Geometry Basics - Grade 10 [CAPS]*

Free High School Science Texts Project

Based on *Geometry Basics*† by

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1 Introduction

The purpose of this chapter is to recap some of the ideas that you learned in geometry and trigonometry in earlier grades. You should feel comfortable with the work covered in this chapter before attempting to move onto the Grade 10 Geometry chapter, the Grade 10 Trigonometry chapter or the Grade 10 Analytical Geometry chapter. This chapter revises:

1. Terminology: vertices, sides, angles, parallel lines, perpendicular lines, diagonals, bisectors, transversals
2. Properties of triangles
3. Congruence
4. Classification of angles into acute, right, obtuse, straight, reflex or revolution
5. Theorem of Pythagoras which is used to calculate the lengths of sides of a right-angled triangle

2 Points and Lines

The two simplest objects in geometry are *points* and *lines*.

A point is a coordinate that marks a position in space (on a number line, on a plane or in three dimensions or even more) and is denoted by a dot. Points are usually labelled with a capital letter. Some examples of how points can be represented are shown in Figure 1.

A line is a continuous set of coordinates in space and can be thought of as being formed when many points are placed next to each other. Lines can be straight or curved, but are always continuous. This means

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that there are never any breaks in the lines (if there are, they would be distinct lines denoted separately). The endpoints of lines are labeled with capital letters. Examples of two lines are shown in Figure 1.

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Figure 1: Examples of some points (labelled P, Q, R and S) and some lines (labelled BC and DE).

Lines are labelled according to the start point and end point. We call the line that starts at a point A and ends at a point B, AB. Since the line from point B to point A is the same as the line from point A to point B, we have that $AB = BA$.

When there is no ambiguity (which is the case throughout this text) the length of the line between points A and B is also denoted AB , the same as the notation to refer to the line itself. So if we say $AB = CD$ we mean that the length of the line between A and B is equal to the length of the line between C and D.

Note: in higher mathematics, where there might be some ambiguity between when we want refer to the length of the line and when we just want to refer to the line itself, the notation $|AB|$ is usually used to refer to the length of the line. In this case, if one says $|AB| = |CD|$, it means the lengths of the lines are the same, whereas if one says $AB = CD$, it means that the two lines actually coincide (i.e. they are the same). Throughout this text, however, this notation will not be used, and $AB = CD$ ALWAYS implies that the lengths are the same.

A line is measured in *units of length*. Some common units of length are listed in Table 1.

Unit of Length	**Abbreviation**
kilometre	km
metre	m
centimetre	cm
millimetre	mm

Table 1: Some common units of length and their abbreviations.

3 Angles

An *angle* is formed when two straight lines meet at a point. The point at which two lines meet is known as a *vertex*. Angles are labelled with a $\hat{}$ called a caret on a letter. For example, in Figure 2 the angle is at $\hat{B}$. Angles can also be labelled according to the line segments that make up the angle. For example, in Figure 2 the angle is made up when line segments CB and BA meet. So, the angle can be referred to as $\angle CBA$ or $\angle ABC$ or, if there is no ambiguity (i.e. there is only one angle at B) sometimes simply $\angle B$. The $\angle$ symbol is a short method of writing angle in geometry.

Angles are measured in *degrees* which is denoted by $^\circ$, a small circle raised above the text in the same fashion as an exponent (or a superscript).

NOTE: Angles can also be measured in radians. At high school level you will only use degrees, but if you decide to take maths at university you will learn about radians.

Figure 2: Angle labelled as $\hat{B}$, $\angle CBA$ or $\angle ABC$

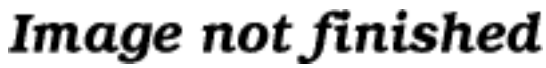

Figure 3: Examples of angles. $\hat{A}=\hat{E}$, even though the lines making up the angles are of different lengths.

3.1 Measuring angles

The size of an angle does not depend on the length of the lines that are joined to make up the angle, but depends only on how both the lines are placed as can be seen in Figure 3. This means that the idea of length cannot be used to measure angles. An angle is a rotation around the vertex.

3.1.1 Using a Protractor

A protractor is a simple tool that is used to measure angles. A picture of a protractor is shown in Figure 4.

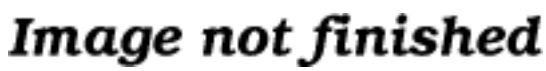

Figure 4: Diagram of a protractor.

Method:
Using a protractor

1. Place the bottom line of the protractor along one line of the angle so that the other line of the angle points at the degree markings.
2. Move the protractor along the line so that the centre point on the protractor is at the vertex of the two lines that make up the angle.

3. Follow the second line until it meets the marking on the protractor and read off the angle. Make sure you start measuring at 0°.

3.1.1.1 Measuring Angles : Use a protractor to measure the following angles:

Image not finished

Figure 5

3.2 Special Angles

What is the smallest angle that can be drawn? The figure below shows two lines (CA and AB) making an angle at a common vertex A. If line CA is rotated around the common vertex A, down towards line AB, then the smallest angle that can be drawn occurs when the two lines are pointing in the same direction. This gives an angle of 0°. This is shown in Figure 6

Image not finished

Figure 6

If line CA is now swung upwards, any other angle can be obtained. If line CA and line AB point in opposite directions (the third case in Figure 6) then this forms an angle of 180°.

> TIP: If three points A, B and C lie on a straight line, then the angle between them is 180°. Conversely, if the angle between three points is 180°, then the points lie on a straight line.

An angle of 90° is called a *right angle*. A right angle is half the size of the angle made by a straight line (180°). We say CA is *perpendicular* to AB or $CA \perp AB$. An angle twice the size of a straight line is 360°. An angle measuring 360° looks identical to an angle of 0°, except for the labelling. We call this a *revolution*.

Image not finished

Figure 7: An angle of 90° is known as a *right angle*.

3.2.1 Angles larger than 360°

All angles larger than 360° also look like we have seen them before. If you are given an angle that is larger than 360°, continue subtracting 360° from the angle, until you get an answer that is between 0°and 360°. Angles that measure more than 360° are largely for mathematical convenience.

TIP:

- *Acute angle*: An angle $\geq 0°$ and $< 90°$.
- *Right angle*: An angle measuring $90°$.
- *Obtuse angle*: An angle $> 90°$ and $< 180°$.
- *Straight angle*: An angle measuring $180°$.
- *Reflex angle*: An angle $> 180°$ and $< 360°$.
- *Revolution*: An angle measuring $360°$.

These are simply labels for angles in particular ranges, shown in Figure 8.

Image not finished

Figure 8: Three types of angles defined according to their ranges.

Once angles can be measured, they can then be compared. For example, all right angles are $90°$, therefore all right angles are equal and an obtuse angle will always be larger than an acute angle.

The following video summarizes what you have learnt so far about angles.

Khan Academy video on angles - 1

This media object is a Flash object. Please view or download it at
<http://www.youtube.com/v/2439OIVBgPg&rel=0&hl=en_US&feature=player_embedded&version=3>

Figure 9

Note that for high school trigonometry you will be using degrees, not radians as stated in the video.

3.3 Special Angle Pairs

In Figure 10, straight lines AB and CD intersect at point X, forming four angles: $\hat{X}_1$ or $\angle BXD$, $\hat{X}_2$ or $\angle BXC$, $\hat{X}_3$ or $\angle CXA$ and $\hat{X}_4$ or $\angle AXD$.

Image not finished

Figure 10: Two intersecting straight lines with vertical angles $\hat{X}_1, \hat{X}_3$ and $\hat{X}_2, \hat{X}_4$.

The table summarises the special angle pairs that result.

Special Angle	Property	Example
adjacent angles	share a common vertex and a common side	$\left(\hat{X_1}, \hat{X_2}\right)$, $\left(\hat{X_2}, \hat{X_3}\right)$, $\left(\hat{X_3}, \hat{X_4}\right)$, $\left(\hat{X_4}, \hat{X_1}\right)$
linear pair (adjacent angles on a straight line)	adjacent angles formed by two intersecting straight lines that by definition add to 180°	$\hat{X_1} + \hat{X_2} = 180°$; $\hat{X_2} + \hat{X_3} = 180°$; $\hat{X_3} + \hat{X_4} = 180°$; $\hat{X_4} + \hat{X_1} = 180°$
opposite angles	angles formed by two intersecting straight lines that share a vertex but do not share any sides	$\hat{X_1} = \hat{X_3}$; $\hat{X_2} = \hat{X_4}$
supplementary angles	two angles whose sum is 180°	
complementary angles	two angles whose sum is 90°	

Table 2

TIP: The opposite angles formed by the intersection of two straight lines are equal. Adjacent angles on a straight line are supplementary.

The following video summarises what you have learnt so far

Khan Academy video on angles - 2

This media object is a Flash object. Please view or download it at
<http://www.youtube.com/v/zrqzG6xKa1A&rel=0&hl=en_US&feature=player_embedded&version=3>

Figure 11

3.4 Parallel Lines intersected by Transversal Lines

Two lines intersect if they cross each other at a point. For example, at a traffic intersection two or more streets intersect; the middle of the intersection is the common point between the streets.

Parallel lines are lines that never intersect. For example the tracks of a railway line are parallel (for convenience, sometimes mathematicians say they intersect at 'a point at infinity', i.e. an infinite distance away). We wouldn't want the tracks to intersect after as that would be catastrophic for the train!

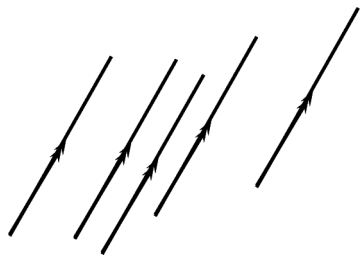

Figure 12

All these lines are parallel to each other. Notice the pair of arrow symbols for parallel.

NOTE: A section of the Australian National Railways Trans-Australian line is perhaps one of the longest pairs of man-made parallel lines.

> ***Longest Railroad Straight*** *(Source: www.guinnessworldrecords.com) The Australian National Railways Trans-Australian line over the Nullarbor Plain, is 478 km (297 miles) dead straight, from Mile 496, between Nurina and Loongana, Western Australia, to Mile 793, between Ooldea and Watson, South Australia.*

A *transversal* of two or more lines is a line that intersects these lines. For example in Figure 13, AB and CD are two parallel lines and EF is a transversal. We say $AB \parallel CD$. The properties of the angles formed by these intersecting lines are summarised in the table below.

Image not finished

Figure 13: Parallel lines intersected by a transversal

Name of angle	Definition	Examples	Notes
interior angles	the angles that lie inside the parallel lines	in Figure 13a, b, c and d are interior angles	the word *interior* means inside
			continued on next page

adjacent angles	the angles share a common vertex point and line	in Figure 13 (a, h) are adjacent and so are (h, g); (g, b); (b, a)	
exterior angles	the angles that lie outside the parallel lines	in Figure 13e, f, g and h are exterior angles	the word *exterior* means outside
alternate interior angles	the interior angles that lie on opposite sides of the transversal	in Figure 13 (a, c) and (b,d) are pairs of alternate interior angles, $a = c$, $b = d$	**Figure**
co-interior angles on the same side	co-interior angles that lie on the same side of the transversal	in Figure 13 (a,d) and (b,c) are interior angles on the same side. $a + d = 180°$, $b + c = 180°$	**Figure**
corresponding angles	the angles on the same side of the transversal and the same side of the parallel lines	in Figure 13(a, e), (b, f), (c, g) and (d, h) are pairs of corresponding angles. $a = e$, $b = f$, $c = g$, $d = h$	**Figure**

Table 3

The following video summarises what you have learnt so far

Khan Academy video on angles - 3

This media object is a Flash object. Please view or download it at
<http://www.youtube.com/v/NLg6hfoKKlE&rel=0&hl=en_US&feature=player_embedded&version=3>

Figure 17

NOTE: **Euclid's Parallel line postulate.** If a straight line falling across two other straight lines makes the two interior angles on the same side less than two right angles (180°), the two straight lines, if produced indefinitely, will meet on that side. This postulate can be used to prove many identities about the angles formed when two parallel lines are cut by a transversal.

TIP:

1. If two parallel lines are intersected by a transversal, the sum of the co-interior angles on the same side of the transversal is 180°.
2. If two parallel lines are intersected by a transversal, the alternate interior angles are equal.
3. If two parallel lines are intersected by a transversal, the corresponding angles are equal.
4. If two lines are intersected by a transversal such that any pair of co-interior angles on the same side is supplementary, then the two lines are parallel.
5. If two lines are intersected by a transversal such that a pair of alternate interior angles are equal, then the lines are parallel.
6. If two lines are intersected by a transversal such that a pair of alternate corresponding angles are equal, then the lines are parallel.

Exercise 1: Finding angles ***(Solution on p. 26.)***

Find all the unknown angles in the following figure:

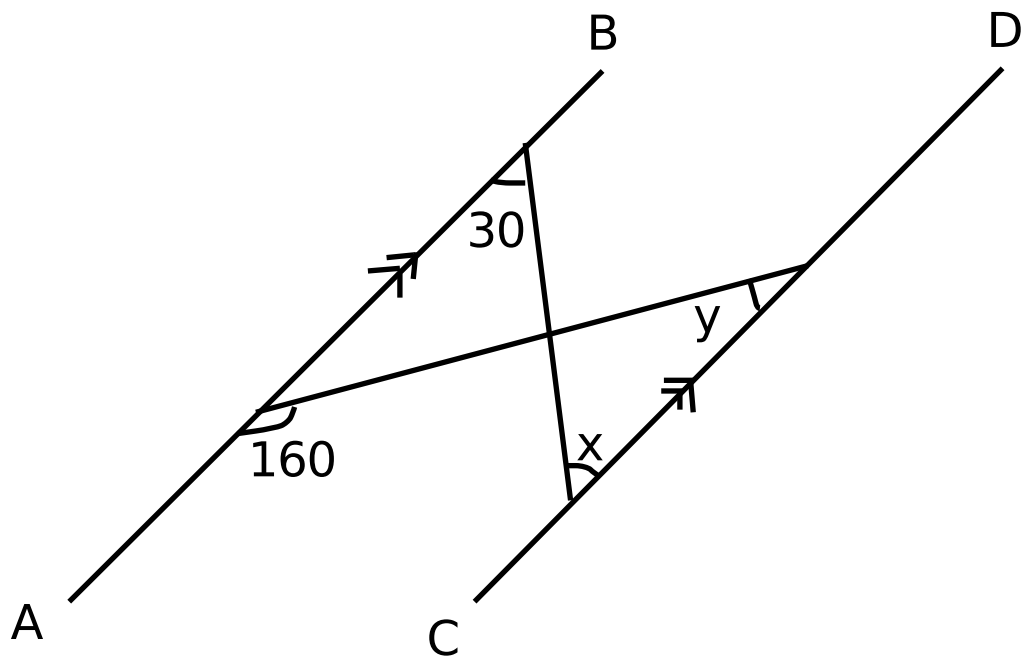

Figure 18

Exercise 2: Parallel lines ***(Solution on p. 26.)***

Determine if there are any parallel lines in the following figure:

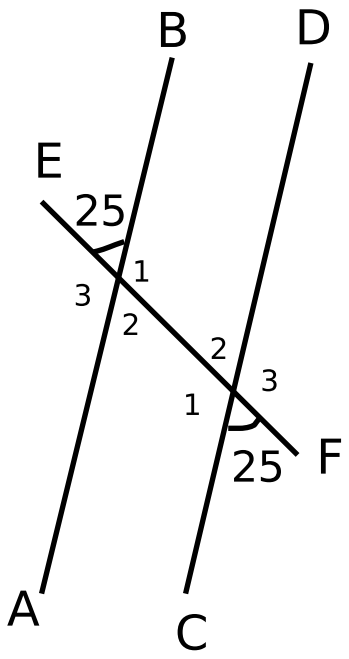

Figure 19

3.4.1 Angles

1. Use adjacent, corresponding, co-interior and alternate angles to fill in all the angles labeled with letters in the diagram below:

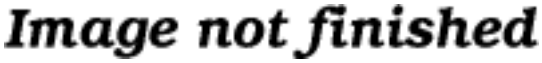

Figure 20

   Click here for the solution[1]
2. Find all the unknown angles in the figure below:

[1] http://www.fhsst.org/lxF

Image not finished

Figure 21

Click here for the solution[2]

3. Find the value of x in the figure below:

Image not finished

Figure 22

Click here for the solution[3]

4. Determine whether there are pairs of parallel lines in the following figures.

a. Image not finished

Figure 23

b. Image not finished

Figure 24

c. Image not finished

Figure 25

Click here for the solution[4]

5. If AB is parallel to CD and AB is parallel to EF, prove that CD is parallel to EF:

[2] http://www.fhsst.org/lxL
[3] http://www.fhsst.org/lxM
[4] http://www.fhsst.org/lxe

Image not finished

Figure 26

Click here for the solution[5]

The following video shows some problems with their solutions

Khan Academy video on angles - 4

This media object is a Flash object. Please view or download it at
<http://www.youtube.com/v/kqU_ymV581c&rel=0&hl=en_US&feature=player_embedded&version=3>

Figure 27

4 Polygons

If you take some lines and join them such that the end point of the first line meets the starting point of the last line, you will get a *polygon*. Each line that makes up the polygon is known as a *side*. A polygon has interior angles. These are the angles that are inside the polygon. The number of sides of a polygon equals the number of interior angles. If a polygon has equal length sides and equal interior angles, then the polygon is called a *regular polygon*. Some examples of polygons are shown in Figure 28.

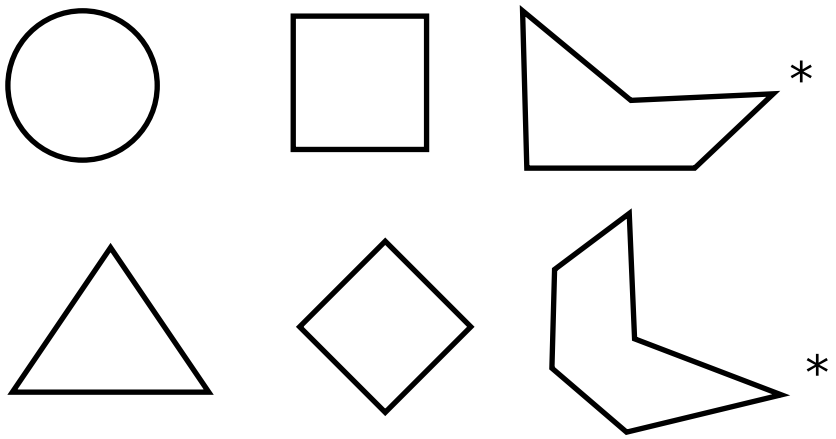

Figure 28: Examples of polygons. They are all regular, except for the one marked *

[5] http://www.fhsst.org/lxt

4.1 Triangles

A triangle is a three-sided polygon. Triangles are usually split into three categories: equilateral, isosceles, and scalene, depending on how many of the sides are of equal length. A fourth category, right-angled triangle (or simply 'right triangle') is used to refer to triangles with one right angle. Note that all right-angled triangles are also either isosceles (if the other two sides are equal) or scalene (it should be clear why you cannot have an equilateral right triangle!). The properties of these triangles are summarised in Table 4.

Name	Diagram
equilateral	*Image not finished* **Figure 29**
isosceles	*Image not finished* **Figure 30**
right-angled	*Image not finished* **Figure 31**
scalene (non-syllabus)	*Image not finished* **Figure 32**

Table 4: Types of Triangles

We use the notation [U+25B5] ABC to refer to a triangle with corners labeled A, B, and C.

4.1.1 Properties of Triangles

4.1.1.1 Investigation : Sum of the angles in a triangle

1. Draw on a piece of paper a triangle of any size and shape

2. Cut it out and label the angles $\hat{A}$, $\hat{B}$ and $\hat{C}$ on both sides of the paper
3. Draw dotted lines as shown and cut along these lines to get three pieces of paper
4. Place them along your ruler as shown to see that $\hat{A} + \hat{B} + \hat{C} = 180°$

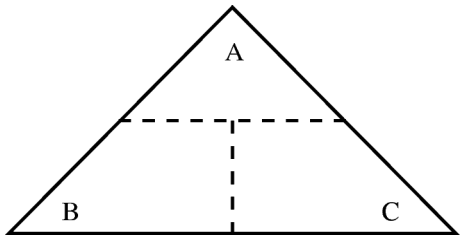

Figure 33

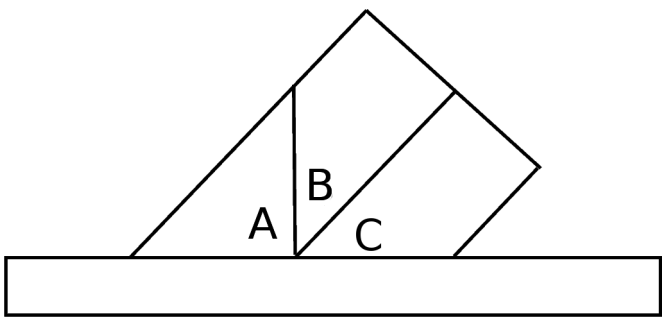

Figure 34

TIP: The sum of the angles in a triangle is 180°.

Image not finished

Figure 35: In any triangle, $\angle A + \angle B + \angle C = 180°$

TIP: Any exterior angle of a triangle is equal to the sum of the two opposite interior angles. An exterior angle is formed by extending any one of the sides.

Image not finished

Figure 36: In any triangle, any exterior angle is equal to the sum of the two opposite interior angles.

4.1.2 Congruent Triangles

Two triangles are called congruent if one of them can be superimposed, that is moved on top of to exactly cover, the other. In other words, if both triangles have all of the same angles and sides, then they are called congruent. To decide whether two triangles are congruent, it is not necessary to check every side and angle. The following list describes various requirements that are sufficient to know when two triangles are congruent.

Label	Description	Diagram
RHS	If the hypotenuse and one side of a right-angled triangle are equal to the hypotenuse and the respective side of another triangle, then the triangles are congruent.	Image Fi
SSS	If three sides of a triangle are equal in length to the same sides of another triangle, then the two triangles are congruent	Image Fi

continued on next page

SAS	If two sides and the included angle of one triangle are equal to the same two sides and included angle of another triangle, then the two triangles are congruent.	*Image* **Fi**
AAS	If one side and two angles of one triangle are equal to the same one side and two angles of another triangle, then the two triangles are congruent.	*Image* **Fi**

Table 5

4.1.3 Similar Triangles

Two triangles are called similar if it is possible to proportionally shrink or stretch one of them to a triangle congruent to the other. Congruent triangles are similar triangles, but similar triangles are only congruent if they are the same size to begin with.

Description	**Diagram**
If all three pairs of corresponding angles of two triangles are equal, then the triangles are similar.	*Image not finished* **Figure 41**

continued on next page

If all pairs of corresponding sides of two triangles are in proportion, then the triangles are similar.	Image not finished **Figure 42** $\frac{x}{p} = \frac{y}{q} = \frac{z}{r}$

Table 6

4.1.4 The theorem of Pythagoras

Image not finished

Figure 43

If [U+25B5]ABC is right-angled ($\hat{B}$= 90°) then $b^2 = a^2 + c^2$

Converse: If $b^2 = a^2 + c^2$, then [U+25B5]ABC is right-angled ($\hat{B}$= 90°).

Exercise 3: Triangles ***(Solution on p. 26.)***

In the following figure, determine if the two triangles are congruent, then use the result to help you find the unknown letters.

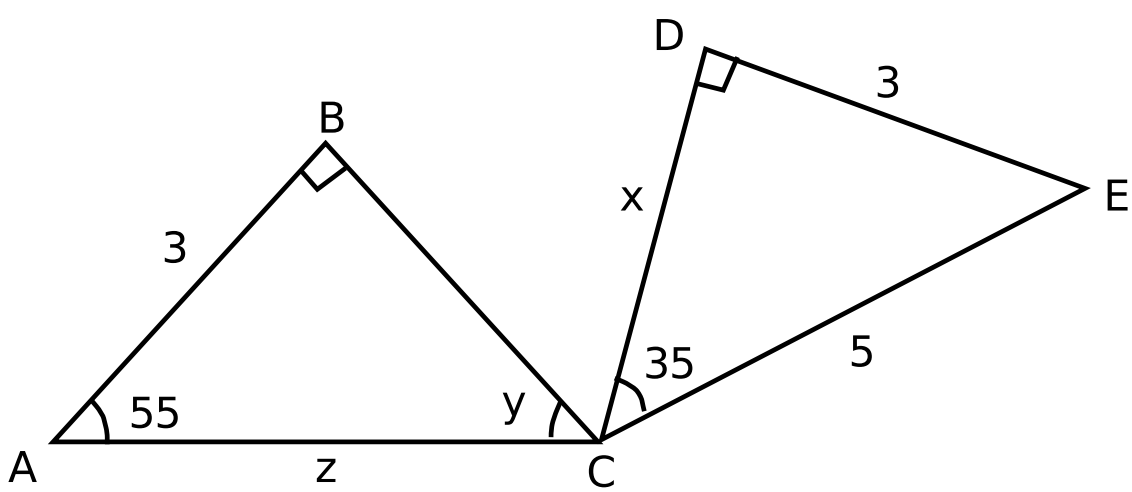

Figure 44

4.1.4.1 Triangles

1. Calculate the unknown variables in each of the following figures. All lengths are in mm.

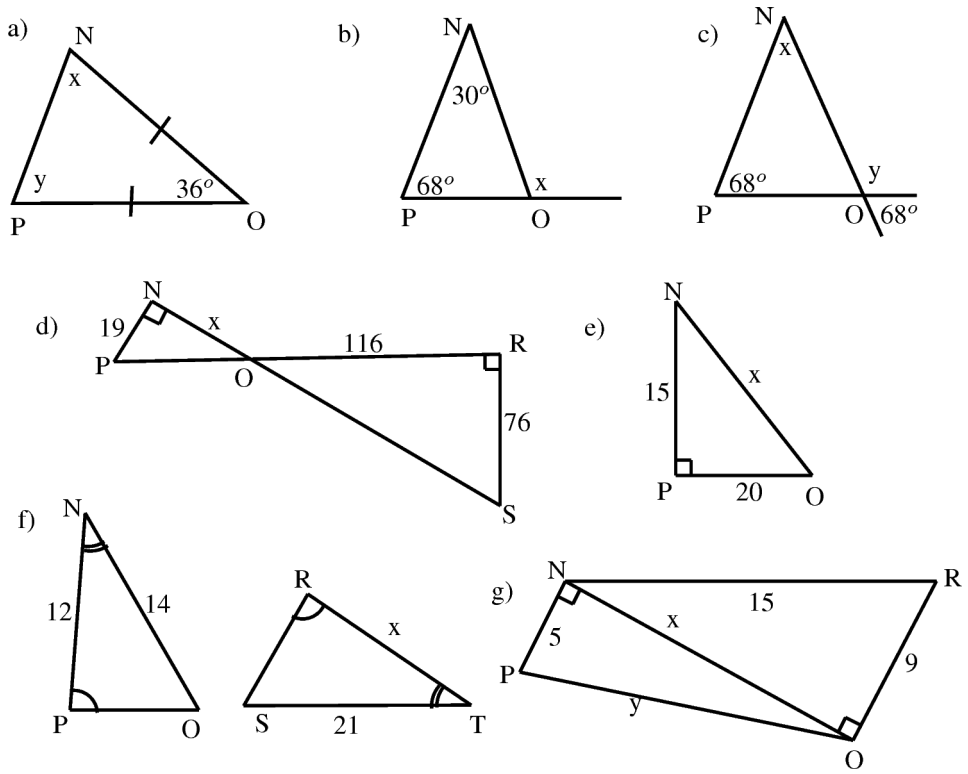

Figure 45

Click here for the solution[6]

2. State whether or not the following pairs of triangles are congruent or not. Give reasons for your answers. If there is not enough information to make a descision, say why.

[6] http://www.fhsst.org/lxz

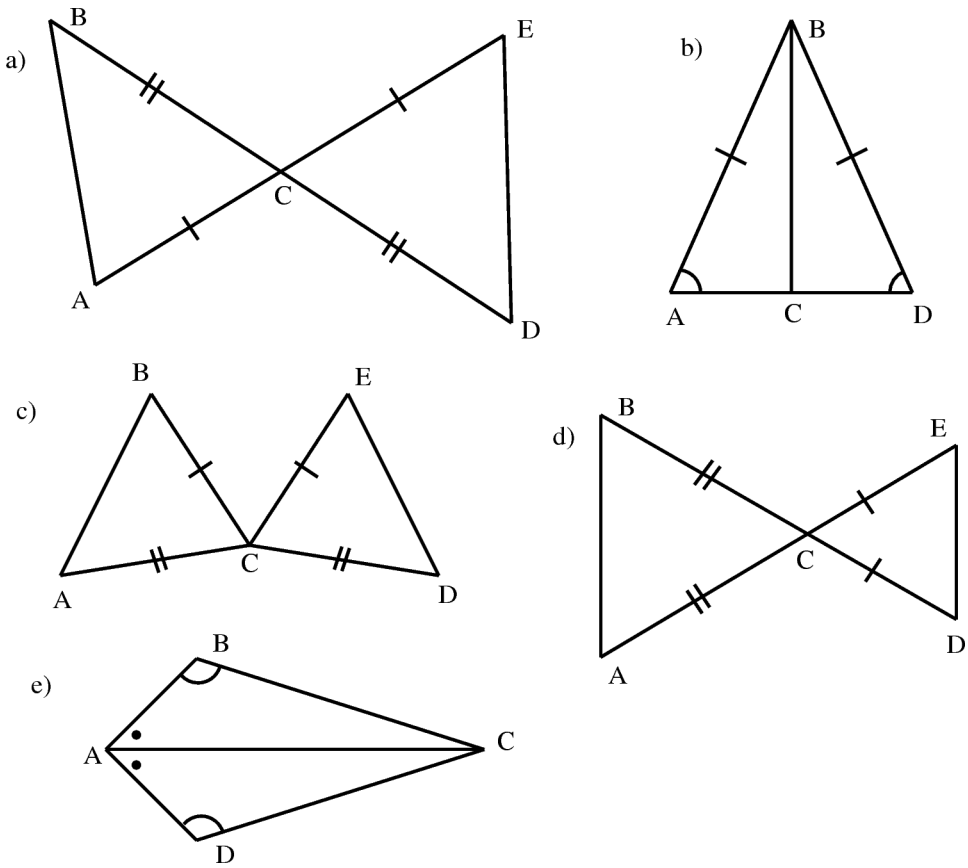

Figure 46

Click here for the solution[7]

4.1.5 Quadrilaterals

A quadrilateral is a four sided figure. There are some special quadrilaterals (trapezium, parallelogram, kite, rhombus, square, rectangle) which you will learn about in Geometry.

4.1.6 Other polygons

There are many other polygons, some of which are given in the table below.

[7] http://www.fhsst.org/lxu

Sides	Name
5	pentagon
6	hexagon
7	heptagon
8	octagon
10	decagon
15	pentadecagon

Table 7: Table of some polygons and their number of sides.

Image not finished

Figure 47: Examples of other polygons.

4.1.7 Angles of Regular Polygons

Polygons need not have all sides the same. When they do, they are called regular polygons. You can calculate the size of the interior angle of a regular polygon by using:

$$\hat{A} = \frac{n-2}{n} \times 180^\circ \tag{1}$$

where n is the number of sides and $\hat{A}$ is any angle.

Exercise 4 ***(Solution on p. 26.)***

Find the size of the interior angles of a regular octagon.

5 Summary

- Make sure you know what the following terms mean: quadrilaterals, vertices, sides, angles, parallel lines, perpendicular lines,diagonals, bisectors and transversals.
- The properties of triangles has been covered.
- Congruency and similarity of triangles
- Angles can be classified as acute, right, obtuse, straight, reflex or revolution
- Theorem of Pythagoras which is used to calculate the lengths of sides of a right-angled triangle
- Angles:
  - Acute angle: An angle 0° and 90°
  - Right angle: An angle measuring 90°
  - Obtuse angle: An angle 90° and 180°
  - Straight angle: An angle measuring 180°
  - Reflex angle: An angle 180° and 360°

- Revolution: An angle measuring 360°

- There are several properties of angles and some special names for these
- There are four types of triangles: Equilateral, isoceles, right-angled and scalene
- The angles in a triangle add up to 180°

6 Exercises

1. Find all the pairs of parallel lines in the following figures, giving reasons in each case.

   a. Image not finished

   Figure 48

   b. Image not finished

   Figure 49

   c. Image not finished

   Figure 50

   Click here for the solution[8]

2. Find angles a, b, c and d in each case, giving reasons.

   a. Image not finished

   Figure 51

   b. Image not finished

   Figure 52

   c. Image not finished

   Figure 53

[8] http://www.fhsst.org/lxh

Click here for the solution[9]

3. Say which of the following pairs of triangles are congruent with reasons.

a. Image not finished

Figure 54

b. Image not finished

Figure 55

c. Image not finished

Figure 56

d. Image not finished

Figure 57

Click here for the solution[10]

4. Identify the types of angles shown below (e.g. acute/obtuse etc):

Image not finished

Figure 58

Click here for the solution[11]

5. Calculate the size of the third angle (x) in each of the diagrams below:

Image not finished

Figure 59

[9] http://www.fhsst.org/laq
[10] http://www.fhsst.org/lai
[11] http://www.fhsst.org/lTb

Click here for the solution[12]

6. Name each of the shapes/polygons, state how many sides each has and whether it is regular (equiangular and equilateral) or not:

Image not finished

Figure 60

Click here for the solution[13]

7. Assess whether the following statements are true or false. If the statement is false, explain why:
   a. An angle is formed when two straight lines meet at a point.
   b. The smallest angle that can be drawn is 5°.
   c. An angle of 90° is called a square angle.
   d. Two angles whose sum is 180° are called supplementary angles.
   e. Two parallel lines will never intersect.
   f. A regular polygon has equal angles but not equal sides.
   g. An isoceles triangle has three equal sides.
   h. If three sides of a triangle are equal in length to the same sides of another triangle, then the two triangles are incongruent.
   i. If three pairs of corresponding angles in two triangles are equal, then the triangles are similar.

Click here for the solution[14]

8. Name the type of angle (e.g. acute/obtuse etc) based on it's size:
   a. 30°
   b. 47°
   c. 90°
   d. 91°
   e. 191°
   f. 360°
   g. 180°

Click here for the solution[15]

9. Using Pythagoras' theorem for right-angled triangles, calculate the length of x:

Image not finished

Figure 61

Click here for the solution[16]

[12] http://www.fhsst.org/lTj
[13] http://www.fhsst.org/lTD
[14] http://www.fhsst.org/lTZ
[15] http://www.fhsst.org/lTB
[16] http://www.fhsst.org/lTK

6.1 Challenge Problem

1. Using the figure below, show that the sum of the three angles in a triangle is 180°. Line DE is parallel to BC.

Image not finished

Figure 62

Click here for the solution[17]

[17] http://www.fhsst.org/laO

Solutions to Exercises in this Module

Solution to Exercise (p. 10)

Step 1. AB ∥ CD. So $x = 30^{\circ}$ (alternate interior angles)
Step 2.

$$\begin{aligned} 160 + y &= 180 \\ y &= 20^{\circ} \end{aligned} \tag{2}$$

(co-interior angles on the same side)

Solution to Exercise (p. 10)

Step 1. Line EF cannot be parallel to either AB or CD since it cuts both these lines. Lines AB and CD may be parallel.
Step 2. We can show that two lines are parallel if we can find one of the pairs of special angles. We know that $\overset{\Theta}{E}_2 = 25^{\circ}$ (opposite angles). And then we note that

$$\begin{aligned} \overset{\Theta}{E}_2 &= \overset{\Theta}{F}_4 \\ &= 25^{\circ} \end{aligned} \tag{3}$$

So we have shown that AB ∥ CD (corresponding angles)

Solution to Exercise (p. 18)

Step 1. $D\overset{\Theta}{E}C = B\overset{\Theta}{A}C = 55^{\circ}$ (angles in a triangle add up to 180°).
$A\overset{\Theta}{B}C = C\overset{\Theta}{D}E = 90^{\circ}$ (given)
DE = AB = 3 (given)

$$\therefore \Delta\mathrm{ABC} \equiv \Delta\mathrm{CDE} \tag{4}$$

Step 2. We use Pythagoras to find x:

$$\begin{aligned} \mathrm{CE}^2 &= \mathrm{DE}^2 + \mathrm{DC}^2 \\ 5^2 &= 3^2 + x^2 \\ x^2 &= 16 \\ x &= 4 \end{aligned} \tag{5}$$

$y = 35^{\circ}$ (angles in a triangle)
$z = 5$ (congruent triangles, AC = CE)

Solution to Exercise (p. 21)

Step 1. An octagon has 8 sides.
Step 2.

$$\begin{aligned} \hat{A} &= \tfrac{n-2}{n} \times 180^{\circ} \\ \hat{A} &= \tfrac{8-2}{8} \times 180^{\circ} \\ \hat{A} &= \tfrac{6}{2} \times 180^{\circ} \\ \hat{A} &= 135^{\circ} \end{aligned} \tag{6}$$