Electric Circuits: Resistance (Grade 11)*

Free High School Science Texts Project

1 Resistance

In Grade 10, you learnt about resistors and were introduced to circuits where resistors were connected in series and circuits where resistors were connected in parallel. In a series circuit there is one path for the current to flow through. In a parallel circuit there are multiple paths for the current to flow through.

Figure 1

1.1 Equivalent resistance

When there is more than one resistor in a circuit, we are usually able to calculate the total combined resitance of all the resistors. The resistance of the single resistor is known as *equivalent resistance*.

1.1.1 Equivalent Series Resistance

Consider a circuit consisting of three resistors and a single cell connected in series.

Figure 2

The first principle to understand about series circuits is that the amount of current is the same through any component in the circuit. This is because there is only one path for electrons to flow in a series circuit.

*Version 1.1: Jul 27, 2011 3:54 am -0500
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From the way that the battery is connected, we can tell which direction the current will flow. We know that current flows from positive to negative, by convention. Current in this circuit will flow in a clockwise direction, from point A to B to C to D and back to A.

So, how do we use this knowledge to calculate the total resistance in the circuit?

We know that in a series circuit the current has to be the same in all components. So we can write:

$$I = I_1 = I_2 = I_3 \tag{1}$$

We also know that total voltage of the circuit has to be equal to the sum of the voltages over all three resistors. So we can write:

$$V = V_1 + V_2 + V_3 \tag{2}$$

Finally, we know that Ohm's Law has to apply for each resistor individually, which gives us:

$$\begin{aligned} V_1 &= I_1 \cdot R_1 \\ V_2 &= I_2 \cdot R_2 \\ V_3 &= I_3 \cdot R_3 \end{aligned} \tag{3}$$

Therefore:

$$V = I_1 \cdot R_1 + I_2 \cdot R_2 + I_3 \cdot R_3 \tag{4}$$

However, because

$$I = I_1 = I_2 = I_3 \tag{5}$$

, we can further simplify this to:

$$\begin{aligned} V &= I \cdot R_1 + I \cdot R_2 + I \cdot R_3 \\ &= I\left(R_1 + R_2 + R_3\right) \end{aligned} \tag{6}$$

Further, we can write an Ohm's Law relation for the entire circuit:

$$V = I \cdot R \tag{7}$$

Therefore:

$$\begin{aligned} V &= I\left(R_1 + R_2 + R_3\right) \\ I \cdot R &= I\left(R_1 + R_2 + R_3\right) \\ \therefore\ R &= R_1 + R_2 + R_3 \end{aligned} \tag{8}$$

Definition 1: Equivalent resistance in a series circuit, R_s

For n resistors in series the equivalent resistance is:

$$R_s = R_1 + R_2 + R_3 + \cdots + R_n \tag{9}$$

You can use the following simulation to test this result and all other results in this chapter.

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Figure 3

run demo[1]

Let us apply this to the following circuit.

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Figure 4

The resistors are in series, therefore:

$$\begin{aligned} R_s &= R_1 + R_2 + R_3 \\ &= 3\,\Omega + 10\,\Omega + 5\,\Omega \\ &= 18\,\Omega \end{aligned} \tag{10}$$

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Figure 5

Exercise 1: Equivalent series resistance I ***(Solution on p. 8.)***
Two 10 kΩ resistors are connected in series. Calculate the equivalent resistance.

Exercise 2: Equivalent series resistance II ***(Solution on p. 8.)***
Two resistors are connected in series. The equivalent resistance is 100 Ω. If one resistor is 10 Ω, calculate the value of the second resistor.

1.1.2 Equivalent parallel resistance

Consider a circuit consisting of a single cell and three resistors that are connected in parallel.

[1]http://phet.colorado.edu/sims/circuit-construction-kit/circuit-construction-kit-dc_en.jnlp

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Figure 6

The first principle to understand about parallel circuits is that the voltage is equal across all components in the circuit. This is because there are only two sets of electrically common points in a parallel circuit, and voltage measured between sets of common points must always be the same at any given time. So, for the circuit shown, the following is true:

$$V = V_1 = V_2 = V_3 \tag{11}$$

The second principle for a parallel circuit is that all the currents through each resistor must add up to the total current in the circuit.

$$I = I_1 + I_2 + I_3 \tag{12}$$

Also, from applying Ohm's Law to the entire circuit, we can write:

$$V = \frac{I}{R_p} \tag{13}$$

where R_p is the equivalent resistance in this parallel arrangement.

We are now ready to apply Ohm's Law to each resistor, to get:

$$\begin{aligned} V_1 &= R_1 \cdot I_1 \\ V_2 &= R_2 \cdot I_2 \\ V_3 &= R_3 \cdot I_3 \end{aligned} \tag{14}$$

This can be also written as:

$$\begin{aligned} I_1 &= \tfrac{V_1}{R_1} \\ I_2 &= \tfrac{V_2}{R_2} \\ I_3 &= \tfrac{V_3}{R_3} \end{aligned} \tag{15}$$

Now we have:

$$\begin{aligned} I &= I_1 + I_2 + I_3 \\ \tfrac{V}{R_p} &= \tfrac{V_1}{R_1} + \tfrac{V_2}{R_2} + \tfrac{V_3}{R_3} \\ &= \tfrac{V}{R_1} + \tfrac{V}{R_2} + \tfrac{V}{R_3} \\ \text{because} &\quad V = V_1 = V_2 = V_3 \\ &= V\left(\tfrac{1}{R_1} + \tfrac{1}{R_2} + \tfrac{1}{R_3}\right) \\ \therefore \quad \tfrac{1}{R_p} &= \left(\tfrac{1}{R_1} + \tfrac{1}{R_2} + \tfrac{1}{R_3}\right) \end{aligned} \tag{16}$$

Definition 2: Equivalent resistance in a parallel circuit, R_p

For n resistors in parallel, the equivalent resistance is:

$$\frac{1}{R_p} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}\right) \tag{17}$$

Let us apply this formula to the following circuit.

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Figure 7

What is the total resistance in the circuit?

$$\begin{array}{rcl} \frac{1}{R_p} & = & \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) \\ & = & \left(\frac{1}{10\,\Omega}+\frac{1}{2\,\Omega}+\frac{1}{1\,\Omega}\right) \\ & = & \left(\frac{1+5+10}{10}\right) \\ & = & \left(\frac{16}{10}\right) \\ \therefore \quad R_p & = & 0,625\,\Omega \end{array} \quad (18)$$

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Figure 8

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Figure 9

1.2 Use of Ohm's Law in series and parallel Circuits

Exercise 3: Ohm's Law ***(Solution on p. 8.)***

Calculate the current (I) in this circuit if the resistors are both ohmic in nature.

Exercise 4: Ohm's Law I ***(Solution on p. 9.)***

Calculate the current (I) in this circuit if the resistors are both ohmic in nature.

Exercise 5: Ohm's Law II ***(Solution on p. 9.)***

Two ohmic resistors (R_1 and R_2) are connected in series with a cell. Find the resistance of R_2, given that the current flowing through R_1 and R_2 is 0,25 A and that the voltage across the cell is 1,5 V. R_1=1 Ω.

1.3 Batteries and internal resistance

Real batteries are made from materials which have resistance. This means that real batteries are not just sources of potential difference (voltage), but they also possess internal resistance. If the total voltage source is referred to as the emf, $\mathcal{E}$, then a real battery can be represented as an emf connected in series with a resistor r. The internal resistance of the battery is represented by the symbol r.

Figure 10

Definition 3: Load
The external resistance in the circuit is referred to as the load.

Suppose that the battery with emf $\mathcal{E}$ and internal resistance r supplies a current I through an external load resistor R. Then the voltage drop across the load resistor is that supplied by the battery:

$$V = I \cdot R \tag{19}$$

Similarly, from Ohm's Law, the voltage drop across the internal resistance is:

$$V_r = I \cdot r \tag{20}$$

The voltage V of the battery is related to its emf $\mathcal{E}$ and internal resistance r by:

$$\begin{aligned} \mathcal{E} &= V + Ir; or \\ V &= \mathcal{E} - Ir \end{aligned} \tag{21}$$

The emf of a battery is essentially constant because it only depends on the chemical reaction (that converts chemical energy into electrical energy) going on inside the battery. Therefore, we can see that the voltage across the terminals of the battery is dependent on the current drawn by the load. The higher the current, the lower the voltage across the terminals, because the emf is constant. By the same reasoning, the voltage only equals the emf when the current is very small.

The maximum current that can be drawn from a battery is limited by a critical value I_c. At a current of I_c, V=0 V. Then, the equation becomes:

$$\begin{aligned} 0 &= \mathcal{E} - I_c r \\ I_c r &= \mathcal{E} \\ I_c &= \frac{\mathcal{E}}{r} \end{aligned} \tag{22}$$

The maximum current that can be drawn from a battery is less than $\frac{\mathcal{E}}{r}$.

Exercise 6: Internal resistance ***(Solution on p. 10.)***
What is the internal resistance of a battery if its emf is 12 V and the voltage drop across its terminals is 10 V when a current of 4 A flows in the circuit when it is connected across a load?

1.3.1 Resistance

1. Calculate the equivalent resistance of:
   a. three 2 Ω resistors in series;
   b. two 4 Ω resistors in parallel;
   c. a 4 Ω resistor in series with a 8 Ω resistor;
   d. a 6 Ω resistor in series with two resistors (4 Ω and 2 Ω) in parallel.
2. Calculate the total current in this circuit if both resistors are ohmic.

Figure 11

3. Two ohmic resistors are connected in series. The resistance of the one resistor is 4 Ω . What is the resistance of the other resistor if a current of 0,5 A flows through the resistors when they are connected to a voltage supply of 6 V.
4. Describe what is meant by the *internal resistance* of a real battery.
5. Explain why there is a difference between the emf and terminal voltage of a battery if the load (external resistance in the circuit) is comparable in size to the battery's internal resistance
6. What is the internal resistance of a battery if its emf is 6 V and the voltage drop across its terminals is 5,8 V when a current of 0,5 A flows in the circuit when it is connected across a load?

Solutions to Exercises in this Module

Solution to Exercise (p. 3)

Step 1. Since the resistors are in series we can use:

$$R_s = R_1 + R_2 \tag{23}$$

Step 2.

$$\begin{array}{rcl} R_s & = & R_1 + R_2 \\ & = & 10\,k\,\Omega + 10\,k\,\Omega \\ & = & 20\,k\,\Omega \end{array} \tag{24}$$

Step 3. The equivalent resistance of two 10 kΩ resistors connected in series is 20 kΩ.

Solution to Exercise (p. 3)

Step 1. Since the resistors are in series we can use:

$$R_s = R_1 + R_2 \tag{25}$$

We are given the value of R_s and R_1.

Step 2.

$$\begin{array}{rrcl} & R_s & = & R_1 + R_2 \\ \therefore & R_2 & = & R_s - R_1 \\ & & = & 100\,\Omega - 10\,\Omega \\ & & = & 90\,\Omega \end{array} \tag{26}$$

Step 3. The second resistor has a resistance of 90 Ω.

Solution to Exercise (p. 5)

Figure 12

Step 1. We are required to calculate the current flowing in the circuit.
Step 2. Since the resistors are Ohmic in nature, we can use Ohm's Law. There are however two resistors in the circuit and we need to find the total resistance.
Step 3. Since the resistors are connected in series, the total resistance R is:

$$R = R_1 + R_2 \tag{27}$$

Therefore,

$$R = 2 + 4 = 6 \ \Omega \tag{28}$$

Step 4.

$$\begin{aligned} V &= R \cdot I \\ \therefore \quad I &= \frac{V}{R} \\ &= \frac{12}{6} \\ &= 2\ A \end{aligned} \tag{29}$$

Step 5. A 2 A current is flowing in the circuit.

Solution to Exercise (p. 5)

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Figure 13

Step 1. We are required to calculate the current flowing in the circuit.
Step 2. Since the resistors are Ohmic in nature, we can use Ohm's Law. There are however two resistors in the circuit and we need to find the total resistance.
Step 3. Since the resistors are connected in parallel, the total resistance R is:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \tag{30}$$

Therefore,

$$\begin{aligned} \frac{1}{R} &= \frac{1}{R_1} + \frac{1}{R_2} \\ &= \frac{1}{2} + \frac{1}{4} \\ &= \frac{2+1}{4} \\ &= \frac{3}{4} \\ Therefore, R &= 1,33\ \Omega \end{aligned} \tag{31}$$

Step 4.

$$\begin{aligned} V &= R \cdot I \\ \therefore \quad I &= \frac{V}{R} \\ &= \frac{12}{\frac{4}{3}} \\ &= 9\ A \end{aligned} \tag{32}$$

Step 5. A 9 A current is flowing in the circuit.

Solution to Exercise (p. 5)

Step 1. Image not finished

Figure 14

Step 2. We can use Ohm's Law to find the total resistance R in the circuit, and then calculate the unknown resistance using:

$$R = R_1 + R_2 \tag{33}$$

because it is in a series circuit.

Step 3.

$$\begin{array}{rcl} V & = & R \cdot I \\ \therefore \quad R & = & \frac{V}{I} \\ & = & \frac{1{,}5}{0{,}25} \\ & = & 6\,\Omega \end{array} \tag{34}$$

Step 4. We know that:

$$R = 6\,\Omega \tag{35}$$

and that

$$R_1 = 1\,\Omega \tag{36}$$

Since

$$R = R_1 + R_2 \tag{37}$$

$$R_2 = R - R_1 \tag{38}$$

Therefore,

$$R_2 = 5\,\Omega \tag{39}$$

Solution to Exercise (p. 6)

Step 1. It is an internal resistance problem. So we use the equation:

$$\mathcal{E} = V + Ir \tag{40}$$

Step 2.

$$\begin{array}{rcl} \mathcal{E} & = & V + Ir \\ 12 & = & 10 + 4\,(r) \\ & = & 0.5 \end{array} \tag{41}$$

Step 3. The internal resistance of the resistor is 0.5 Ω.