vectors B and A approaches zero.

As you can see from Figure 2 (p. 11) , regardless of the angle between vectors B and A, the difference vector is always perpendicular to the sum vector.

As you also can also see from Figure 2 (p. 11) , for very small angles, the angle of the sum vector is very close to the angles of the other two vectors. (They almost overlay one another.) Therefore, for very small angles, the difference vector is very close to being perpendicular to each of the vectors being subtracted.

As I mentioned earlier, this conclusion will be very important in a future module dealing with circular motion.

5 Run the script

I encourage you to run the script that I presented in this lesson to confirm that you get the same results. Confirm some of those results with your graph board.

Copy the code for the script into a text file with an extension of html. Then open that file in your browser. Experiment with the code, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

7 Miscellaneous

This section contains a variety of miscellaneous information.

NOTE: **Housekeeping material**

- Module name: Vector Subtraction for Blind Students
- File: Phy1230.htm
- Keywords:
  - physics
  - accessible
  - accessibility
  - blind
  - graph board
  - protractor
  - screen reader
  - refreshable Braille display
  - JavaScript
  - trigonometry
  - vector

NOTE: **Disclaimers:** **Financial** : Although the Connexions site makes it possible for you to download a PDF file for this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

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Affiliation : I am a professor of Computer Information Technology at Austin Community College in Austin, TX.

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