Vector Multiplication[*]

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Abstract

This module explains vector multiplication in a format that is accessible to blind students.

1 Table of Contents

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2 Preface

2.1 General

This module is part of a collection (see http://cnx.org/content/col11294/latest/ [1]) of modules designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

This module explains vector multiplication in a format that is accessible to blind students.

2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [2]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [3]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [4] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [5]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch? [6]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [7]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [8]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [9] and http://www.w3schools.com/js/default.asp [10]).
- An understanding of all of the material covered in the earlier modules in this collection.

2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures while you are reading about them.

2.3.1 Figures

- Figure 1 (p. 6) . Facts worth remembering for the dot product.
- Figure 2 (p. 9) . Facts worth remembering for the cross product.

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[1] http://cnx.org/content/col11294/latest/
[2] http://www.youtube.com/watch?v=c8plj9UsJbg
[3] http://www.youtube.com/watch?v=v-F06HgiUpw
[4] http://www.nvda-project.org/
[5] http://www.userite.com/ecampus/lesson1/tools.php
[6] http://www.youtube.com/watch?v=c8plj9UsJbg
[7] http://www.youtube.com/watch?v=v-F06HgiUpw
[8] http://www.clarku.edu/~djoyce/trig/
[9] http://www.dickbaldwin.com/tocjscript1.htm
[10] http://www.w3schools.com/js/default.asp

2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [11] .

3 Discussion

You learned about vector addition and vector subtraction in earlier modules. It is also possible to multiply vectors. That capability will become important in future modules.

There is more than one way to multiply vectors (see http://mathworld.wolfram.com/VectorMultiplication.html [12]). I will explain two of those ways in this module:

- dot product
- cross product

3.1 Dot product, inner product, or scalar product

I will begin with some background information on the dot product of two vectors.

3.1.1 Background

The terms dot product, inner product, and scalar product all mean the same thing and are used in various context's by different authors.

The term *dot product* derives from the fact that a vector product of this sort is often written as the names of the two vectors separated by a dot. However, that special dot character is probably not compatible with your Braille display. Therefore, I will write the dot product of the vectors named A and B as

(A dot B)

The term scalar product derives from the fact that a vector product of this sort more closely resembles scalar arithmetic than vector arithmetic. In particular, unlike the cross product (that will be discussed later), the result of the dot product does not have a direction.

3.1.2 Create a vector diagram on your graph board

In order for you to better understand the nature of a vector dot product, I recommend that you create a Cartesian coordinate system on your graph board, and draw the following two vectors.

NOTE: **A vector diagram for your graph board**

```
    Draw the first vector from the origin to a point at
x = 1
y = 1.73
Label this vector A.

Draw a second vector from the origin to a point at
x = 2.9
y = 0.78
Label this vector B.
```

References to the vector coordinates

I will refer to the coordinates at the tip of vector A as ax and ay. Similarly, I will refer to the coordinates at the tip of vector B as bx and by.

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[11] http://www.dickbaldwin.com/toc.htm
[12] http://mathworld.wolfram.com/VectorMultiplication.html

3.1.3 The dot product

Using this nomenclature , the dot product of any two vectors is given by

(A dot B) = (ax * bx) + (ay * by)

where

- (A dot B) represents the dot product of the vectors named A and B
- ax, ay, bx, and by are the coordinates of the tips of the vectors named A and B respectively

So what?

By now you are probably saying "So what? Why should I care?"

Although it isn't obvious from what you see above, the dot product of two vectors is also equal to the product of their magnitudes and the cosine of the angle between them. In other words,

(A dot B) = Amag*Bmag*cosine(angle between A and B)

where

- Amag is the magnitude of vector A
- Bmag is the magnitude of vector B

3.1.4 The projection of A onto B

If you divide the dot product of A and B by the magnitude of B, the result is equal to the projection of vector A onto vector B. In other words,

(A dot B)/(Bmag) = projection of A onto B

This sort of projection operation is an operation that occurs frequently in physics. For example, the horizontal component of a velocity vector is the projection of the velocity vector onto the horizontal axis. Similarly, the vertical component of a velocity vector is the projection of the velocity vector onto the vertical axis.

Let's work through some numbers

Substituting your coordinate values into the expression given above (p. 4) yields

(A dot B) = (ax * bx) + (ay * by), or

(A dot B) = (1.0*2.9) + (1.73*0.78), or

(A dot B) = 4.2494

We will make use of this value a little later in this module.

3.1.5 Compute the projection

Suppose your problem calls for the projection of vector A onto vector B. Let's compute the value of that projection. For this, we need to know the magnitude of vector B, which we can compute using the Pythagorean theorem:

Bmag = sqrt(2.9*2.9 + 0.78*0.78), or

Bmag = 3.0

The projection of A onto B is equal to

Projection = (A dot B)/Bmag, or

Projection = 4.294/3 = 1.43

We will also have more to say about this value a little later in this module.

What do we mean by the projection?

Using the vectors on your graph board, draw a line segment beginning at the tip of vector A. Make that line perpendicular to vector B and mark the point on vector B where that line intersects vector B.

The distance from the origin to that point is the value of the projection of vector A onto vector B.

According to the above arithmetic, that distance should be equal to 1.43 units. Hopefully when you measure it on your graph board, you will get approximately the same value.

3.1.6 Compute the angle between the vectors

Suppose instead that your problem calls for the angle between the two vectors. Given that the dot product is equal to the product of the magnitudes and the cosine of the angle between the vectors, the cosine of the angle between the vectors is equal to

cosine(angle) = (A dot B)/(Amag *Bmag)

You may or may not know this from your earlier experience with trigonometry, but the angle in the above expression is the arccosine of the cosine value.

To compute the angle, we first need to compute the magnitude of vector A. Once again using the Pythagorean theorem,

Amag = sqrt(1.0*1.0+1.73*1.73), or

Amag = 2

Now compute the angle between the vectors

We now have all of the information that we need to computer the angle between the vectors. Using Google calculator nomenclature,

Angle = arccos((A dot B)/(Amag *Bmag)) in degrees, or

angle = arccos(4.294/(2*3)) in degrees, or

angle = 44.30 degrees

(Actually, I chose the original values in hopes of causing this final answer to come out to 45 degrees, but the round off errors along the way threw things off a bit.)

What have we learned?

For these two vectors, we have learned that

- The angle between them is 44.3 degrees
- The projection of vector A onto vector B is equal to 1.43 units

3.1.7 Perpendicular or parallel vectors

Now consider what happens as the angle varies between 90 degrees (perpendicular vectors) and 0 degrees (parallel vectors) for a given pair of vectors.

As the angle approaches 90 degrees, the cosine of the angle approaches 0, and the dot product of the vectors approaches 0.

As the angle approaches 0 degrees, the cosine of the angle approaches 1.0 and the dot product approaches a value that is the product of the magnitudes of the two vectors.

For a given pair of vectors, the dot product can be thought of as a measure of the extent to which they are parallel. The closer they are to parallel, the greater will be the value of the dot product.

A check on the projection value

We can check our earlier projection value from a different viewpoint now that we know the angle between the vectors.

From the drawing on your graph board, you should see that vector A forms the hypotenuse of a right triangle and the projection of vector A onto vector B forms the base of that triangle. You should know from the earlier module on trigonometry that the length of the base is

base = Amag * cos(angle), or

base = 2 * cos(44.3 degrees), or

base = 1.43 units

which matches the length of the projection that we computed earlier.

3.1.8 Summary for vector dot product

Figure 1 (p. 6) contains some facts worth remembering about the vector dot product.

Facts worth remembering for the dot product.

```
Given two vectors, A and B with their tails at the origin and
their tips at ax, ay, bx, and by respectively

(A dot B) = (ax * bx) + (ay * by)

where
     (A dot B) represents the dot product of the vectors
     named A and B

also

(A dot B) = Amag*Bmag*cosine(angle between A and B)

where
     Amag is the magnitude of vector A
     Bmag is the magnitude of vector B

The projection of A onto B = (A dot B)/(Bmag)

The angle between A and B = arccos((A dot B)/(Amag *Bmag))

For a given pair of vectors, the dot product can be thought
of as a measure of the extent to which they are parallel.
The closer they are to parallel, the greater will be the
value of the dot product.
```

Figure 1: Facts worth remembering for the dot product.

3.2 Cross product

Let's begin our discussion of the vector cross product with some background information.

3.2.1 Background

The *cross product* , sometimes called a *vector product* , is an operation on two vectors in three-dimensional space. The operation results in a vector that is perpendicular to both of the vectors being multiplied.

The name of the operation

The name "cross product" derives from the fact that a special character that looks like an "x" is often used to indicate the nature of the operation.

I doubt that the special character will display properly on your Braille display. In this module, therefore, I will use an actual "x" character instead of the special character that is typically used. For example, I will indicate the cross product between vectors A and B as

AxB

A cross product with a zero result

If either of the vectors being multiplied has a magnitude of 0, the cross product will be zero. Also if the vectors being multiplied are parallel, their cross product will be zero.

The area of a parallelogram

Except for the case of perpendicular vectors, the magnitude of the cross product between two vectors equals the area of a parallelogram with the vectors forming two sides of the parallelogram. For the case of perpendicular vectors, the parallelogram becomes a rectangle and the magnitude of the product is the area of that rectangle.

The direction of the resultant vector

As mentioned earlier, the result of the cross product is a vector that is perpendicular to both of the vectors being multiplied. The resultant vector can satisfy that requirement and point in ether of two directions. The actual direction depends on certain orientation conventions as described by the *right-hand rule* .

3.2.2 The right-hand rule

For a "right-handed" coordinate system, the direction of the resultant vector for AxB can be determined as follows:

Point the forefinger of the right hand in the direction of A and point the second finger in the direction of B. The thumb will then point in the direction of the resultant vector.

The cross product is not commutative

If you think about this, you should realize that the cross product is not commutative. That is to say that AxB is not the same as BxA because the direction of the resultant vector would not be the same.

3.2.3 Create a vector diagram on your graph board

Once again, in order for you to better understand the nature of a vector cross product, I recommend that you create a Cartesian coordinate system on your graph board, and draw the following two vectors.

NOTE: **A vector diagram for your graph board**

```
    Draw the first vector from the origin to a point at
x = 1
y = 1.73
Label this vector A.

Draw a second vector from the origin to a point at
x = 2.9
y = 0.78
Label this vector B.
```

3.2.4 The cross product

The cross product, **AxB is defined as**

AxB = Amag*Bmag*sin(angle)

where

- Amag is the magnitude of the vector A

- Bmag is the magnitude of the vector B
- angle is the angle between the two vectors, which must be less than or equal to180 degrees

3.2.5 The area of the parallelogram

Use the vectors that you have drawn on your graph board to construct a parallelogram and see if you can estimate the area of that parallelogram.

Even if you were a sighted student having the parallelogram drawn on high-quality graph paper, it would be something of a chore to manually determine the area of the parallelogram.

Let's work through some numbers

Let's use the cross product to determine the area of the parallelogram.

Given the definition (p. 7) of the cross product, we see that there are three values that we need:

- Amag
- Bmag
- angle

Same vectors as before

If we were starting out with two new vectors, we could compute the magnitude of each vector using the Pythagorean theorem. We could also determine the angle by computing the vector dot product that I explained earlier in this module.

As you may have noticed, these are the same two vectors that we used earlier, and we computed those three values earlier. Going back and recovering those three values, we have

- Amag = 2.0
- Bmag = 3.0
- angle = 45 degrees (at least that is what I intended for it to be)

The area of the parallelogram

Using the earlier definition (p. 7) and the nomenclature for the Google calculator,

AxB = Amag*Bmag*sin(angle), or

AxB = 2.0*3.0*sin(45 degrees), or

AxB = 4.24 square units

3.2.6 The direction of the resultant vector

If you place the end of your thumb at the origin of your Cartesian coordinate system, you should be able, with reasonable comfort, to point your forefinger in the direction of A and your second finger in the direction of B.

According to the right-hand rule (p. 7) , this means that the direction of the resultant vector is the direction that your thumb is pointing, or straight down into the graph board.

3.2.7 Perpendicular or parallel vectors

Now consider what happens as the angle varies between 90 degrees (perpendicular vectors) and 0 degrees (parallel vectors) for a given pair of vectors.

As the angle approaches 0 degrees, the sine of the angle approaches 0, and the cross product of the vectors approaches 0.

As the angle approaches 90 degrees, the sine of the angle approaches 1.0 and the cross product approaches a value that is the product of the magnitudes of the two vectors.

Thus, for a given pair of vectors, the cross product can be thought of as a measure of the extent to which they are perpendicular to one another. The closer they are to perpendicular, the greater will be the value of the cross product.

Hence, the dot product is a measure of the extent to which two vectors are parallel to one another, while the cross product is a measure of the extent to which two vectors are perpendicular to one another.

3.2.8 Summary for vector cross product

Figure 2 (p. 9) contains some facts worth remembering about the vector cross product.

Facts worth remembering for the cross product.

```
Given two vectors, A and B with their tails at the origin

AxB = Amag*Bmag*sin(angle between A and B)

where
    AxB represents the cross product of the vectors
      named A and B
    Amag is the magnitude of vector A
    Bmag is the magnitude of vector B

The cross product is a vector operation. The resultant vector
is perpendicular to the two vectors being multiplied. The
direction of the resultant vector can be determined by the
right-hand rule.

The magnitude of the product equals the area of a
parallelogram with the vectors forming two sides of the
parallelogram.

The cross product is not commutative. In other words, AxB
is not equal to BxA.

For a given pair of vectors, the cross product can be
thought of as a measure of the extent to which they are
perpendicular to one another. The closer they are to
perpendicular, the greater will be the value of the
cross product.
```

Figure 2: Facts worth remembering for the cross product.

4 Repeat the computations

I encourage you to repeat the computations that I have presented in this lesson to confirm that you get the same results. Then do similar computations for pairs of different vectors. For example, swap the positions of vectors A and B and see what this does to the direction of the resultant vector for a cross product.

5 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

6 Miscellaneous

This section contains a variety of miscellaneous information.

NOTE: **Housekeeping material**

- Module name: Vector Multiplication for Blind Students
- File: Phy1310.htm
- Keywords:
  - physics
  - accessible
  - accessibility
  - blind
  - graph board
  - protractor
  - screen reader
  - refreshable Braille display
  - JavaScript
  - trigonometry
  - dot product
  - inner product
  - scalar product
  - cross product
  - vector product
  - projection

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I also want you to know that I receive no financial compensation from the Connexions website even if you purchase the PDF version of the module.

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