Game Theory: Homework*

UniqU, LLC

Based on *Applied Finite Mathematics: Chapter 10*† by

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Abstract

This chapter covers principles of game theory. After completing this chapter students should be able to: solve strictly determined games and solve games involving mixed strategies.

1 STRICTLY DETERMINED GAMES

Exercise 1 ***(Solution on p. 9.)***

Determine whether the games are strictly determined. If the games are strictly determined, find the optimal strategies for each player and the value of the game.

a. $\begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$

b. $\begin{bmatrix} 6 & 3 \\ 2 & 1 \end{bmatrix}$

c. $\begin{bmatrix} -1 & -3 & 2 \\ 0 & 3 & -1 \\ 1 & -2 & 4 \end{bmatrix}$

d. $\begin{bmatrix} 2 & 0 & -4 \\ 3 & 4 & 2 \\ 0 & -2 & -3 \end{bmatrix}$

e. $\begin{bmatrix} 0 & 2 \\ -1 & -1 \\ -1 & 1 \\ 3 & 2 \end{bmatrix}$

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f. $\begin{bmatrix} 5 & -3 & 2 \\ 3 & -1 & 4 \end{bmatrix}$

Exercise 2

Two players play a game which involves holding out one or two fingers simultaneously. If the sum of the fingers is more than 2, Player II pays Player I the sum of the fingers; otherwise, Player I pays Player II the sum of the fingers.

a. Write a payoff matrix for Player I.
b. Find the optimal strategies for each player and the value of the game.

Exercise 3 ***(Solution on p. 9.)***

A mayor of a large city is thinking of running for re-election, but does not know who his opponent is going to be. It is now time for him to take a stand for or against abortion. If he comes out against abortion rights and his opponent is for abortion, he will increase his chances of winning by 10%. But if he is against abortion and so is his opponent, he gains only 5%. On the other hand, if he is for abortion and his opponent against, he decreases his chance by 8%, and if he is for abortion and so is his opponent, he decreases his chance by 12%.

a. Write a payoff matrix for the mayor.
b. Find the optimal strategies for the mayor and his opponent.

Exercise 4

A man accused of a crime is not sure whether anybody saw him do it. He needs to make a choice of pleading innocent or pleading guilty to a lesser charge. If he pleads innocent and nobody comes forth, he goes free. However, if a witness comes forth, the man will be sentenced to 10 years in prison. On the other hand, if he pleads guilty to a lesser charge and nobody comes forth, he gets a sentence of one year and if a witness comes forth, he gets a sentence of 3 years.

a. Write a payoff matrix for the accused.
b. If you were his attorney, what strategy would you advise?

2 NON-STRICTLY DETERMINED GAMES

Exercise 5 ***(Solution on p. 9.)***

Determine the optimal strategies for both the row player and the column player, and find the value of the game.

a. $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & -1 \\ -4 & 0 \end{bmatrix}$

c. $\begin{bmatrix} 3 & -2 \\ 2 & 4 \end{bmatrix}$

d. $\begin{bmatrix} -3 & 2 \\ 1 & -4 \end{bmatrix}$

Exercise 6

Find the expected payoff for the given game matrix G if the row player plays strategy R, and column player plays strategy C.

a. $G = \begin{bmatrix} -3 & 2 \\ 1 & -4 \end{bmatrix}$ $R = \begin{bmatrix} 2/3 & 1/3 \end{bmatrix}$ $C = \begin{bmatrix} 1/4 \\ 3/4 \end{bmatrix}$

b. $G = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$ $R = \begin{bmatrix} 1/3 & 2/3 \end{bmatrix}$ $C = \begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}$

Exercise 7 ***(Solution on p. 9.)***

Two players play a game which involves holding out one or two fingers simultaneously. If the sum of the fingers is even, Player II pays Player I the sum of the fingers. If the sum of the fingers is odd, Player I pays Player II the sum of the fingers.

a. Write a payoff matrix for Player I.
b. Find the optimal strategies for both the row player and the column player, and the value of the game.

Exercise 8

In December 1995, President Clinton ordered the first of 20,000 U. S. troops to be sent into Bosnia-Herzegovina as a peace keeping force. Unfortunately, the heavy fog made visibility very poor at the Tuzla airfield, and at the same time increased the threat of sniper attacks from the Serbian forces. U. S. Air Force Col. Neal Patton, and Lt. Col. Sid Kooyman, the advance specialists, had two choices: either to send in the troops by air with the difficulties already described or by road thus exposing the troops to ambush by the Serbian forces. The Serbian army, with its limited resources, had a choice of deploying its forces near the airport or along the road route.

If the U. S. lands its troops on the airfield in the fog while the Serbs are concentrating on the road route, the payoff for U. S. is 20 points. But if the U. S. lands its troops on the airfield, and Serbians are there hiding in the fog, U. S. wins only 5 points. On the other hand, if U. S. transports its troops by road and avoids Serbs its payoff is 35 points, but if U. S. meets Serb resistance on the road route, it loses 50 points.

a. Write a payoff matrix for the game.
b. If you were Air Force Col. Neal Patton's advisor, what advice would you give him?

3 REDUCTION BY DOMINANCE

Reduce the payoff matrix by dominance. Find the optimal strategy for each player and the value of the game.

Exercise 9 ***(Solution on p. 10.)***

$$\begin{bmatrix} 2 & -1 \\ 0 & 3 \\ -2 & 0 \end{bmatrix} \tag{1}$$

Exercise 10

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 5 & 4 \end{bmatrix} \tag{2}$$

Exercise 11 ***(Solution on p. 10.)***

$$\begin{bmatrix} 1 & 3 & -2 \\ -2 & 9 & 4 \\ -5 & 0 & 1 \end{bmatrix} \tag{3}$$

Exercise 12

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 4 \\ 3 & 1 & 3 \end{bmatrix} \tag{4}$$

Exercise 13 ***(Solution on p. 10.)***

$$\begin{bmatrix} 2 & 0 & -4 & 8 \\ 0 & -6 & -6 & 2 \\ 2 & -2 & 4 & 6 \\ -2 & -4 & -8 & 0 \end{bmatrix} \tag{5}$$

Exercise 14

$$\begin{bmatrix} -1 & 3 & 2 & 4 \\ 1 & 2 & 0 & 5 \end{bmatrix} \tag{6}$$

Exercise 15 ***(Solution on p. 10.)***

$$\begin{bmatrix} -5 & -1 & -1 & 3 \\ -10 & 1 & 2 & -8 \\ 4 & 0 & 1 & 5 \\ 3 & -8 & 0 & 5 \end{bmatrix} \tag{7}$$

Exercise 16

$$\begin{bmatrix} 1 & -3 & -4 & 1 \\ 1 & -4 & -1 & 3 \\ 1 & 1 & -1 & 1 \\ 0 & -1 & 1 & 1 \end{bmatrix} \tag{8}$$

4 CHAPTER REVIEW

Exercise 17 ***(Solution on p. 10.)***

Determine whether the games are strictly determined. If the games are strictly determined, find the optimal strategies for each player and the value of the game.

a. $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$

b. $\begin{bmatrix} 0 & 3 & -1 \\ 1 & 3 & -2 \\ -1 & 2 & -5 \end{bmatrix}$

c. $\begin{bmatrix} 3 & 2 & -1 \\ 5 & 3 & 4 \end{bmatrix}$

d. $\begin{bmatrix} 4 & 2 \\ -1 & 3 \\ 4 & 3 \\ 1 & -3 \end{bmatrix}$

Exercise 18 ***(Solution on p. 10.)***

Two players play a game which involves holding out a nickel or a dime simultaneously. If the sum of the coins is more than 10 cents, Player I gets both the coins; otherwise, Player II gets both the coins.

a. Write a payoff matrix for Player I.
b. Find the optimal strategies for each player and the value of the game.

Exercise 19 ***(Solution on p. 11.)***

Lacy's department store is thinking of having a major sale in the month of February, but does not know if its competitor store Hordstrom's is also planning one. If Lacy's has a sale and Hordstrom's does not, Lacy's sales go up by 30%, but if both stores have a sale simultaneously, Lacy's sales go up by only 5%. On the other hand, if Lacy's does not have a sale and Hordstrom's does, Lacy's loses 5% of its sales to Hordstrom's, and if neither of the stores has a sale, Lacy's experiences no gain in sales.

a. Write a payoff matrix for Lacy's.
b. Find the optimal strategies for both stores.

Exercise 20 *(Solution on p. 11.)*

Mr. Halsey has a choice of three investments: Investment A, Investment B, and Investment C. If the economy booms, then Investment A yields 14% return, Investment B returns 8%, and Investment C 11%. If the economy grows moderately, then Investment A yields 12% return, Investment B returns 11%, and Investment C 11%. If the economy experiences a recession, then Investment A yields a 6% return, Investment B returns 9%, and Investment C 10%.

a. Write a payoff matrix for Mr. Halsey.
b. What would you advise him?

Exercise 21 *(Solution on p. 11.)*

Mr. Thaggert is trying to decide whether to invest in stocks or in CD's(Certificate of deposit). If he invests in stocks and the interest rates go up, his stock investments go down by 2%, but he gains 1% in his CD's. On the other hand if the interest rates go down, he gains 3% in his stock investments, but he loses 1% in his CD's.

a. Write a payoff matrix for Mr. Thaggert.
b. If you were his investment advisor, what strategy would you advise?

Exercise 22 *(Solution on p. 11.)*

Determine the optimal strategies for both the row player and the column player, and find the value of the game.

a. $\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$
b. $\begin{bmatrix} -2 & 2 \\ 5 & 0 \end{bmatrix}$
c. $\begin{bmatrix} 3 & 5 \\ 4 & -1 \end{bmatrix}$
d. $\begin{bmatrix} -2 & 5 \\ 4 & -3 \end{bmatrix}$

Exercise 23 *(Solution on p. 11.)*

Find the expected payoff for the given game matrix G if the row player plays strategy R, and the column player plays strategy C.

a. $G = \begin{bmatrix} 3 & 5 \\ 4 & -1 \end{bmatrix}$ $R = \begin{bmatrix} 1/2 & 1/2 \end{bmatrix}$ $C = \begin{bmatrix} 1/4 \\ 3/4 \end{bmatrix}$
b. $G = \begin{bmatrix} -2 & 5 \\ 4 & -3 \end{bmatrix}$ $R = \begin{bmatrix} 2/3 & 1/3 \end{bmatrix}$ $C = \begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix}$

Exercise 24 *(Solution on p. 11.)*

A group of thieves are planning to burglarize either Warehouse A or Warehouse B. The owner of the warehouses has the manpower to secure only one of them. If Warehouse A is burglarized the owner will lose \$20,000, and if Warehouse B is burglarized the owner will lose \$30,000. There is a 40% chance that the thieves will burglarize Warehouse A and 60% chance they will burglarize

Warehouse B. There is a 30% chance that the owner will secure Warehouse A and 70% chance he will secure Warehouse B. What is the owner's expected loss?

Exercise 25 ***(Solution on p. 11.)***

Two players play a game which involves holding out a nickel or a dime. If the sum of the coins is odd, Player I gets both the coins, and if the sum of the coins is even, Player II gets both the coins. Determine the optimal strategies for both the row player and the column player, and find the expected payoff.

Exercise 26 ***(Solution on p. 12.)***

A football quarterback has to choose between a pass play or a run play depending on how the defending team is going to react. If he chooses a pass play and the defending team is expecting a pass, he expects to gain 4 yards, but if the defending team is expecting a run, he gains 20 yards. On the other hand, if he calls a run play and the defending team expects a pass, he gains 7 yards, and if he calls a run play and the defending team expects a run, he loses 2 yards. If you were the quarterback, what would your strategy be?

Exercise 27 ***(Solution on p. 12.)***

The Watermans go fishing every weekend either at Eel River or at Snake River. Unfortunately, so do the Nelsons. If both families show up at Eel River, the Watermans can hope to catch only 3 fish, but if the Watermans fish at Eel River and the Nelsons at Snake River, the Watermans can catch as many as 12 fish. On the other hand, if both families fish at Snake river, the Watermans can catch about 5 fish, and if Watermans fish at Snake river while the Nelsons fish at Eel river, the Watermans can catch up to 15 fish. Determine a mixed strategy for the Watermans, and the expected payoff.

Exercise 28 ***(Solution on p. 12.)***

Terry knows there is a quiz tomorrow, but does not remember whether it is in his math class or in his biology class. He has time to study for only one subject. If he studies math and there is a quiz in it, he gains 10 points and even if there is no quiz he gains two points for acquiring the extra knowledge which he will apply towards the final exam. If he studies biology and there is a quiz in it, he gains ten points but there is no gain if there is no quiz. Determine a mixed strategy for Terry, and the expected payoff.

Exercise 29 ***(Solution on p. 12.)***

Reduce the payoff matrix by dominance. Find the optimal strategy for each player and the value of the game.

a. $\begin{bmatrix} -3 & 1 & 2 \\ -3 & 5 & 3 \\ 2 & 4 & -1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 4 \\ 2 & 3 & 4 \\ 1 & 2 & 2 \end{bmatrix}$

c. $\begin{bmatrix} 4 & 3 & 9 & 7 \\ -7 & -5 & -3 & 5 \\ -1 & 4 & 5 & 8 \\ -3 & -5 & 1 & -1 \end{bmatrix}$

d. $\begin{bmatrix} 2 & 3 & 1 & 5 \\ -2 & 2 & 1 & 3 \end{bmatrix}$

e. $\begin{bmatrix} 0 & 3 & 2 & 1 \\ 0 & 2 & 1 & -7 \\ -4 & -9 & 5 & 4 \\ 4 & -7 & 6 & 6 \end{bmatrix}$

f. $\begin{bmatrix} 1 & 0 & 2 & 2 \\ 2 & 2 & 0 & 2 \\ 2 & -3 & 0 & 4 \\ 2 & -2 & -3 & 2 \end{bmatrix}$

Solutions to Exercises in this Module

Solution to Exercise (p. 1)

a. The game is strictly determined. Optimal strategy for the row player is to always play row 1 and never row 2. In other words, his strategy is $\begin{bmatrix} 1 & 0 \end{bmatrix}$. The optimal strategy for the column player is to always to play column 1 and never play column 2. We write it as $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$. When both players play their optimal strategy, the value of the game is 1.
b.
c. The game has no saddle point, therefore, it is not strictly determined.
d.
e. The game is strictly determined. The optimal strategy for the row player is to always play row 4, and never play any other row. We write his strategy as $\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$. The column player's strategy is $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$. The value of the game is 2.
f.

Solution to Exercise (p. 2)

a. $\begin{bmatrix} .05 & .10 \\ -.08 & -.12 \end{bmatrix}$
b. The optimal strategy for the mayor is $\begin{bmatrix} 1 & 0 \end{bmatrix}$ and for his opponent is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$. In other words, both candidates should oppose abortion rights.

Solution to Exercise (p. 2)

a. The optimal strategy for the row player is $\begin{bmatrix} 1/2 & 1/2 \end{bmatrix}$. The optimal strategy for the column player is $\begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$. The value of the game is 0.
b.
c. Optimal strategy for the row player is $\begin{bmatrix} 2/7 & 5/7 \end{bmatrix}$. The optimal strategy for the column player is $\begin{bmatrix} 6/7 \\ 1/7 \end{bmatrix}$. The value of the game is 16/7.
d.

Solution to Exercise (p. 3)

a. $\begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix}$
b. Optimal strategy for the row player is $\begin{bmatrix} 7/12 & 5/12 \end{bmatrix}$. The optimal strategy for the column player is $\begin{bmatrix} 7/12 \\ 5/12 \end{bmatrix}$. The value of the game is $-1/12$.

Solution to Exercise (p. 3)

$$\begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}, \; R = \begin{bmatrix} 1/2 & 1/2 & 0 \end{bmatrix}, \; C = \begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}, \; \text{The value} = 1 \tag{9}$$

Solution to Exercise (p. 4)

$$\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}, \; R = \begin{bmatrix} 2/3 & 1/3 & 0 \end{bmatrix}, \; C = \begin{bmatrix} 2/3 \\ 0 \\ 1/3 \end{bmatrix}, \; \text{The value} = 0 \tag{10}$$

Solution to Exercise (p. 4)

$$\begin{bmatrix} 0 & -4 \\ -2 & 4 \end{bmatrix}, \; R = \begin{bmatrix} .6 & 0 & .4 & 0 \end{bmatrix}, \; C = \begin{bmatrix} 0 \\ .8 \\ .2 \\ 0 \end{bmatrix}, \; \text{The value} = -0.8 \tag{11}$$

Solution to Exercise (p. 4)

$$\begin{bmatrix} 2 & -8 \\ 1 & 5 \end{bmatrix}, \; R = \begin{bmatrix} 0 & 2/7 & 5/7 & 0 \end{bmatrix}, \; C = \begin{bmatrix} 0 \\ 0 \\ 13/14 \\ 1/14 \end{bmatrix}, \; \text{The value} = 9/7 \tag{12}$$

Solution to Exercise (p. 5)

a. $R = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, value $= 3$

b. $R = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $v = -1$

c. $R = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, value $= 3$

d. $R = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 \\ 1 \end{bmatrix} v = 3$

Solution to Exercise (p. 5)

a. $\begin{bmatrix} -5 & 10 \\ 5 & 10 \end{bmatrix}$

b. $R = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, value $= 5$ cents

Solution to Exercise (p. 5)

a. $\begin{bmatrix} 5 & 30 \\ -5 & 0 \end{bmatrix}$

b. $R = \begin{bmatrix} 1 & 0 \end{bmatrix} C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, value $= 5\%$

Solution to Exercise (p. 6)

a. $\begin{bmatrix} .14 & .08 & .11 \\ .12 & .11 & .11 \\ .06 & .09 & .10 \end{bmatrix}$

b. $\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, value $= .11$

Solution to Exercise (p. 6)

a. $\begin{bmatrix} -.02 & .03 \\ .01 & -.01 \end{bmatrix}$

b. stocks $= 2/7$, CD's $= 5/7$

Solution to Exercise (p. 6)

a. $\begin{bmatrix} 1/2 & 1/2 \end{bmatrix}, \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$, value $= 0$

b. $\begin{bmatrix} 5/9 & 4/9 \end{bmatrix}, \begin{bmatrix} 2/9 \\ 7/9 \end{bmatrix}$, value $= 10/9$

c. $\begin{bmatrix} 5/7 & 2/7 \end{bmatrix}, \begin{bmatrix} 6/7 \\ 1/7 \end{bmatrix}$, value $= 23/7$

d. $\begin{bmatrix} 1/2 & 1/2 \end{bmatrix}, \begin{bmatrix} 4/7 \\ 3/7 \end{bmatrix}$, value $= 1$

Solution to Exercise (p. 6)

a. 19/8
b. 14/9

Solution to Exercise (p. 6)
$11,000
Solution to Exercise (p. 7)

$$\begin{bmatrix} 1/2 & 1/2 \end{bmatrix}, \begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}, v = 0 \tag{13}$$

Solution to Exercise (p. 7)

$$\text{Pass} = 9/25,\ \text{Run} = 16/25 \tag{14}$$

Solution to Exercise (p. 7)

$$\begin{bmatrix} 10/19 & 9/19 \end{bmatrix},\ \text{payoff} = 9.58 \quad \text{fish} \tag{15}$$

Solution to Exercise (p. 7)

$$\begin{bmatrix} 1 & 0 \end{bmatrix},\ \text{payoff} = 2 \quad \text{points} \tag{16}$$

Solution to Exercise (p. 7)

a. $\begin{bmatrix} -3 & 3 \\ 2 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1/3 & 2/3 \end{bmatrix}, \begin{bmatrix} 4/9 \\ 0 \\ 5/9 \end{bmatrix}$, value $= 1/3$

b. $\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}, \begin{bmatrix} 0 & 1/4 & 3/4 & 0 \end{bmatrix}, \begin{bmatrix} 1/2 \\ 1/2 \\ 0 \end{bmatrix}$, value $= 5/2$

c. $\begin{bmatrix} 4 & 3 \\ --1 & 4 \end{bmatrix}, \begin{bmatrix} 5/6 & 0 & 1/6 & 0 \end{bmatrix}, \begin{bmatrix} 1/6 \\ 5/6 \\ 0 \\ 0 \end{bmatrix}$, value $= 19/6$

d. $\begin{bmatrix} 2 & 1 \\ -2 & 1 \end{bmatrix}, \begin{bmatrix} 3/4 & 1/4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, value $= 1$

e. $\begin{bmatrix} 0 & 3 \\ 4 & -7 \end{bmatrix}, \begin{bmatrix} 11/14 & 0 & 0 & 3/14 \end{bmatrix}, \begin{bmatrix} 10/14 \\ 4/14 \\ 0 \\ 0 \end{bmatrix}$, value $= 6/7$

f. $\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1/2 \\ 1/2 \\ 0 \end{bmatrix}$, value $= 1$